

First Exam for the Course
Constraint Technology for Solving Combinatorial Problems
Summer 2004

Magnus Ågren and Justin Pearson

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Cover Sheet

This is a closed book exam, no written or printed material is allowed. This cover sheet should be handed in together with the exam. A mark from 50% to 84% earns a G passing grade, and a mark from 85% to 100% earns a VG passing grade. For each answer, you *must* show how you reached the result if nothing else is stated in the question. Unreadable or unclear answers will obtain 0 points. Each problem should be answered on a separate sheet. Write your name on each sheet. Indicate below which questions you have answered.

Problem no.	Solution provided	Max	Your points
1		10	
2		15	
3		25	
4		20	
5		20	
Total:		90	

Name :

Pers.no. :

– Modelling Instructions –

You may write CSP models in the language of your choice such as Koalog, SICStus Prolog or FaCiLe. You may also write in pseudocode as you have seen during the lectures but please do not use any “magic” constraints that are not available in any of the solvers you have seen. You may use standard mathematical and logic notations, such as $M[i, j]$ (the element in row i and column j of the matrix M), \sum (sum), $\forall i \in S$ (for all i in S), $\exists i \in S$ (there exists i in S), $\&$ (and), \vee (or), \rightarrow (implies), and so on.

The instance data, as well as the decision variables and their domains must be declared, possibly in mathematical notation. Use only individual variables (whose domains are sets), do not use set variables (whose domains are sets of sets).

– Questions –

1. Constraint Satisfaction Problems (10 Points)

Consider the CSP $P = \langle \mathcal{C}; \mathcal{DE} \rangle$ where

- $\mathcal{C} = \{x_1 < x_2, x_1 \neq x_3, x_1 + x_2 \geq x_3\}$, and
- $\mathcal{DE} = \langle x_1 \in \{0, 1, 2\}, x_2 \in \{1, 2\}, x_3 \in \{2, 3, 4\} \rangle$

- What is the solution set of $x_1 \neq x_3$? (**2 Points**)
- What is the solution set of $x_1 + x_2 \geq x_3$? (**2 Points**)
- What is the solution set of P ? (**2 Points**)
- Give a constraint c such that the solution set of $P' = \langle \mathcal{C} \cup c; \mathcal{DE} \rangle$:
 - contains only one element. (**2 Points**)
 - is empty. (**2 Points**)

2. Consistency (15 Points)

- Consider a CSP with the following constraints:

$$x + y \leq 3, y + z \leq 4, x < y, y < z$$

where, initially, $x, y, z \in \{0, \dots, 10\}$. After arc consistency is applied, what will the domains of x , y and z be? (**5 Points**)

$$x \in \{0, 1\}, y \in \{1, 2\}, z \in \{2, 3\}$$

You have to go a few iterations to get to the fixed point.

- What is the width of the CSP from part (a)? (**2 Points**) It is 1.
- How can a solution to the CSP from part (a) be found in a backtrack-free manner? (**2 Points**) Since the width is 1, make the problem arc-consistent then backtrack freeness follows.
- Is the following CSP

$$\langle x - y \leq 4, y - z \leq 5, x - z \leq 10; x \in \{0, \dots, 10\}, y \in \{0, \dots, 10\}, z \in \{0, \dots, 10\} \rangle$$

path consistent? Why or why not? If it is not path consistent, modify the constraints of the CSP so that it becomes path consistent. (**4 Points**)

It is not path consistent. The third constraint needs to be changed to $x - z \leq 9$

(e) Make the following CSP

$$\langle x + y = 2z ; x \in \{1, \dots, 10\}, y \in \{1, \dots, 10\}, z \in \{1, 2, 3\} \rangle$$

hyper-arc consistent by modifying the domains of the variables. **(2 Points)** $x \in \{1, \dots, 5\}$, $y \in \{1, \dots, 5\}$

3. Modelling (25 Points)

- (a) You are faced with a combinatorial problem specified in a natural language (e.g. English or Swedish) and you want to find the solution(s) to the problem. Explain *briefly* how you would proceed when trying to solve the problem using a constraint solver such as Koalog. Explain also what difficulties (at least 2) you could encounter and how you would try to resolve those difficulties. **(10 Points)**
- (b) Consider a boss at a warehouse. Today, she has m tasks that must be performed. At her service she has n employees where each employee e is able to perform a maximum of $tasks[e]$ tasks in one day. Each task t requires exactly $demand[t]$ employees to be completed. Below is an example instance where $m = 8$ and $n = 4$:

demand							
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
1	3	2	4	2	1	1	2

tasks			
Per	Linda	Lisa	Karl
6	4	10	8

In this example, $demand[t_1] = 1$, i.e., task t_1 only needs one employee to be completed, $demand[t_4] = 4$ so task t_4 needs 4 employees to be completed, etc. Also, $tasks[Per] = 6$ means that Per is able to perform a maximum of 6 tasks in one day, $tasks[Lisa] = 10$ means that Lisa is able to perform a maximum of 10 tasks in one day, and so on.

Unfortunately, some employees may not want to work together with some other employees on the same task. If a person is forced to do this anyway, there will be a conflict and the overall wellbeing in the warehouse will decrease. The possible conflicts for our example instance are shown in the $n \times n$ matrix below where $conflict[p_1][p_2] = 1$ means that p_1 does not want to work together with p_2 .

conflict	Per	Linda	Lisa	Karl
Per	0	1	0	0
Linda	1	0	1	1
Lisa	0	0	0	1
Karl	0	0	0	0

So, in our example, if Lisa must work on the same task as Karl there will be a conflict increase of 1 since Lisa does not want to work with Karl. If Per must work with Linda, however, there will be a conflict increase of 2 since Per does not want to work with Linda **and** Linda does not want to work with Per either.

Your task is to come up with a constraint based model for solving this problem. Given the number of employees n , the number of tasks m , the arrays $demand$ and $tasks$, and the matrix $conflict$ it should assign to each task a set of people such that:

- Each task t is assigned exactly the number of people, $demand[t]$, it needs to be completed.
- Each employee e is assigned at most $tasks[e]$ tasks.
- The number of conflicts is minimised.

(15 Points)

4. Search (20 Points)

- (a) Consider the CSP $P = \langle x < y, x \neq z, z < y ; x \in \{0, 1, 2\}, y \in \{0, 1, 2\}, z \in \{0, 1, 2\} \rangle$ and the ordering on the variables $x \prec y \prec z$.
- Draw the *partial look ahead* search tree of P . (5 Points)
 - Draw the *full look ahead* search tree of P . (5 Points)
- (b) Consider the COP defined by the CSP $P = \langle \mathcal{C}; \mathcal{DE} \rangle$ and the objective function $obj : Sol(P) \rightarrow \mathbb{R}$ where:
- $\mathcal{C} = \{x_5 \neq x_4, x_4 = x_3, x_2 < x_3, x_2 \neq x_1\}$, and
 - $\mathcal{DE} = \langle x_1 \in \{1, 3\}, x_2 \in \{2, 3\}, x_3 \in \{3, 5\}, x_4 \in \{3, 5, 6\}, x_5 \in \{4, 5\} \rangle$, and
 - $obj(d_1, \dots, d_5) = \sum_{i=1}^5 d_i$.
- Assume that we want to find a solution to P that *maximises* the function obj by using branch and bound search. Define a heuristic function $h : \mathcal{P}(D_1) \times \dots \times \mathcal{P}(D_5) \rightarrow \mathbb{R}$ for the search, where D_i is the domain of the variable x_i . (2 Points)
 - Use the heuristic function defined above to draw the branch and bound search tree for the COP. Your tree should be drawn with respect to the following instructions:
 - Enumerate the variables in the order $x_1 \prec x_2 \prec x_3 \prec x_4 \prec x_5$.
 - Assign values to the variables starting with their **highest** value first.
 - There should be no propagation, i.e., a constraint such as $x < y$ may only fail when both x and y are assigned. (And of course only if $x \geq y$ under that assignment.)
 - Indicate in the tree
 - where a branch is cut because of a failed constraint.
 - where a branch is cut because of the branch and bound search cannot find an optimal solution in it.
 - where the optimal solution is found.
- (8 Points)

5. AllDifferent Filtering Algorithm (20 Points)

Consider the variables with their respective domains $x_0 \in \{0, 1\}$, $x_1 \in \{1, 2\}$, $x_2 \in \{2, 3\}$, $x_3 \in \{1, 3\}$, $x_4 \in \{0, 3, 4, 5\}$, and $x_5 \in \{5, 6\}$. Your task in this question is to go through one filtering of Régin's algorithm for the constraint $all_different(\{x_0, x_1, x_2, x_3, x_4, x_5\})$.

- Draw the variable value graph for the constraint $all_different(\{x_0, x_1, x_2, x_3, x_4, x_5\})$. (4 Points)
- Draw the graph in (a) again but this time orient the edges (give them arrows) with respect to a maximal matching of your choice. (4 Points)
- Draw the graph in (b) again but this time show only the edges that take part in an alternating path (mark these with a 'p') or an alternating cycle (mark these with a 'c'). (6 Points)
- Draw the graph in (a) again but this time remove the edges that cannot take part in a solution with respect to Régin's algorithm. (4 Points)
- What are the domains of the variables after the filtering you just went through? (2 Points)

Good Luck!

Course Evaluation: If you haven't done so already, *please* fill in the course evaluation (it is attached) when you are finished and put it in Magnus's box (number 55 on the fourth floor).