UPPSALA UNIVERSITET

Summation-by-parts in Time

A stable second-derivative formulation in time with SBP-P

The Problem

A higher-order finite difference SBP implementation on a second-order initial-value-problem (IVP) has proved to be a difficult numerical problem. The aim of this project is to combine SBP operators with the Projection method to yield an A-stable and potentially efficient implementation on stiff problems on the form:

$$u_{tt} + Au_t + Bu = F(t), \qquad t > 0,$$
$$u = f, \qquad u_t = g, \qquad t = 0.$$

Results and Discussion

A stable SBP-P discretization was successfully implemented. Convergence and efficiency were studied for a stiff IVP, where *A* was set much larger than *B* in the aforementioned equation.

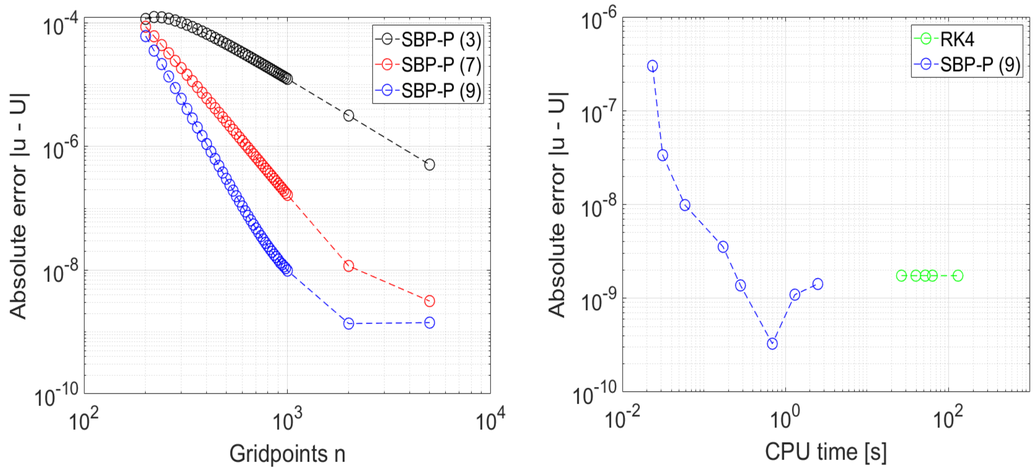

Figure 1. Analysis of convergence (left) and efficiency (right) using SBP-P on a stiff IVP.

The method was extended to initial-boundary-value-problems (IBVP) and implemented on the dynamic beam equation:

$$u_{tt} = \alpha u_{xxxx}.$$

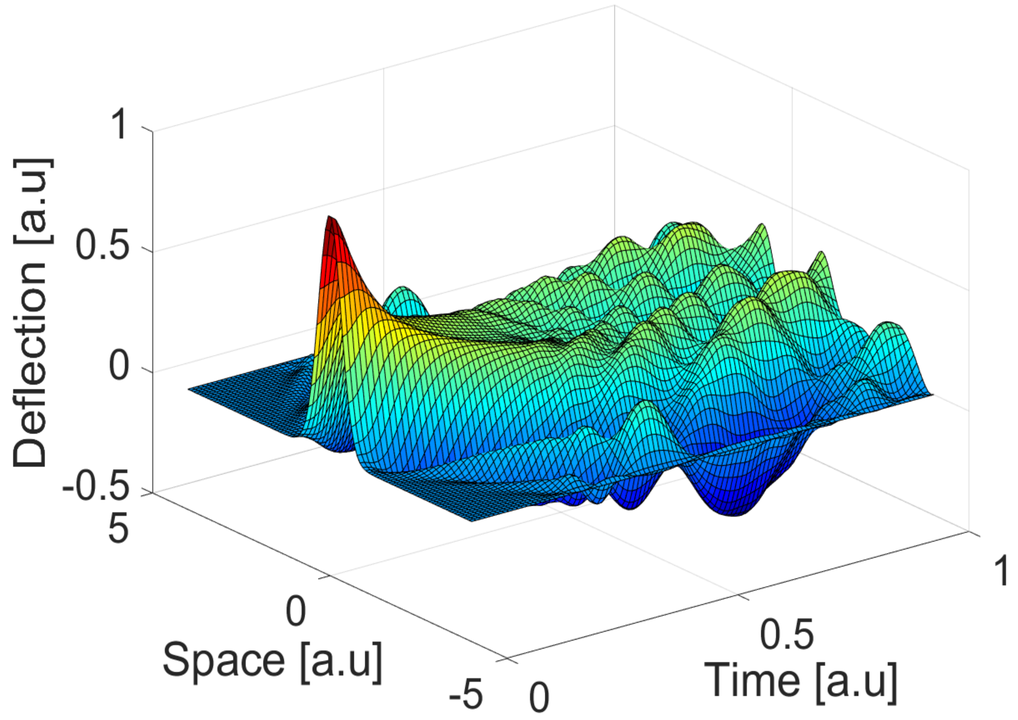

Figure 2. Numerical solution to the dynamic beam equation using the SBP-P method in both space and time.

Future Work

- Optimize performance with multistage implementation.
- Study non-linear coefficient problems.
- Extension to higher dimensional IBVP.

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