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PROJECT REPORT

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Temperature of steel in hot rolling mill using combination of finite element and empirical methods

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Abstract

This project suggests a method to simulate the temperature of steel during hot steel rolling. The method uses a combination of finite element method to simulate the heat transfer and empirical approach to the deformation energy. An optimized Johnson-Cook material model was used to calculate the flow stress. The results showed that the temperature distribution could be calculated reasonably fast with results conforming with previous work.

Keywords: Hot steel rolling, Finite element method, Johnson-Cook

1 Introduction

The process of producing long steel products such as beams, rails and wire is performed in a hot rolling mill. The starting material consists of long rectangular blocks called billets. The material travels through a series of rolls which deforms and elongates it to the desired form. The complete process is highly complex. There are two main ways to model the complete mill, one is an empirically based model which yields high accuracy on a global scale but is not accurate in terms of dimensions and temperature locally between stands[1].

The other approach is a full finite element simulation using a coupled thermo-mechanical model of the mill. This has been shown to yield both good local and global results. However at a high computational cost due to the complexity and non-linearity of the equations [2]. The full finite element approach requires solving a system of:

- A heat transfer partial differential equation (PDE)
- A PDE involving plastic deformations
- A microstructural model

One of the challenges of simulating metal-shaping processes is finding a balance between an accurate solution to the problem and the computational cost of the simulation. In this project, the temperature of steel during the hot rolling process is simulated using Finite element method (FEM) in combination with an empirical model.

Modeling shape rolling is a multi-faceted process. The steel is pressed though a series of rolls that reshape the billet. When the hot steel moves through the steel mill the heat transfers to

the air follows Newton's law of cooling combined with Stefan-Boltzmann's radiation law. When the steel passes the rolls, the plastic deformation generates energy in the form of heat. Water is sprayed during the contact with the rolls together with the direct contact with the roll material, which leads to a higher heat transfer coefficient (α). In this project the heat contribution is calculated using empirical models and then distributed over the steel cross section.

2 Numerical method

The implemented model uses a 2D cross-section traveling along the mill as time progresses. The solution from the previous roll-stand is interpolated as a new initial condition for the next stretch. For the boundary condition, a combination of radiation and convection is used, relative to the surrounding temperature. The deformation energy is locally related to the perceived strain in conjunction with the flow stress of the material. To calculate the flow stress a material model called Johnson-Cook is used to describe how the material behaves under a certain strain, the model will be further explained in Section 2.2.2.

We implement the empirical modelm given in Eq (8). In order to use the model we must also calculate the mean strain rate and the mean flow stress during the deformation process. The Johnson-Cook equation material model needs several material-specific constants to be defined. These were received from a previous research project called OptiRoll [3].

The added heat is then distributed over the steel cross-section in a distribution that tries to mimic a plastic deformation simulation. The distribution pattern can then be studied in order to see how it behaves with time due to diffusion inside the material. With this approach we try to produce a method that is usable through out the whole process of hot steel rolling but still is not computationally expensive.

2.1 Derivation of the heat equation

2.1.1 Heat transfer model

Previous studies show that heat transfer in the rolling direction is negligible and a 2D model is sufficiently accurate for temperature calculation [2]. The generalized Fourier diffusion equation with conduction and a forcing function Q reads as follows.

$$c\rho\frac{\partial T}{\partial t} = -\nabla(k\nabla)T + Q, \tag{1}$$

where k is the temperature-dependent heat conduction coefficient, Q is the temperature-dependent generated heat due to plastic deformations. c is the specific heat and ρ is the temperature-dependent material density.

2.1.2 Boundary conditions

If the heat flux is assumed to be zero then the Neuman-Hankel boundary condition

$$k\frac{\partial T}{\partial n} + \alpha(T - T_0) + q,$$

simply becomes Newton's law of cooling,

$$k\nabla T \cdot \hat{\mathbf{n}} + \alpha(T - T_0),$$

see [4] for details. T_0 can the be set to the temperature of the surrounding environment (rolls, water, air), with α being the heat transfer coefficient between the materials. The coefficient q is the heat flux through the deformation boundary.

According to Stefan Boltzmann's law of black body radiance emittance the radiation amount scales with the surface temperature as T^4 and is described as follows [5]:

$$I_e(T) = \epsilon_e \sigma_b (T^4 - T_0^4). \tag{2}$$

Here I_e is the emittance from a gray body, ϵ_e is the emissivity and σ_b is the Stefan-Boltzman constant. For non-polished steel alloys emissivity vary between 0.7 and 0.8 [6]. We choose to set the parameter σ_b at a average value of 0.75. Including this term into Newton's law of cooling boundary condition leads to:

$$-k\frac{\partial T}{\partial n} = \alpha(T - T_0) + \epsilon_e \sigma_b (T^4 - T_0^4). \tag{3}$$

2.1.3 Determining the heat specific parameters

Thermal heat conduction k, density ρ and the specific heat capacity c_p are all dependent on the temperature of the steel making the PDE non-linear. The temperature dependence is based on experimental measurements meaning it is not a simple analytic function but a series of measured data points. Adaptable functions such as $y = Ax^2 + Bx + C$ or $y = Ax + B$ which fit the data without over-fitting and then the constants A, B, C were tuned using Levenberg-Marquardt optimization algorithm [7]. This method introduces a small amount of error in the material coefficients but also increased the speed of computation while simplifying the implementation.

2.1.4 Variational formulation

In order to solve Eq (4) in FEniCS we must formulate a suitable space and find the variational formulation. The variational form can then be rewritten as a minimization function and solved using Newton-Raphson's method.

$$\begin{cases} c\rho u_t & = -\nabla \cdot (k\nabla \cdot u) + Q & , x \in \Omega \\ -k(\nabla \cdot u)\hat{\mathbf{n}} & = \alpha(u - u_0) + \epsilon\sigma(u^4 - u_0^4) & , x \in \partial\Omega \end{cases} \tag{4}$$

Let $V \in H^1(\Omega)$ and $\hat{V} \in H^1(\Omega)$ and let $v \in V$ be test functions , $u \in \hat{V}$ be trial functions. We can multiply Eq (4) with v and integrate using Greens formula and then impose the boundary conditions to yield the variational formulation.

Find $u \in \hat{V}$ such that:

$$\begin{aligned} c\rho(u_t, v) &= \int_\Omega k\nabla u \nabla v d\mathbf{x} - \int_{\partial\Omega} k\nabla u \hat{\mathbf{n}} v d\mathbf{x} + \int_\Omega Q v d\mathbf{x} \\ c\rho(u_t, v) &= -\int_\Omega k\nabla u \nabla v d\mathbf{x} + \int_{\partial\Omega} (\alpha(u - u_0) + \epsilon\sigma(u^4 - u_0^4)) v d\mathbf{x} + \int_\Omega Q v d\mathbf{x} \quad , \forall v \in \hat{V} \end{aligned} \tag{5}$$

We must now discretize the variational formulation in time in order to implement it in FEniCS. The constants are not isotropic and depend both on space and temperature, which means that Eq (5) is non-linear and must be solved using Newton-Rahpson's method. This means we assume the solution u can be written as $u = u^0 + \delta u$. The solution is then iterated until a sufficiently

small tolerance is achieved, see [8] for details. In FEniCS there is no need to derive the equation in terms of u^0 and δ, instead we express the variational formulation as a minimization problem shown in Eq (7) and FEniCS does the rest of the work.

2.1.5 Discretization in time

The heat PDE can be discretized using implicit Euler. We let u^n denote the solution at the current time step and u^{n+1} denote the solution in the next time step.

$$\begin{cases} (c\rho\frac{\partial u}{\partial t})^{n+1} & = -\nabla(k\nabla)u^{n+1} + Q^{n+1} \approx c\rho\frac{u^{n+1}-u^n}{\Delta t} \\ u^0 = u_0 \end{cases} \tag{6}$$

We insert this into the the variational formulation in Eq (7). In Eq (7) we have also created the minimization function $F(uv) = 0$.

$$\begin{cases} F_{n+1}(u,v) & = \int\limits_\Omega \left(u^{n+1}v - u^n v + \frac{k(u^{n+1})\Delta t}{c(u^{n+1})\rho(u^{n+1})}\nabla u^{n+1}\nabla v + fv\right) d\mathbf{x} \\ & \quad + \int\limits_{\partial\Omega} \frac{\alpha\Delta t}{c(u^{n+1})\rho(u^{n+1})}(u^{n+1} - u_{amb})v d\mathbf{s} \\ & \quad + \int\limits_{\partial\Omega} \frac{\sigma_b\epsilon_e\Delta t}{c(u^{n+1})\rho(u^{n+1})}((u^{n+1})^4 - u_{amb}^4)v d\mathbf{s} = 0 \\ f(u,x) & = 0, \quad \alpha = \alpha_{air} \quad \text{if between rolls} \\ f(u,x) & = Q, \quad \alpha = \alpha_{roll} \quad \text{if in contact with rolls} \end{cases} \tag{7}$$

Here u^{n+1} denotes the temperature solution and u^n denotes the solution in the previous time step. As seen in Eq (7), there are two different boundary conditions applied, one for contact with surrounding air and one for contact with the rolls. This means that two different partial differential equations must be solved.

2.2 Approximating deformation and deformation energy Q

To properly describe the heat transfer behavior we need a way to approximate the rate of generated heat due to the released deformation energy during the rolling process. One such suggested expression is [9]:

$$Q = \chi\frac{\bar{\sigma}\bar{\dot{\epsilon}}}{\rho(u)C_p(u)}, \tag{8}$$

where χ represents the fraction of plastic deformation work that is converted to heat, $\bar{\sigma}$ is the mean flow stress and $\bar{\dot{\epsilon}}$ is the mean strain rate experienced during the rolling.

In order to find Q we must first determine $\bar{\dot{\epsilon}}$ by calculating the expected deformation. We also implement a material model to find the mean flow stress parameter $\bar{\sigma}$.

2.2.1 Strain and contact time with rolls

We used pre-calculated dimensional changes from a mill setup using the process design tool Wicon which uses a modified Wusatowski spread formula [10] and Lendl's method [11] to approximate the dimensions with equivalent rectangles [12].

With elastic and shear strain neglected, we can assume the volume to stay constant meaning the changes in width and height, due to plastic deformation, are proportional to the increase

in elongation, as described in [13].

$$\begin{aligned} \epsilon_3 &= -\epsilon_1 - \epsilon_2, \\ \epsilon_1 &= \ln(\frac{W_i}{W_p}), \\ \epsilon_2 &= \ln(\frac{H_i}{H_p}). \end{aligned} \tag{9}$$

In Eq (9), W_i, H_i are the initial width and height dimensions and W_p, H_p are the dimensions after the roll pair interaction (called pass). These closed-form approximations in the principal axis can be expressed as a total mean strain in a a pass:

$$\bar{\epsilon_p} = [\frac{2}{3}(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)]^{1/2}. \tag{10}$$

We can retrieve the average strain rate as the total strain over some time interval related to the pass:

$$\bar{\dot{\epsilon}} = \frac{\bar{\epsilon}}{t_p}. \tag{11}$$

For this we need an expression for the time interval and the estimated contact length $\bar{L}$:

$$t_p = \frac{60\bar{L}}{2\pi N R_{eff}}[\text{sec}], \tag{12}$$

$$\bar{L} = \sqrt{[R_{max} - \frac{H_p - G}{2}](H_i - H_p)}. \tag{13}$$

R_{eff} is the effective roll radius, R_{max} the maximum roll radius, N is the roller rotations per minute at the pass.

2.2.2 Johnson-Cook material model

The Johnson-Cook model presumes that the flow stress obeys a simple power law relationship. The model is completely empirical and the constants have been determined using experimental data [14],

$$\sigma = (A + B\epsilon^n)\,(1 + C\ln\dot{\epsilon})\,(1 - (T^*)^m)\,. \tag{14}$$

The constants A, B, C, n and m are optimized values and have been obtained at Morgårdshammar AB from previous research in a project called OptiRoll [3]. T^* is related to the melting temperature of the steel,

$$T^* = \frac{T_{mean} - T_{amb}}{T_{melt} - T_{amb}}.$$

2.2.3 Distribution of deformation energy

The deformation is not distributed uniformly over the domain. The added deformation energy depends on the local strain in the cross section. For an accurate distribution pattern we would need to solve the full thermo-mechanical model as suggested by [2] and [1]. We choose to distribute it as a simple pattern in order to study how quickly it diffuses away inside the material.

A very simple distribution depending on the angle θ is defined in Eq (15). Here $\theta = \pi/2$ and $3\pi/2$ is equivalent with the contact point of the top and bottom rolls,

$$P(\theta) = \sin(\theta) * \cos(\theta). \tag{15}$$

The angular distribution can then be combined with a 2D Gaussian function shown in Eq (16). The two distributions are then scaled and the deformation energy is multiplied distributed over the domain,

$$A \cdot \exp(-B(x^2 + y^2). \tag{16}$$

Combining Eq (15) and Eq (16) yields the distribution shown in Figure 1. Eq (15) and (16) must also be normalized before the scalar deformation energy Q_{def} is multiplied with the distribution, otherwise the added energy will not be correct when projected and added onto the solution.

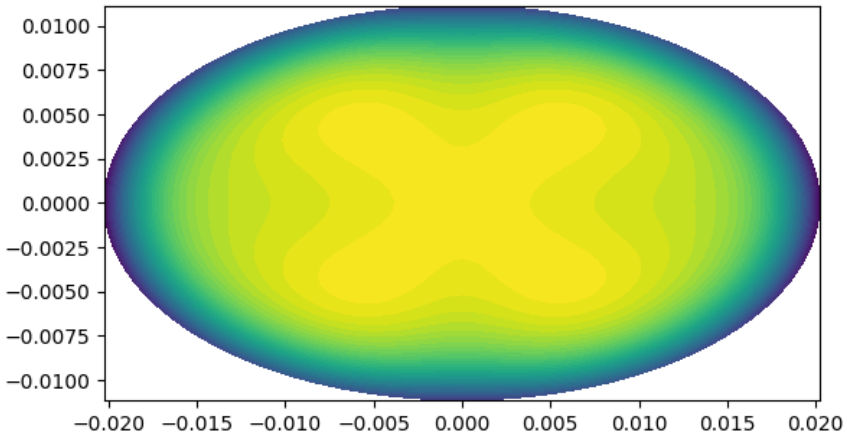

Figure 1: Plot showing the distribution of the deformation energy Q after a short amount of time.

2.3 Method of implementation

2.3.1 Interpolation of solution

When the steel is reshaped by the rolls the old heat distribution must be refitted to a new mesh. We solve this problem using the Levenberg-Marquardt optimization algorithm to fit the heat distribution to a Gaussian function. The new solution is then tuned to give a consistent mean temperature. To simplify the function fitting we interpolated the solution before we add the heat from the deformation energy Q. When studying how the implemented model behaves it can be seen that if sufficient time has passed the diffusion inside material in conjunction with the boundary conditions will lead to a Gaussian distribution.

The purpose of interpolating using this method is to keep accuracy in terms of the mean temperature, while also retaining information about the surface temperature. Examining Figure 2 it can be seen that the total area is decreased but the width has increased. This means it exists spatial coordinates outside of the region of the previous mesh, other examined methods lead to large errors in this region.

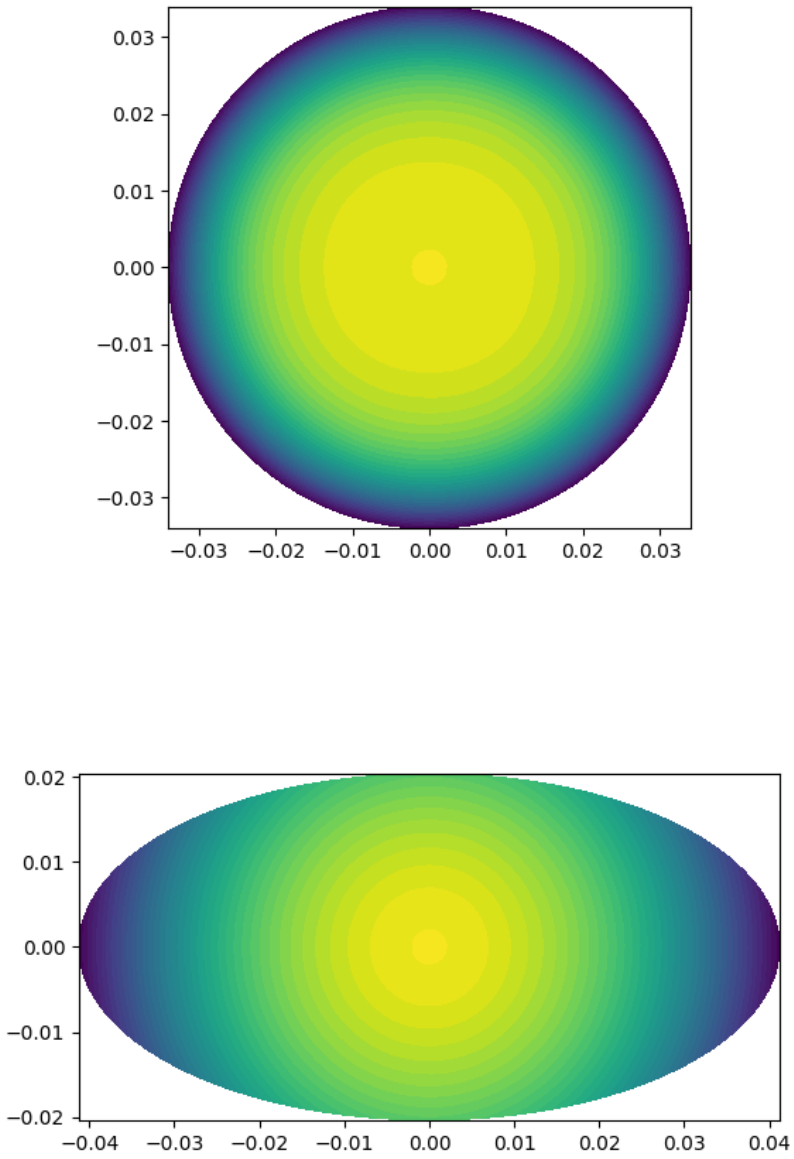

Figure 2: Plot showing the remeshing of a solution from a round to oval shape. The dimensions of the axis are in meters.

2.3.2 Convergence

It is preferred to prove convergence of the implemented model by using the Method of Manufactured Solution (MMS). We had problems finding a suitable analytic solution to our non-linear problem. Instead we opted to test the implementation using one time-dependent linear MMS and one time-independent non-linear MMS. Both methods showed satisfactory convergence.

Furthermore, we wanted the model to show a rate of convergence when increasing the number of nodal points and/or increasing the number of time steps. One way to test this is to compute the tolerance between the previous solution and the next with increased number of time steps or number of nodal points as seen in Eq (17). The measured convergence is presented in Tables 1 and 2.

$$|u_{h-1} - u_h| \tag{17}$$

Table 1: Convergence when increasing number of time steps

nr. time steps	10	25	50	100
tol	5.293	0.403	0.527	0.205

Table 2: Convergence when the number of nodal points is increased

nodal points	32	128	512	2048	8192	32768	131072
tol [C]	10.070	2.527	2.522	1.851	0.454	0.391	0.198

Here, the tolerance denotes the pass with maximum tolerance. The tables were generated by running a series of four passes with large dimensions. When increasing from 25 to 50 the tolerance increased a small amount, we were not able to find the root cause for this behavior.

3 Results

The model focuses on computing the temperature of the steel cross-section, it computes the added deformation energy but does not compute the actual plastic deformation. Instead, we used data from a real mill using the software tool Wicon, which contained parameters needed to perform the calculations described in Section 2. Wicon is a rolling simulation software developed at Morgårdshammar AB [12].

Two sets of simulations were conducted each for two types of steel, one called 803J and one called 330CB. The first simulation was early rolling process when the speed of the material travels slower and its dimensions are larger, initial size was 102×56[mm] and final size 63×35[mm] with the speeds varying between 0.4 and 0.8[m/s]. The second case was later in the rolling process where the speed is higher and dimensions are smaller, initial size was 34×33[mm] and final size 22×22[mm] with speeds between 1.9 and 3.6[m/s].

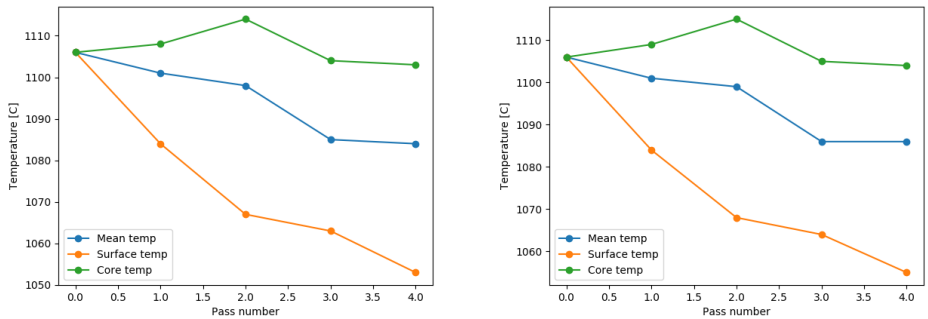

Figure 3: Plot showing the temperature of steeltype 803J(left) and 330CB (right) when the dimensions of the steel is large.

When the dimensions are large we can see in Figure 3 that the surface temperature quickly lowers its temperature while the core and mean has a relative stable temperature. This behavior is apparent in both steel types. We can see that its the heat equation that is the dominating factor during this part of the rolling.

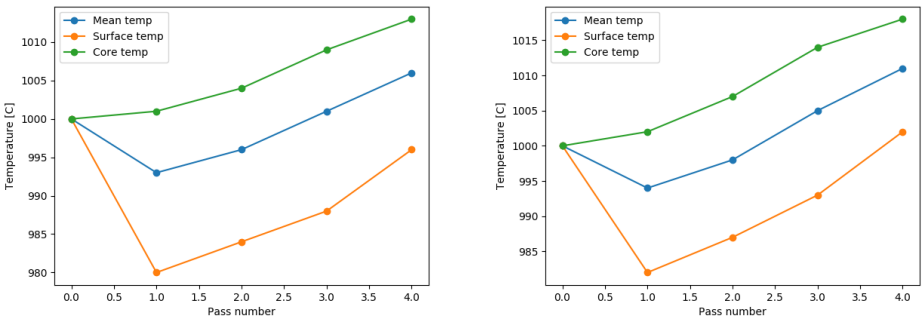

Figure 4: Temperature for for 803J (left) and 330CB (right) when the dimensions of the steel are small.

In the later stages of rolling the temperature rises due to a larger energy addition from the deformation. This can be seen in figure 4. This means that the deformation energy is the dominating factor. The drastic decrease of temperature between the start of the simulation and the first pass is due to the long waiting time before the steel comes in contact with the first roll pair. This is due to the design of the modeled mill.

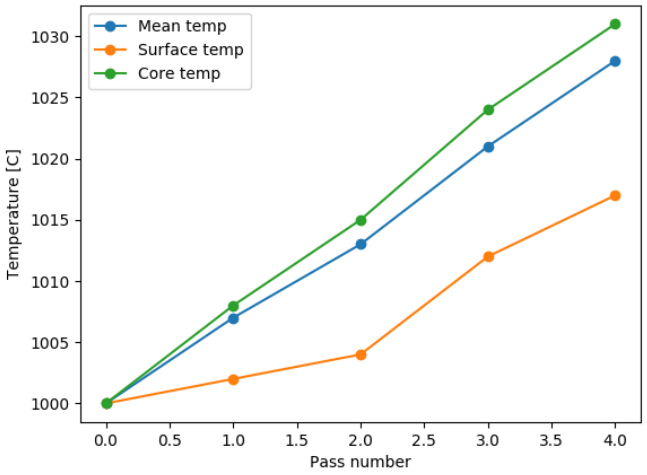

Figure 5: Plot showing the temperature of steeltype 803J with small dimensions when there is very little downtime between rolls.

Evaluating the accuracy of the model is difficult. We would have preferred to compare our results with a full thermo-mechanical model. We had no data from such a model, so comparison was not possible. The characteristics of the model were compared with results from Wicon and a program called RollSim FE developed in the OptiRoll project [3], and was shown to be in agreement even though the exact solutions was not comparable due to the difference in approach between the three models. The Wicon solutions showed a similar behavior to our computation

of the surface temperature when the speeds were slower, while the RollSim FE solutions showed similarities when the speeds were higher and dimensions smaller.

4 Discussion

During the later stages in steel rolling the deformation energy is the dominating factor for the temperature of the steel. This means that to get accurate results for these stages in the rolling one should use a coupled thermo-mechanical model. In the early stages of hot steel rolling the heat equation is the dominating factor. Therefore, a model that solves the heat equation using a finite element solver with a properly tuned empirical model to determine the deformation energy temperature should give sufficiently accurate results. However, further study of thermo-mechanical simulations must be done in order to verify this approach conducting proper comparison of errors. A further note, the distribution is symmetric, as shown in Figure 2, and therefore one could solve the problem for a quarter of the cross-section to save computational time. This optimization was omitted for easier boundary treatment.

5 Summary and conclusions

This project examines a new method to simulate the temperature of steel during hot steel rolling. The method uses a combination of FEM to simulate the heat emission and empirical Johnson-Cook method to simulate the heat generation from the deformation. The results indicate, that the method is viable for the earlier stages of hot steel rolling when the heat equation is the dominating factor.

The interpolation of the results to initial conditions for the next pass needs improvements. In reality it is expected that the surface temperature on the north and south boundaries are to be maintained. More work can be done by studying results from more advanced models and optimizing the distribution parameters. Also the simple distribution of deformation energy could be improved by training a distribution pattern from thermo-mechanical simulations.

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