# Introduction to Lexical Analysis

## Outline

- Informal sketch of lexical analysis
  - Identifies tokens in input string
- Issues in lexical analysis
  - Lookahead
  - Ambiguities
- Specifying lexical analyzers (lexers)
  - Regular expressions
  - Examples of regular expressions



## Lexical Analysis

- What do we want to do? Example:

```
if (i == j)
then
   z = 0;
else
   z = 1;
```

- The input is just a string of characters:
  if (i == j)\nthen\n\tz = 0;\n\telse\n\t\tz = 1;
- Goal: Partition input string into substrings
  - where the substrings are tokens
  - and classify them according to their role



## What's a Token?

- A syntactic category
  - In English:
    noun, verb, adjective, …
  - In a programming language:
    Identifier, Integer, Keyword, Whitespace, …

## Tokens

- Tokens correspond to sets of strings
  - these sets depend on the programming language

- Identifier: *strings of letters or digits, starting with a letter*
- Integer: *a non-empty string of digits*
- Keyword: *"else" or "if" or "begin" or …*
- Whitespace: *a non-empty sequence of blanks, newlines, and tabs*



## What are Tokens Used for?

- Classify program substrings according to role

- Output of lexical analysis is a stream of tokens . . .

- . . . which is input to the parser

- Parser relies on token distinctions
  - An identifier is treated differently than a keyword



## Designing a Lexical Analyzer: Step 1

- Define a finite set of tokens
  - Tokens describe all items of interest
  - Choice of tokens depends on language, design of parser
- Recall

  `if (i == j)\nthen\n\tz = 0;\n\telse\n\t\tz = 1;`
- Useful tokens for this expression:

  Integer, Keyword, Relation, Identifier, Whitespace, (, ), =, ;



## Designing a Lexical Analyzer: Step 2

- Describe which strings belong to each token

- Recall:
  - Identifier: *strings of letters or digits, starting with a letter*
  - Integer: *a non-empty string of digits*
  - Keyword: *"else" or "if" or "begin" or …*
  - Whitespace: *a non-empty sequence of blanks, newlines, and tabs*

## Lexical Analyzer: Implementation

An implementation must do two things:

1. Recognize substrings corresponding to tokens

2. Return the value or lexeme of the token
   - The lexeme is the substring



## Example

- Recall:

  if (i == j)\nthen\n\tz = 0;\n\telse\n\t\tz = 1;

- Token-lexeme groupings:
  - Identifier: i, j, z
  - Keyword: if, then, else
  - Relation: ==
  - Integer: 0, 1
  - (, ), =, ; single character of the same name



## Why do Lexical Analysis?

- Dramatically simplify parsing
  - The lexer usually discards "uninteresting" tokens that don't contribute to parsing
    - E.g. Whitespace, Comments
  - Converts data early
- Separate out logic to read source files
  - Potentially an issue on multiple platforms
  - Can optimize reading code independently of parser



## True Crimes of Lexical Analysis

- Is it as easy as it sounds?

- Not quite!

- Look at some programming language history . . .

## Lexical Analysis in FORTRAN

- FORTRAN rule: Whitespace is insignificant
- E.g., **VAR1** is the same as **VA  R1**

FORTRAN whitespace rule was motivated by inaccuracy of punch card operators



## A terrible design! Example

- Consider
  - `DO 5 I = 1,25`
  - `DO 5 I = 1.25`
- The first is DO 5 I = 1 , 25
- The second is DO5I = 1.25
- Reading left-to-right, the lexical analyzer cannot tell if DO5I is a variable or a DO statement until after "," is reached



## Lexical Analysis in FORTRAN. Lookahead.

Two important points:

1. The goal is to partition the string
   - This is implemented by reading left-to-right, recognizing one token at a time
2. "Lookahead" may be required to decide where one token ends and the next token begins
   - Even our simple example has lookahead issues

     `i` vs. `if`

     `=` vs. `==`



## Another Great Moment in Scanning History

PL/1: Keywords can be used as identifiers:

```
IF THEN THEN THEN = ELSE; ELSE ELSE = IF
```

can be difficult to determine how to label lexemes

## More Modern True Crimes in Scanning

Nested template declarations in C++

```
vector<vector<int>> myVector

vector < vector < int >> myVector

(vector < (vector < (int >> myVector)))
```



## Review

- The goal of lexical analysis is to
  - Partition the input string into *lexemes* (the smallest program units that are individually meaningful)
  - Identify the token of each lexeme
- Left-to-right scan ⇒ lookahead sometimes required



## Next

- We still need
  - A way to describe the lexemes of each token
  - A way to resolve ambiguities
    - Is `if` two variables `i` and `f`?
    - Is `==` two equal signs `=` `=`?



## Regular Languages

- There are several formalisms for specifying tokens
- *Regular languages* are the most popular
  - Simple and useful theory
  - Easy to understand
  - Efficient implementations

## Languages

**Def.** Let $\Sigma$ be a set of characters. A *language* $\Lambda$ *over* $\Sigma$ is a set of strings of characters drawn from $\Sigma$

($\Sigma$ is called the *alphabet* of $\Lambda$)



## Examples of Languages

- Alphabet = English characters
- Language = English sentences
- Not every string on English characters is an English sentence

- Alphabet = ASCII
- Language = C programs
- Note: ASCII character set is different from English character set



## Notation

- Languages are sets of strings
- Need some notation for specifying which sets of strings we want our language to contain
- The standard notation for regular languages is *regular expressions*



## Atomic Regular Expressions

- Single character

$$'c' = \{"c"\}$$

- Epsilon

$$\varepsilon = \{""\}$$

## Compound Regular Expressions

- Union

$$A+B=\{s \mid s \in A \text{ or } s \in B\}$$

- Concatenation

$$AB=\{ab \mid a \in A \text{ and } b \in B\}$$

- Iteration

$$A^*=\bigcup_{i\geq 0} A^i \quad \text{where} \quad A^i = A \ldots i \text{ times} \ldots A$$



## Regular Expressions

- **Def**. The *regular expressions over* $\Sigma$ are the smallest set of expressions including

| | |
|---|---|
| $\varepsilon$ | |
| $'c'$ | where $c \in \Sigma$ |
| $A+B$ | where $A, B$ are rexp over $\Sigma$ |
| $AB$ | " " " |
| $A^*$ | where $A$ is a rexp over $\Sigma$ |



## Syntax vs. Semantics

- To be careful, we should distinguish syntax and semantics (meaning) of regular expressions

$$
\begin{aligned}
L(\varepsilon) &= \{""\} \\
L('c') &= \{"c"\} \\
L(A+B) &= L(A) \cup L(B) \\
L(AB) &= \{ab \mid a \in L(A) \text{ and } b \in L(B)\} \\
L(A^*) &= \bigcup_{i \geq 0} L(A^i)
\end{aligned}
$$



## Example: Keyword

Keyword: *"else" or "if" or "begin" or …*

$$\text{'else'} + \text{'if'} + \text{'begin'} + \cdots$$

Note: 'else' abbreviates 'e''l''s''e'

## Example: Integers

Integer: *a non-empty string of digits*

digit = '0'+'1'+'2'+'3'+'4'+'5'+'6'+'7'+'8'+'9'

integer = digit digit$^*$

Abbreviation: $A^+ = AA^*$



## Example: Identifier

Identifier: *strings of letters or digits, starting with a letter*

letter = 'A' +...+'Z'+'a'+...+'z'

identifier = letter (letter + digit)$^*$

Is (letter$^*$ + digit$^*$) the same?



## Example: Whitespace

Whitespace: *a non-empty sequence of blanks, newlines, and tabs*

(' ' + '\n' + '\t')$^+$



## Example 1: Phone Numbers

- Regular expressions are all around you!
- Consider +46(0)18-471-1056

$\Sigma$ = digits $\cup$ {+,–,(,)}
country = digit digit
city = digit digit
univ = digit digit digit
extension = digit digit digit digit
phone_num = ‘+’country’(’0‘)’city’–’univ’–’extension

## Example 2: Email Addresses

- Consider *kostis@it.uu.se*

$$\Sigma = \text{letters} \cup \{.,@\}$$

$$\text{name} = \text{letter}^+$$

$$\text{address} = \text{name '@' name '.' name '.' name}$$



## Summary

- Regular expressions describe many useful languages
- Regular languages are a language specification
  - We still need an implementation
- Next: Given a string *s* and a regular expression *R*, is

$$s \in L(R)?$$

- A yes/no answer is not enough!
- Instead: partition the input into tokens
- We will adapt regular expressions to this goal



# Implementation of Lexical Analysis

## Outline

- Specifying lexical structure using regular expressions
- Finite automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)
- Implementation of regular expressions

  RegExp ⇒ NFA ⇒ DFA ⇒ Tables

## Notation

- For convenience, we will use a variation (we will allow user-defined abbreviations) in regular expression notation

- Union: $A + B \equiv A \mid B$
- Option: $A + \varepsilon \equiv A?$
- Range: 'a'+'b'+…+'z' ≡ [a-z]
- Excluded range:

  complement of [a-z] ≡ [^a-z]



## Regular Expressions ⇒ Lexical Specifications

1. Select a set of tokens
   - Integer, Keyword, Identifier, LeftPar, ...

2. Write a regular expression (pattern) for the lexemes of each token
   - Integer = digit +
   - Keyword = 'if' + 'else' + …
   - Identifier = letter (letter + digit)*
   - LeftPar = '('
   - …



## Regular Expressions ⇒ Lexical Specifications

3. Construct R, a regular expression matching all lexemes for all tokens

   R = Keyword + Identifier + Integer + …

   $= R_1 + R_2 + R_3 + \ldots$

Facts: If $s \in L(R)$ then $s$ is a lexeme

- Furthermore $s \in L(R_j)$ for some "j"
- This "j" determines the token that is reported



## Regular Expressions ⇒ Lexical Specifications

4. Let input be $x_1 \ldots x_n$
   - ($x_1 \ldots x_n$ are characters in the language alphabet)
   - For $1 \le i \le n$ check

     $x_1 \ldots x_i \in L(R)$ ?

5. It must be that

   $x_1 \ldots x_i \in L(R_j)$ for some $i$ and $j$

   (if there is a choice, pick a smallest such $j$)

6. Report token $j$, remove $x1 \ldots x_i$ from input and go to step 4

## How to Handle Spaces and Comments?

1. We could create a token Whitespace

   Whitespace = (' ' + '\n' + '\t')$^+$

   - We could also add comments in there
   - An input "   \t\n  555   " is transformed into

     Whitespace Integer Whitespace

2. Lexical analyzer skips spaces (preferred)
   - Modify step 5 from before as follows:

     It must be that $x_k \ldots x_i \in L(R_j)$ for some $j$ such that $x_1 \ldots x_{k-1} \in L(Whitespace)$
   - Parser is not bothered with spaces



## Ambiguities (1)

- There are ambiguities in the algorithm
- How much input is used? What if
  - $x_1 \ldots x_i \in L(R)$ and also $x_1 \ldots x_K \in L(R)$
- The "maximal munch" rule: Pick the longest possible substring that matches R



## Ambiguities (2)

- Which token is used? What if
  - $x_1 \ldots x_i \in L(R_j)$ and also $x_1 \ldots x_i \in L(R_k)$
- Rule: use rule listed first ($j$ if $j < k$)
- Example:
  - $R_1$ = Keyword and $R_2$ = Identifier
  - "if" matches both
  - Treats "if" as a keyword not an identifier



## Error Handling

- What if

  No rule matches a prefix of input ?
- Problem: Can't just get stuck …
- Solution:
  - Write a rule matching all "bad" strings
  - Put it last
- Lexical analysis tools allow the writing of:

  $R = R_1 + \ldots + R_n + Error$
  - Token Error matches if nothing else matches

## Summary

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known (next)
  - Require only single pass over the input
  - Few operations per character (table lookup)



## Regular Languages & Finite Automata

**Basic formal language theory result**:

*Regular expressions and finite automata both define the class of regular languages.*

Thus, we are going to use:

- Regular expressions for specification
- Finite automata for implementation (automatic generation of lexical analyzers)



## Finite Automata

A finite automaton is a *recognizer* for the strings of a regular language

A finite automaton consists of

- A finite input alphabet $\Sigma$
- A set of states S
- A start state n
- A set of accepting states $F \subseteq S$
- A set of transitions state $\to^{\text{input}}$ state



## Finite Automata

- Transition

$$s_1 \to^a s_2$$

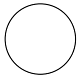
- Is read

  In state $s_1$ on input "a" go to state $s_2$

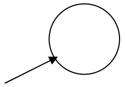
- If end of input
  - If in accepting state $\Rightarrow$ accept
- Otherwise
  - If no transition possible $\Rightarrow$ reject

## Finite Automata State Graphs

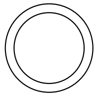
- A state



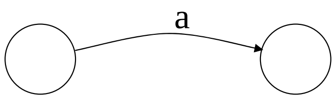
- The start state



- An accepting state



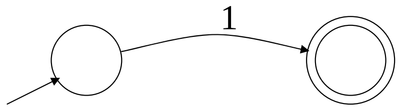
- A transition






## A Simple Example

- A finite automaton that accepts only "1"

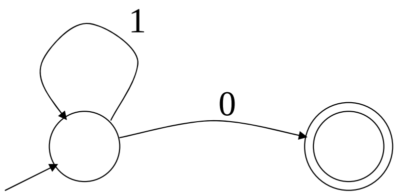



- A finite automaton accepts a string if we can follow transitions labeled with the characters in the string from the start to some accepting state



## Another Simple Example

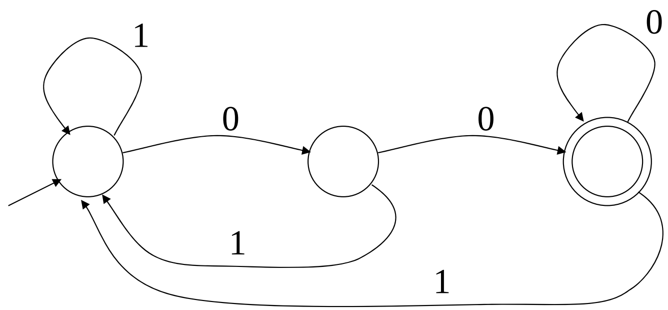
- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: {0,1}





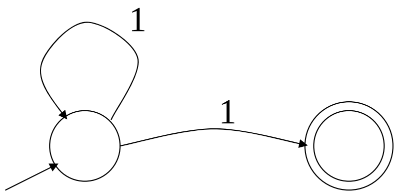

## And Another Example

- Alphabet {0,1}
- What language does this recognize?



## And Another Example

- Alphabet still { 0, 1 }

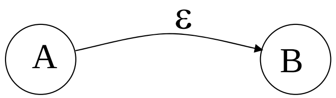



- The operation of the automaton is not completely defined by the input
  - On input "11" the automaton could be in either state



## Epsilon Moves

- Another kind of transition: ε-moves




- Machine can move from state A to state B without reading input



## Deterministic and Non-Deterministic Automata

- **Deterministic Finite Automata (DFA)**
  - One transition per input per state
  - No ε-moves
- **Non-deterministic Finite Automata (NFA)**
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
- Finite automata have finite memory
  - Enough to only encode the current state

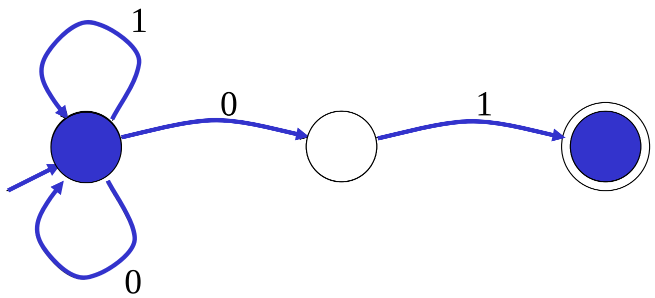


## Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

## Acceptance of NFAs

- An NFA can get into multiple states




- Input: 1 0 1
- Rule: NFA accepts an input if it <u>can</u> get in a final state



## NFA vs. DFA (1)

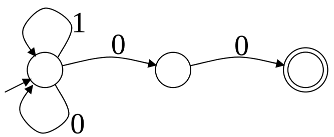
- NFAs and DFAs recognize the same set of languages (regular languages)
- DFAs are easier to implement
  - There are no choices to consider



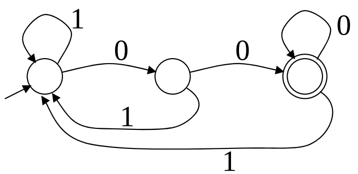
## NFA vs. DFA (2)

- For a given language the NFA can be simpler than the DFA

NFA




DFA

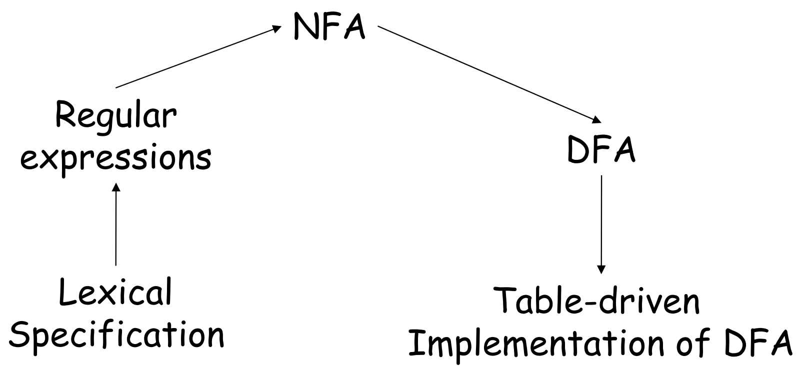



- DFA can be exponentially larger than NFA (contrary to what is shown in the above example)



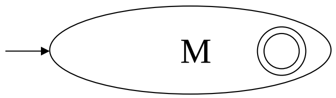
## Regular Expressions to Finite Automata

- High-level sketch



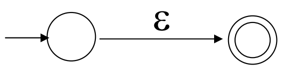
## Regular Expressions to NFA (1)

- For each kind of reg. expr, define an NFA
  - Notation: NFA for regular expression M

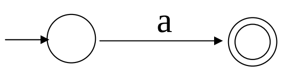



i.e. our automata have **one** start and **one** accepting state

- For ε



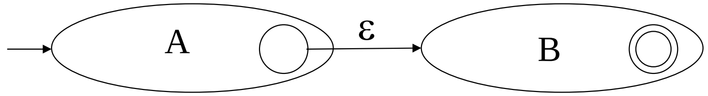

- For input a



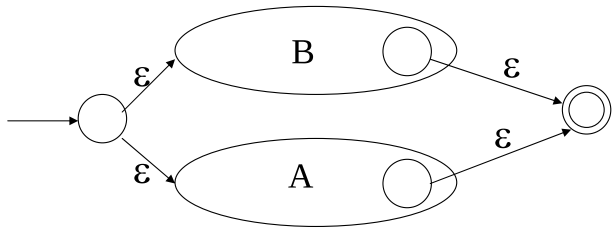



## Regular Expressions to NFA (2)

- For AB



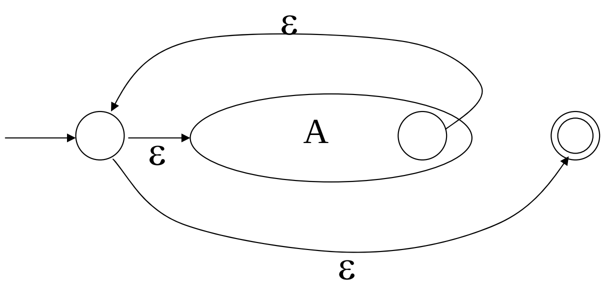

- For A + B






## Regular Expressions to NFA (3)

- For A*



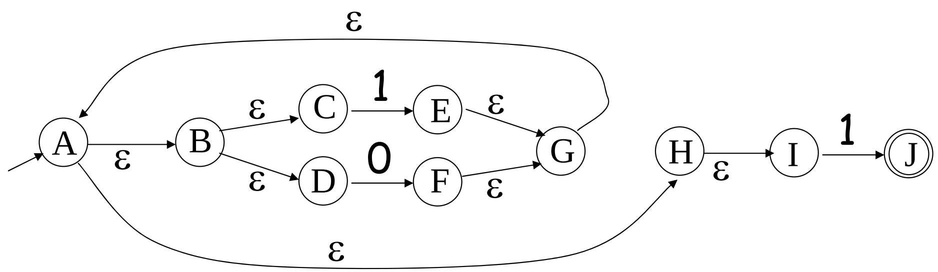



## Example of Regular Expression → NFA conversion

- Consider the regular expression

  (1+0)*1

- The NFA is



## NFA to DFA. The Trick

- Simulate the NFA
- Each state of DFA
  = a non-empty subset of states of the NFA
- Start state
  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \to^a S'$ to DFA iff
  - $S'$ is the set of NFA states reachable from <u>any</u> state in $S$ after seeing the input a
    - considering ε-moves as well

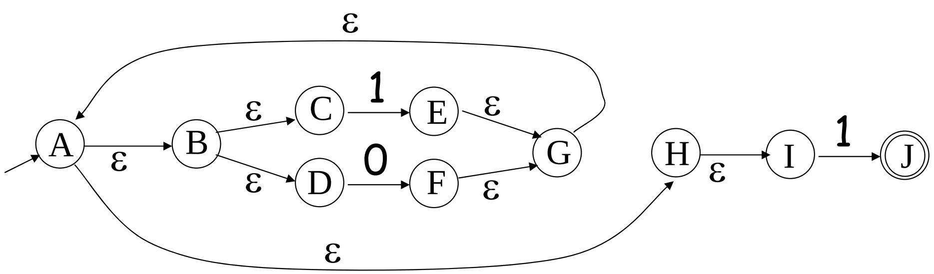


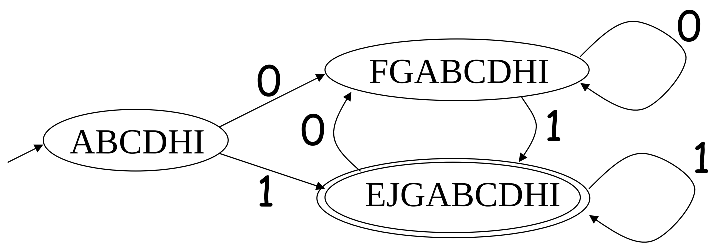
## NFA to DFA. Remark

- An NFA may be in many states at any time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets are there?
  - $2^N - 1$ = finitely many



## NFA to DFA Example



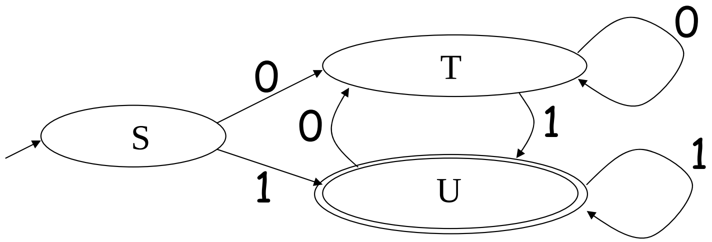






## Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is "states"
  - Other dimension is "input symbols"
  - For every transition $S_i \to^a S_k$ define T[i,a] = k
- DFA "execution"
  - If in state $S_i$ and input a, read T[i,a] = k and skip to state $S_k$
  - Very efficient

## Table Implementation of a DFA




| | 0 | 1 |
|---|---|---|
| S | T | U |
| T | T | U |
| U | T | U |



## Implementation (Cont.)

- NFA → DFA conversion is at the heart of tools such as lex, ML-Lex, flex or jlex
- But, DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations



## Theory vs. Practice

Two differences:

- DFAs *recognize* lexemes. A lexer must return a *type of acceptance* (token type) rather than simply an accept/reject indication.
- DFAs consume the complete string and accept or reject it. A lexer must *find* the end of the lexeme in the input stream and then find the *next* one, etc.