

# Joule heating

## Computer Lab 3

When a current flows through a device it induces Joule heating. This effect is used to move gears and ratchet mechanisms (e.g., for mirror positioning) on a microscopic scale. In this Lab we will study Joule heating in a U-beam, see Figure 1. A voltage is applied to the left boundary of the U-beam with  $V = 1$  on the small part on the top and  $V = 0$  on the larger part in the bottom. This induces a current that flows through the device which leads to heating of the device. The heating then leads to thermal stresses which will force the device to bend due to the fact that the upper part is more narrow than the lower part. In this Lab we will only consider the effect of electric field and heat transfer, i.e. we will skip the thermal stresses which leads to bending.

## The Joule heating problem

We use the same notations as in COMSOL Multiphysics. This problem is a system of two coupled partial differential equations on the form: Find electric potential  $V$  and temperature  $T$  such that,

$$\begin{cases} -\nabla \cdot \sigma \nabla V = 0 & \text{in } \Omega, \\ V = 1 & \text{for } x = -1, \quad 0.6 \leq y \leq 0.8, \\ V = 0 & \text{for } x = -1, \quad -0.8 \leq y \leq 0.4, \\ n \cdot J = n \cdot \sigma \nabla V = 0, & \text{otherwise,} \end{cases} \quad (1)$$

where  $\sigma$  is electrical conductivity,  $J$  is the current, and,

$$\begin{cases} -\nabla \cdot k \nabla T = Q(\sigma, V) = \frac{\sigma}{|J|^2} = \sigma |\nabla V|^2 & \text{in } \Omega, \\ n \cdot k \nabla T = h(T_{\text{inf}} - T) & \text{on } \partial\Omega, \end{cases} \quad (2)$$

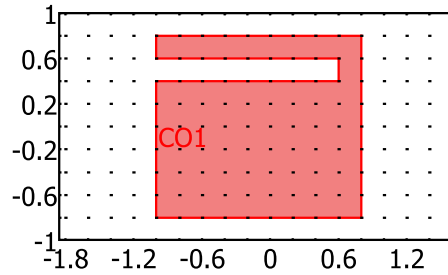


Figure 1: The geometry of the U-beam.

where  $k$  is thermal conductivity,  $Q$  is resistance heating,  $h$  is the heat transfer coefficient, and  $T_{\text{inf}}$  is the external temperature.

**Problem 1.** This is a coupled system of non-linear partial differential equations (since  $|\nabla V|^2$  is present in the right hand side of the heat equation). However, the coupling is weak since  $\sigma$  is independent of  $T$ . Is it possible to solve the equations one by one and thereby avoiding the problem of solving the fully non-linear problems iteratively?

Now start COMSOL Multiphysics using the Matlab7\_comsol icon followed by Comsol. Choose 2D, COMSOL Multiphysics, electro-thermal interaction, and Joule-heating, then ok. Draw the geometry described in Figure 1. Note that you can change which equation you work with (conductive media or heat transfer) in the model tree to the left or under multiphysics.

**Problem 2.** Only modify the subdomain settings for the conductive media equation. Choose  $\sigma$  to be isotropic. This choice also makes it independent of  $T$ . Then change the boundary settings under Physics for both equations to match the boundary conditions described in equations (1) and (2). Let  $h = 1$  and  $T_{\text{inf}} = 273.15$  (pick heat flux under boundary conditions), and for the conductive media, electric insulation for the Neumann boundaries and electric potential with  $V_0$  equal to 1 or 0 on the Dirichlet boundary segments. Construct a mesh and solve the problem, triangle and equality sign. Plot the temperature and electric potential under postprocessing, plot parameters, predefined quantities.

**Problem 3.** Now instead let  $h = 10^4$  and solve the problem again. Plot the temperature. In what way is the quality of the solution effected? What is the physical interpretation of letting  $h = 10^4$ ? Plot the potential, why is it not affected by the change in  $h$ ?

**Problem 4.** Try to improve the solution by refining the mesh. Also try to use adaptivity after first going back to the coarse mesh and choose adaptive mesh refinement under solver parameters. Does the solution improve? Each equation and boundary condition need its own error analysis. It is not clear what COMSOL Multiphysics does here. However, you can improve the solution by refining once everywhere before you start the adaptive loop.

**Problem 5.** So far the system has been one way coupled, why? Now go to physics domain settings for the conductive media equation and let  $\sigma$  be T-dependent. What is the physical interpretation of this? Let  $T_0$  be 273.15 and vary  $a$  what happens? Why?

**Problem 6.** Now solve the Joule heating problem in 3D. Press New... under File and choose 3D instead of 2D. To construct the geometry choose block with default value then make another block with length (0.9, 1.0, 0.1) and use (0, 0, 0.8) as axis base point. Mark the domain with left and right mouse click and use difference to get the desired geometry. Pick boundary conditions as before an use the original coarse mesh, otherwise it will take to much time. Solve the problem and study the solution.

Feel free to keep on investigating COMSOL Multiphysics on your own!