Introduction to Lexical Analysis

Outline

- Informal sketch of lexical analysis
  - Identifies tokens in input string
- Issues in lexical analysis
  - Lookahead
  - Ambiguities
- Specifying lexical analyzers (lexers)
  - Regular expressions
  - Examples of regular expressions

Lexical Analysis

- What do we want to do? Example:

```
if (i == j)
then
   z = 0;
else
   z = 1;
```

- The input is just a string of characters:

  if (i == j)\nthen\n\tz = 0;\n\telse\n\t\tz = 1;

- Goal: Partition input string into substrings
  - where the substrings are tokens
  - and classify them according to their role

What's a Token?

- A syntactic category
  - In English:

    noun, verb, adjective, …

  - In a programming language:

    Identifier, Integer, Keyword, Whitespace, …

Tokens

- Tokens correspond to sets of strings
  - these sets depend on the programming language

- Identifier: *strings of letters or digits, starting with a letter*
- Integer: *a non-empty string of digits*
- Keyword: *"else" or "if" or "begin" or …*
- Whitespace: *a non-empty sequence of blanks, newlines, and tabs*

What are Tokens Used for?

- Classify program substrings according to role
- Output of lexical analysis is a stream of tokens . . .
- . . . which is input to the parser
- Parser relies on token distinctions
  - An identifier is treated differently than a keyword

Designing a Lexical Analyzer: Step 1

- Define a finite set of tokens
  - Tokens describe all items of interest
  - Choice of tokens depends on language, design of parser
- Recall

  if (i == j)\nthen\n\tz = 0;\n\telse\n\t\tz = 1;
- Useful tokens for this expression:

  Integer, Keyword, Relation, Identifier, Whitespace, (,), =, ;

Designing a Lexical Analyzer: Step 2

- Describe which strings belong to each token

- Recall:
  - Identifier: *strings of letters or digits, starting with a letter*
  - Integer: *a non-empty string of digits*
  - Keyword: *"else" or "if" or "begin" or …*
  - Whitespace: *a non-empty sequence of blanks, newlines, and tabs*

Lexical Analyzer: Implementation

An implementation must do two things:

1. Recognize substrings corresponding to tokens

2. Return the value or <u>lexeme</u> of the token
   - The lexeme is the substring

Example

- Recall:

  if (i == j)\nthen\n\tz = 0;\n\telse\n\t\tz = 1;

- Token-lexeme groupings:
  - Identifier: i, j, z
  - Keyword: if, then, else
  - Relation: ==
  - Integer: 0, 1
  - (,), =, ; single character of the same name

Why do Lexical Analysis?

- Dramatically simplify parsing
  - The lexer usually discards “uninteresting” tokens that don’t contribute to parsing
    - E.g. Whitespace, Comments
  - Converts data early
- Separate out logic to read source files
  - Potentially an issue on multiple platforms
  - Can optimize reading code independently of parser

True Crimes of Lexical Analysis

- Is it as easy as it sounds?
- Not quite!
- Look at some programming language history . . .

Lexical Analysis in FORTRAN

- FORTRAN rule: Whitespace is insignificant

- E.g., **`VAR1`** is the same as **`VA   R1`**

FORTRAN whitespace rule was motivated by inaccuracy of punch card operators

A terrible design! Example

- Consider
  - **`DO 5 I = 1,25`**
  - **`DO 5 I = 1.25`**

- The first is DO 5 I = 1 , 25
- The second is DO5I = 1.25

- Reading left-to-right, the lexical analyzer cannot tell if DO5I is a variable or a DO statement until after "," is reached

Lexical Analysis in FORTRAN. Lookahead.

Two important points:

1. The goal is to partition the string
   - This is implemented by reading left-to-right, recognizing one token at a time

2. "Lookahead" may be required to decide where one token ends and the next token begins
   - Even our simple example has lookahead issues

     `i` vs. `if`

     `=` vs. `==`

Another Great Moment in Scanning History

PL/1: Keywords can be used as identifiers:

```
IF THEN THEN THEN = ELSE; ELSE ELSE = IF
```

can be difficult to determine how to label lexemes

More Modern True Crimes in Scanning

Nested template declarations in C++

```
vector<vector<int>> myVector

vector < vector < int >> myVector

(vector < (vector < (int >> myVector)))
```

Review

- The goal of lexical analysis is to
  - Partition the input string into *lexemes* (the smallest program units that are individually meaningful)
  - Identify the token of each lexeme

- Left-to-right scan $\Rightarrow$ lookahead sometimes required

Next

- We still need
  - A way to describe the lexemes of each token
  - A way to resolve ambiguities
    - Is **`if`** two variables **`i`** and **`f`**?
    - Is **`==`** two equal signs **`=`** **`=`**?

Regular Languages

- There are several formalisms for specifying tokens

- *Regular languages* are the most popular
  - Simple and useful theory
  - Easy to understand
  - Efficient implementations

Languages

Def. Let Σ be a set of characters. A *language* Λ *over* Σ is a set of strings of characters drawn from Σ

(Σ is called the *alphabet* of Λ)

Examples of Languages

- Alphabet = English characters
- Language = English sentences

- Not every string on English characters is an English sentence

- Alphabet = ASCII
- Language = C programs

- Note: ASCII character set is different from English character set

Notation

- Languages are sets of strings
- Need some notation for specifying which sets of strings we want our language to contain
- The standard notation for regular languages is *regular expressions*

Atomic Regular Expressions

- Single character

$$'c' = \{"c"\}$$

- Epsilon

$$\varepsilon = \{""\}$$

Compound Regular Expressions

- Union

$$A + B = \{ s \mid s \in A \text{ or } s \in B \}$$

- Concatenation

$$AB = \{ ab \mid a \in A \text{ and } b \in B \}$$

- Iteration

$$A^* = \bigcup_{i \geq 0} A^i \quad \text{where } A^i = A \ldots i \text{ times} \ldots A$$

Regular Expressions

- **Def**. The *regular expressions over* Σ are the smallest set of expressions including

ε	
$'c'$	where $c \in \Sigma$
$A + B$	where A, B are rexp over Σ
AB	" " "
A^*	where A is a rexp over Σ

Syntax vs. Semantics

- To be careful, we should distinguish syntax and semantics (meaning) of regular expressions

$$
\begin{aligned}
L(\varepsilon) &= \{""\} \\
L('c') &= \{"c"\} \\
L(A+B) &= L(A) \cup L(B) \\
L(AB) &= \{ab \mid a \in L(A) \text{ and } b \in L(B)\} \\
L(A^*) &= \bigcup_{i \geq 0} L(A^i)
\end{aligned}
$$

Example: Keyword

Keyword: *"else" or "if" or "begin" or …*

$$'\mathrm{else}' + '\mathrm{if}' + '\mathrm{begin}' + \cdots$$

Note: 'else' abbreviates 'e''l''s''e'

Example: Integers

Integer: *a non-empty string of digits*

digit = '0'+'1'+'2'+'3'+'4'+'5'+'6'+'7'+'8'+'9'

integer = digit digit*

Abbreviation: $A^+ = AA^*$

Example: Identifier

Identifier: *strings of letters or digits, starting with a letter*

$$\text{letter} = \text{'A'} + \ldots + \text{'Z'} + \text{'a'} + \ldots + \text{'z'}$$

$$\text{identifier} = \text{letter}\ (\text{letter} + \text{digit})^*$$

Is $(\text{letter}^* + \text{digit}^*)$ the same?

Example: Whitespace

Whitespace: *a non-empty sequence of blanks, newlines, and tabs*

$$(\text{' '} + \text{'\textbackslash n'} + \text{'\textbackslash t'})^{+}$$

Example 1: Phone Numbers

- Regular expressions are all around you!
- Consider +46(0)18-471-1056

Σ = digits ∪ {+,–,(,)}
country = digit digit
city = digit digit
univ = digit digit digit
extension = digit digit digit digit
phone_num = '+'country'('0')'city'–'univ'–'extension

Example 2: Email Addresses

- Consider *kostis@it.uu.se*

$$\Sigma \quad = \quad \text{letters} \cup \{.,@\}$$

$$\text{name} \quad = \quad \text{letter}^{+}$$

$$\text{address} \quad = \quad \text{name '@' name '.' name '.' name}$$

Summary

- Regular expressions describe many useful languages
- Regular languages are a language specification
  - We still need an implementation

- Next: *Given a string s and a regular expression R*, is

$$s \in L(R)?$$

- *A* yes/no answer is not enough!
- Instead: partition the input into tokens
- We will adapt regular expressions to this goal

Implementation of Lexical Analysis

Outline

- Specifying lexical structure using regular expressions

- Finite automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)

- Implementation of regular expressions

  RegExp $\Rightarrow$ NFA $\Rightarrow$ DFA $\Rightarrow$ Tables

Notation

- For convenience, we will use a variation (we will allow user-defined abbreviations) in regular expression notation

- Union: A + B ≡ A | B
- Option: A + ε ≡ A?
- Range: 'a'+'b'+…+'z' ≡ [a-z]
- Excluded range:

  complement of [a-z] ≡ [^a-z]

Regular Expressions ⇒ Lexical Specifications

1. Select a set of tokens
   - Integer, Keyword, Identifier, LeftPar, ...

2. Write a regular expression (pattern) for the lexemes of each token
   - Integer = digit +
   - Keyword = 'if' + 'else' + …
   - Identifier = letter (letter + digit)*
   - LeftPar = '('
   - …

Regular Expressions ⇒ Lexical Specifications

3. Construct R, a regular expression matching all lexemes for all tokens

$$R = \text{Keyword} + \text{Identifier} + \text{Integer} + \ldots$$
$$= R_1 + R_2 + R_3 + \ldots$$

Facts: If $s \in L(R)$ then s is a lexeme

- Furthermore $s \in L(R_j)$ for some "j"
- This "j" determines the token that is reported

Regular Expressions $\Rightarrow$ Lexical Specifications

4. Let input be $x_1 \ldots x_n$
   - ($x_1 \ldots x_n$ are characters in the language alphabet)
   - For $1 \le i \le n$ check

     $x_1 \ldots x_i \in L(R)$?

5. It must be that

   $x_1 \ldots x_i \in L(R_j)$ for some i and j

   (if there is a choice, pick a smallest such j)

6. Report token j, remove $x1 \ldots x_i$ from input and go to step 4

How to Handle Spaces and Comments?

1. We could create a token Whitespace

   Whitespace = (' ' + '\n' + '\t')$^+$

   - We could also add comments in there
   - An input "   \t\n   555   " is transformed into

     Whitespace Integer Whitespace

2. Lexical analyzer skips spaces (preferred)
   - Modify step 5 from before as follows:

     It must be that $x_k \dots x_i \in L(R_j)$ for some j such that $x_1 \dots x_{k-1} \in L(\text{Whitespace})$
   - Parser is not bothered with spaces

Ambiguities (1)

- There are ambiguities in the algorithm

- How much input is used? What if
  - $x_1 \ldots x_i \in L(R)$ and also $x_1 \ldots x_K \in L(R)$
- The "maximal munch" rule: Pick the longest possible substring that matches R

Ambiguities (2)

- Which token is used? What if
  - $x_1 \ldots x_i \in L(R_j)$ and also $x_1 \ldots x_i \in L(R_k)$
- Rule: use rule listed first (j if $j < k$)

- Example:
  - R_1 = Keyword and R_2 = Identifier
  - "if" matches both
  - Treats "if" as a keyword not an identifier

Error Handling

- What if

  No rule matches a prefix of input ?
- Problem: Can't just get stuck …
- Solution:
  - Write a rule matching all "bad" strings
  - Put it last
- Lexical analysis tools allow the writing of:

  $R = R_1 + \ldots + R_n + \text{Error}$
  - Token Error matches if nothing else matches

Summary

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known (next)
  - Require only single pass over the input
  - Few operations per character (table lookup)

Regular Languages & Finite Automata

Basic formal language theory result:

Regular expressions and finite automata both define the class of regular languages.

Thus, we are going to use:

- Regular expressions for specification
- Finite automata for implementation (automatic generation of lexical analyzers)

Finite Automata

A finite automaton is a *recognizer* for the strings of a regular language

A finite automaton consists of

- A finite input alphabet Σ
- A set of states S
- A start state n
- A set of accepting states $F \subseteq S$
- A set of transitions state $\rightarrow^{input}$ state

Finite Automata

- Transition

$$s_1 \rightarrow^a s_2$$

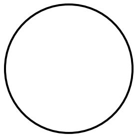
- Is read

  In state s_1 on input "a" go to state s_2

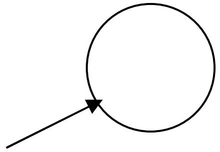
- If end of input
  - If in accepting state $\Rightarrow$ accept
- Otherwise
  - If no transition possible $\Rightarrow$ reject

Finite Automata State Graphs

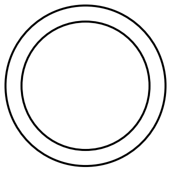
- A state



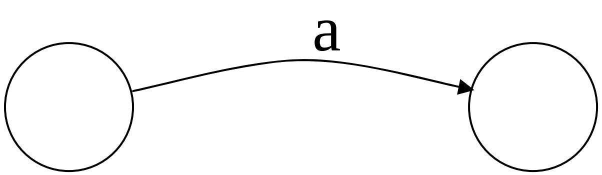
- The start state



- An accepting state

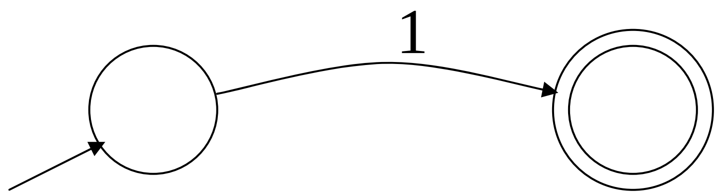


- A transition



A Simple Example

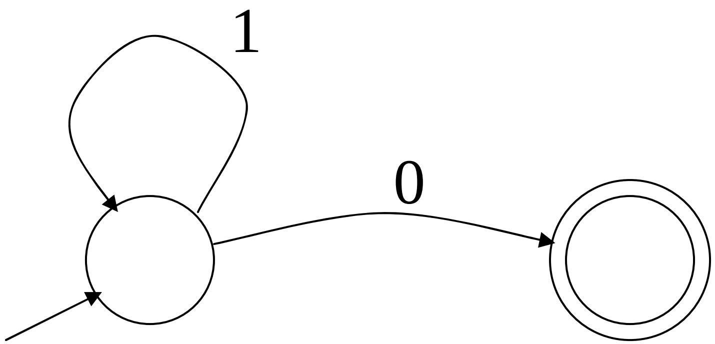
- A finite automaton that accepts only "1"



- A finite automaton accepts a string if we can follow transitions labeled with the characters in the string from the start to some accepting state

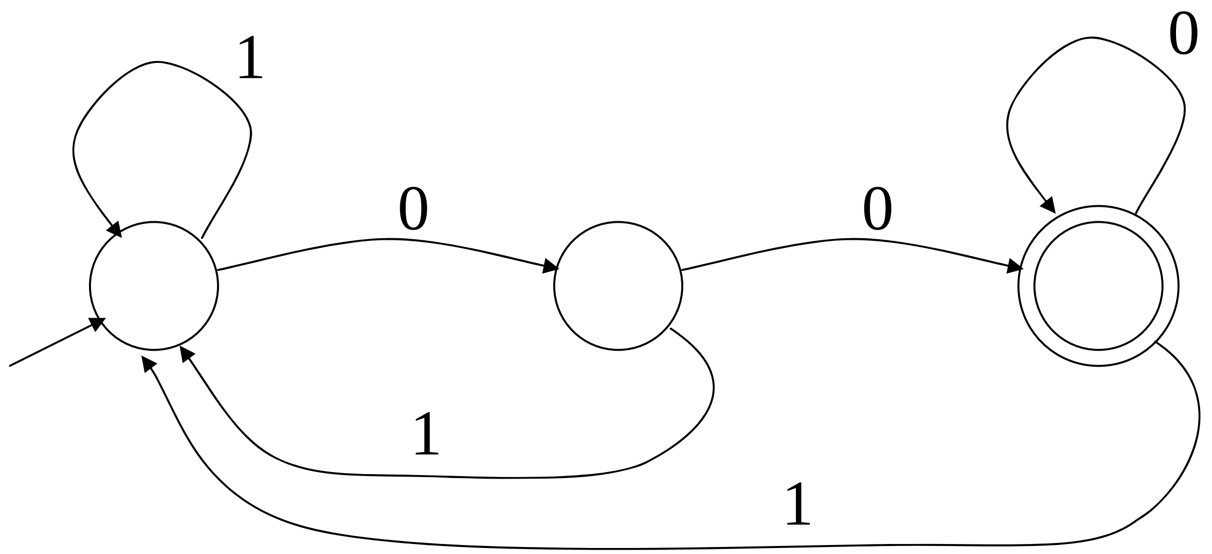
Another Simple Example

- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: {0,1}



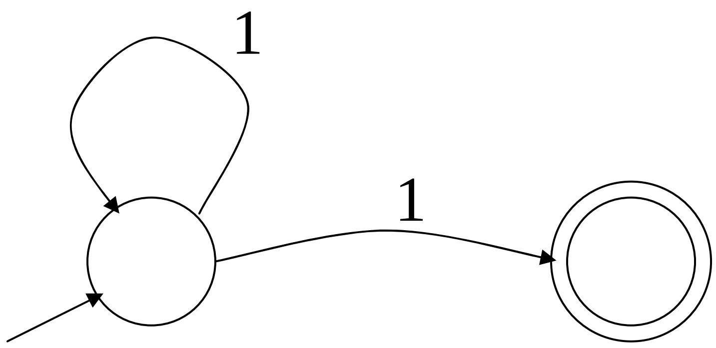
And Another Example

- Alphabet {0,1}
- What language does this recognize?



And Another Example

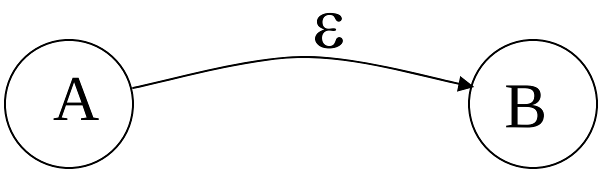
- Alphabet still { 0, 1 }



- The operation of the automaton is not completely defined by the input
  - On input "11" the automaton could be in either state

Epsilon Moves

- Another kind of transition: ε-moves



- Machine can move from state A to state B without reading input

Deterministic and Non-Deterministic Automata

- **Deterministic Finite Automata (DFA)**
  - One transition per input per state
  - No ε-moves
- **Non-deterministic Finite Automata (NFA)**
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
- Finite automata have finite memory
  - Enough to only encode the current state

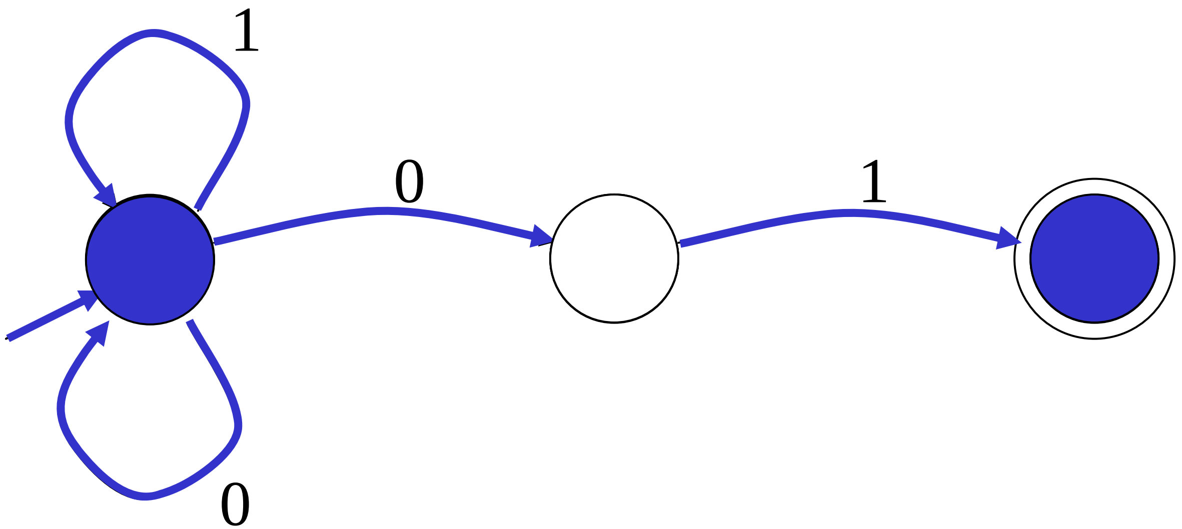
Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input

- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

Acceptance of NFAs

- An NFA can get into multiple states



- Input: 1 0 1
- Rule: NFA accepts an input if it can get in a final state

NFA vs. DFA (1)

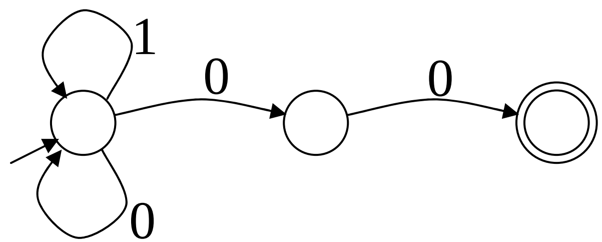
- NFAs and DFAs recognize the same set of languages (regular languages)

- DFAs are easier to implement
  - There are no choices to consider

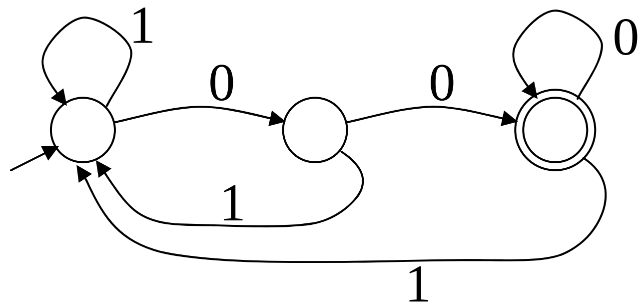
NFA vs. DFA (2)

- For a given language the NFA can be simpler than the DFA

NFA



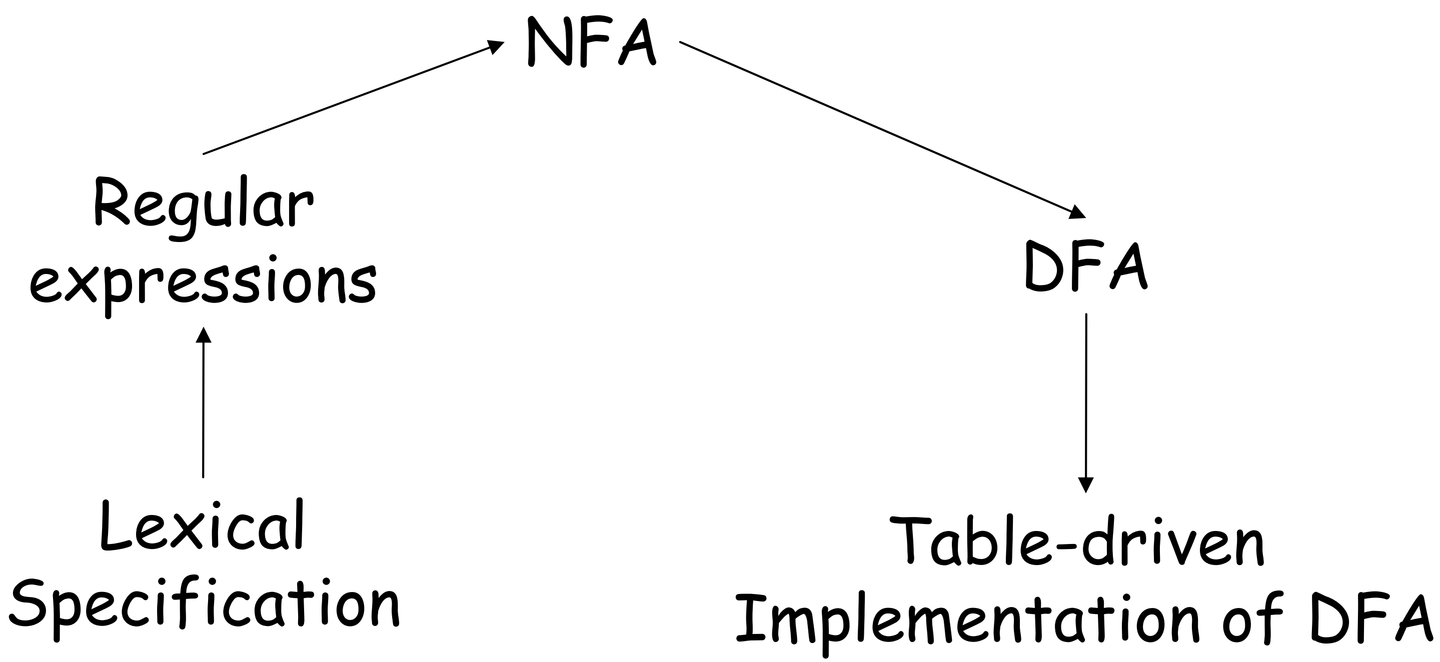
DFA



- DFA can be exponentially larger than NFA
(contrary to what is shown in the above example)

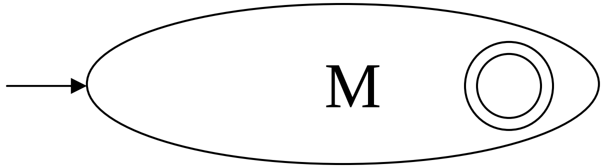
Regular Expressions to Finite Automata

- High-level sketch



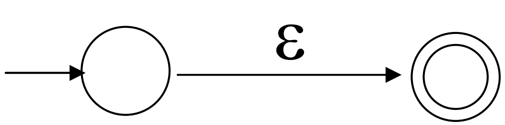
Regular Expressions to NFA (1)

- For each kind of reg. expr, define an NFA
  - Notation: NFA for regular expression M

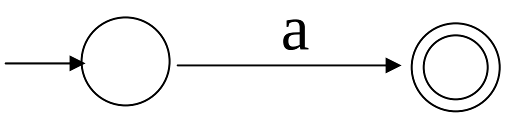


i.e. our automata have **one** start and **one** accepting state

- For ε

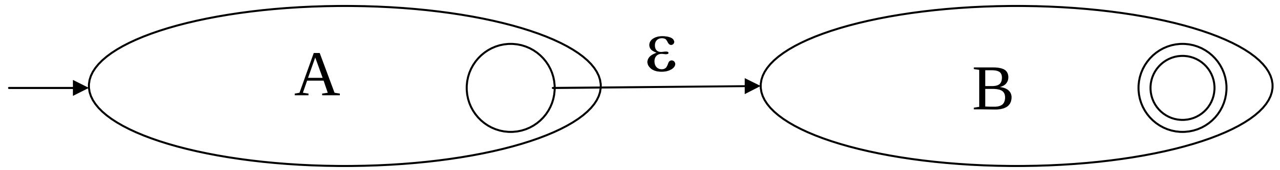


- For input a

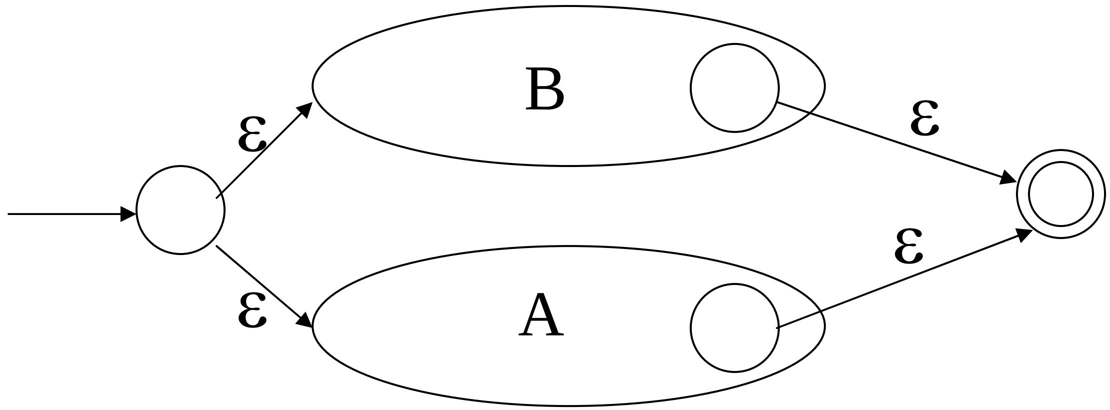


Regular Expressions to NFA (2)

- For *AB*

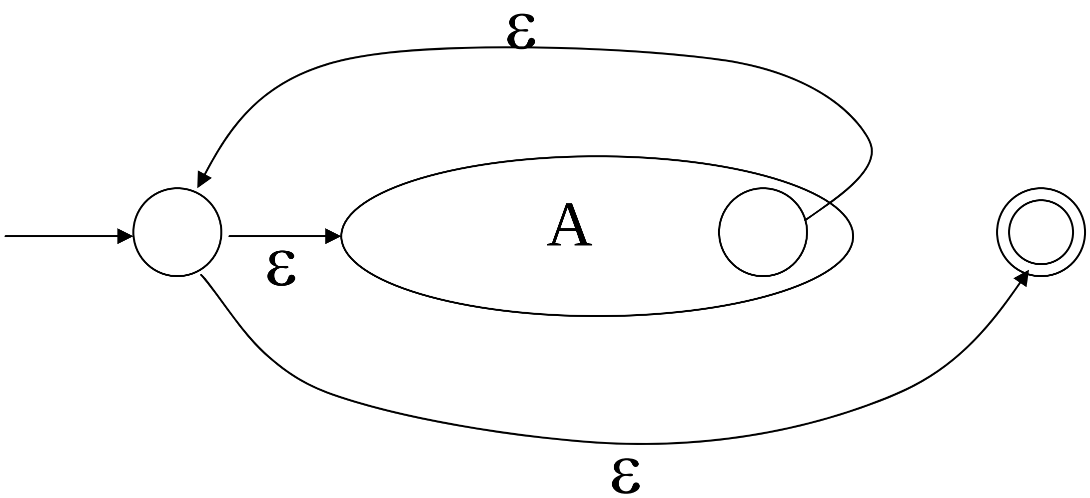


- For *A + B*



Regular Expressions to NFA (3)

- For *A**

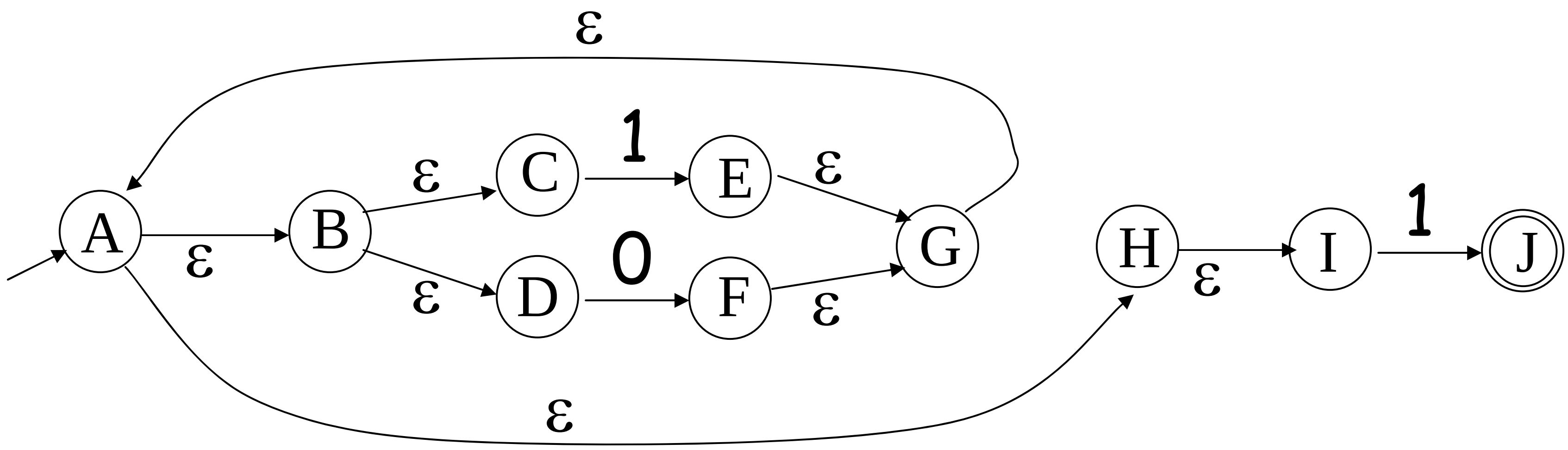


Example of Regular Expression → NFA conversion

- Consider the regular expression

$$(1+0)^*1$$

- The NFA is



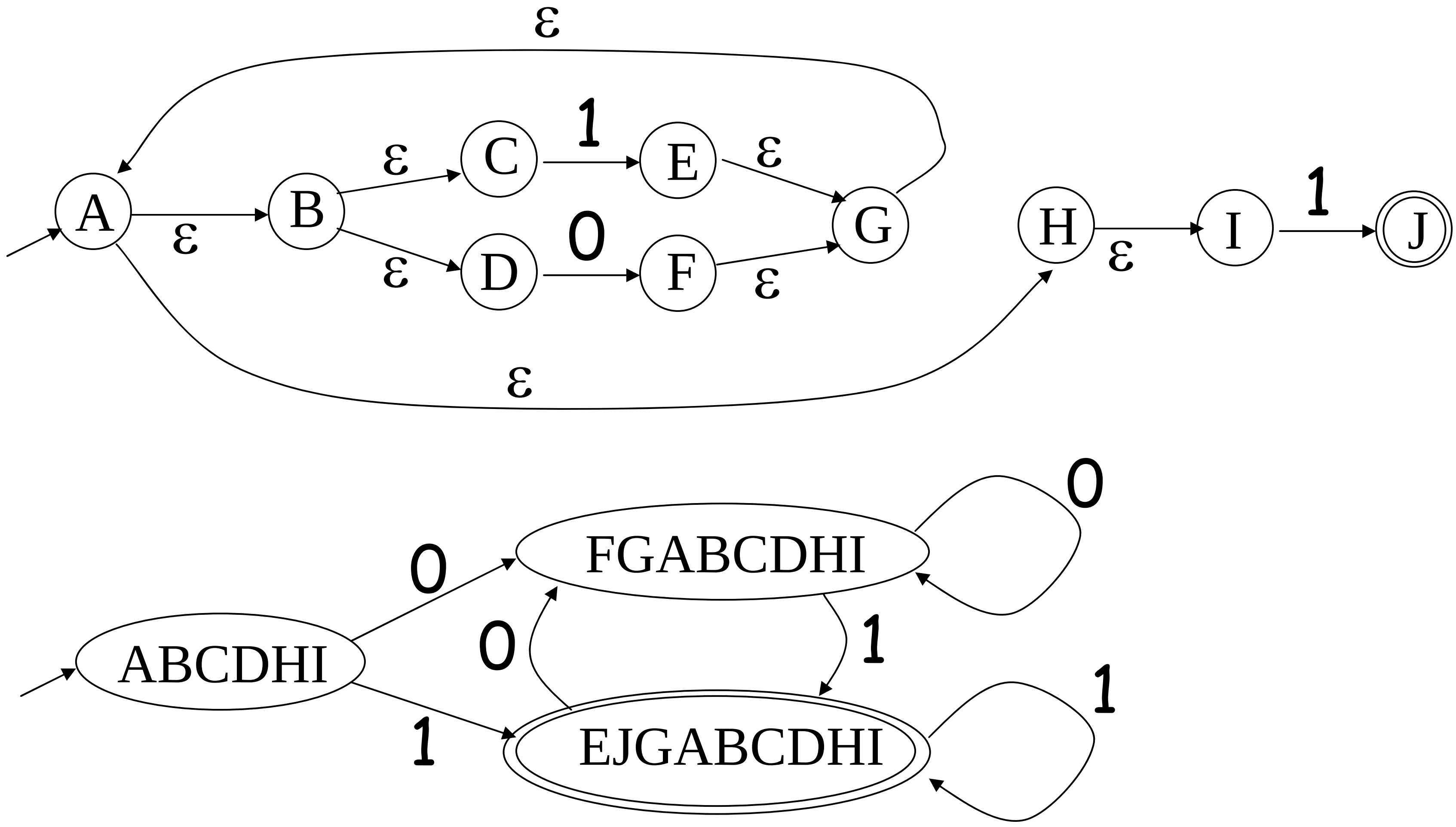
NFA to DFA. The Trick

- Simulate the NFA
- Each state of DFA
  = a non-empty subset of states of the NFA
- Start state
  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \to^{a} S'$ to DFA iff
  - S' is the set of NFA states reachable from any state in S after seeing the input a
    - considering ε-moves as well

NFA to DFA. Remark

- An NFA may be in many states at any time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets are there?
  - $2^N - 1$ = finitely many

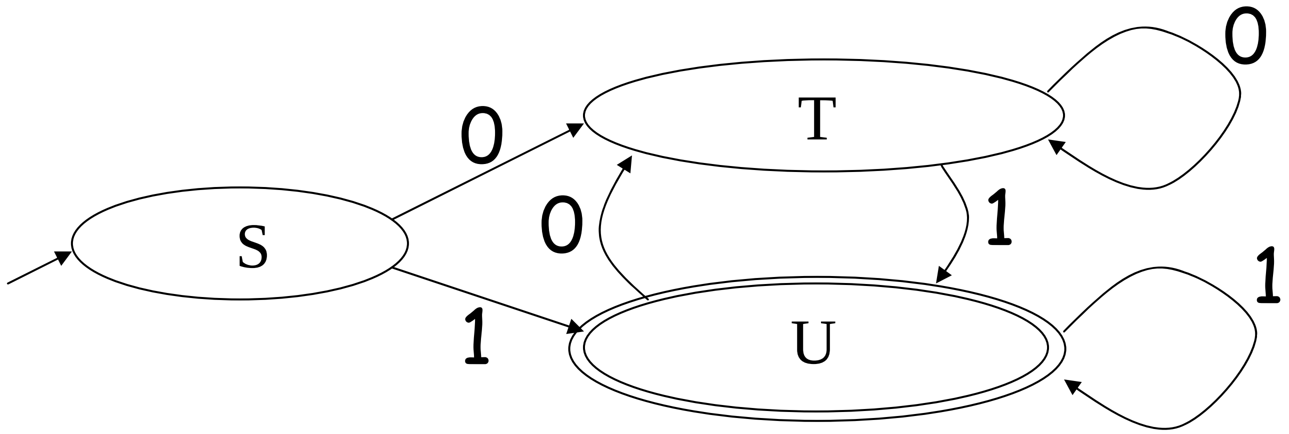
NFA to DFA Example



Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is "states"
  - Other dimension is "input symbols"
  - For every transition $S_i \rightarrow^a S_k$ define T[i,a] = k
- DFA "execution"
  - If in state S_i and input a, read T[i,a] = k and skip to state S_k
  - Very efficient

Table Implementation of a DFA



	0	1
S	T	U
T	T	U
U	T	U

Implementation (Cont.)

- NFA → DFA conversion is at the heart of tools such as lex, ML-Lex, flex or jlex

- But, DFAs can be huge

- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations

Theory vs. Practice

Two differences:

- DFAs *recognize* lexemes. A lexer must return a *type of acceptance* (token type) rather than simply an accept/reject indication.

- DFAs consume the complete string and accept or reject it. A lexer must *find* the end of the lexeme in the input stream and then find the *next* one, etc.