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Predicting the Steady State Solutions of the 1D Shallow Water Equations

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Introduction

Geological processes evolve over thousands of years, making simulations time-consuming. Examples of such processes are water flows in rivers and channels. Here we investigate the possibility of predicting the steady state solutions of the 1D shallow water equations, by training a neural network on data generated by a numerical solver.

Work process

PDE
- Problem formulation
- Shallow water equations

FDM
- MacCormack finite difference scheme
- Produce training data set

ANN
- Feedforward neural network
- Trained to speed up solving process

The 1D shallow water equations

PDE

$$\begin{cases} h_t + (hu)_x = 0, \\ (hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = -gh\left(b_x + S_f\right). \end{cases}$$

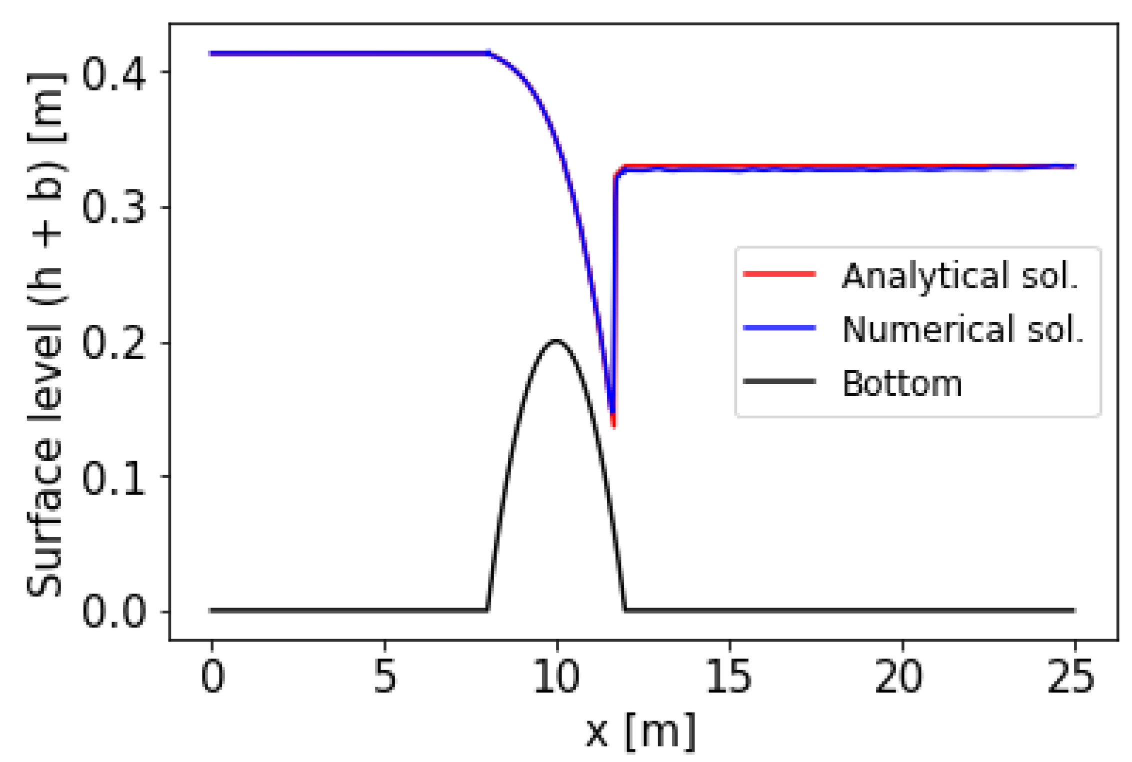
- h - water depth
- u - average water velocity
- hu - discharge or flow
- g - gravitational acc.
- b_x - slope of bottom
- S_f - bottom friction

The equations are a system of hyperbolic PDEs representing conservation of mass and momentum. We want to solve for the water depth and the discharge. The solution waves are driven by gravity and depend on the slope of the bottom and friction from the bottom.

Numerical solver

FDM

A TVD-MacCormack scheme is used to solve the equations numerically. The solver is verified on test problems with known analytical solutions.

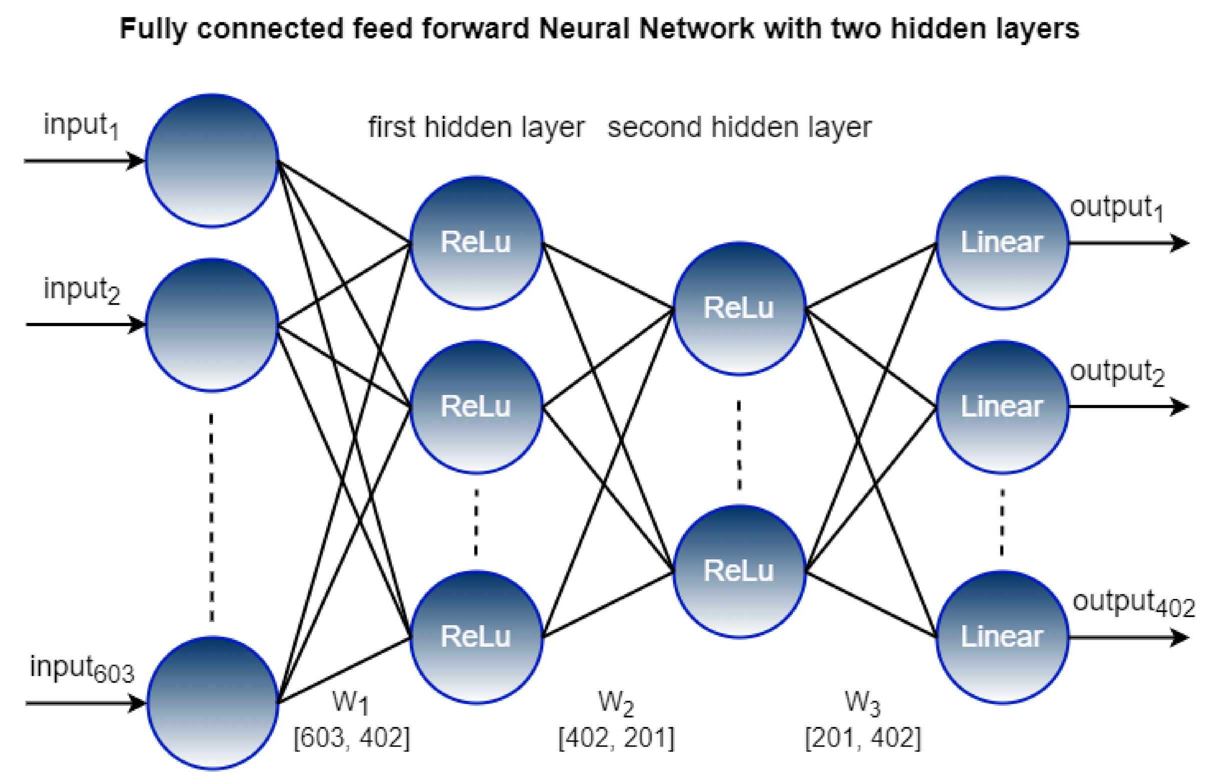


The scheme is able to capture the shock occurring after the hump. The shock is a result of the initial condition and the bottom topography.

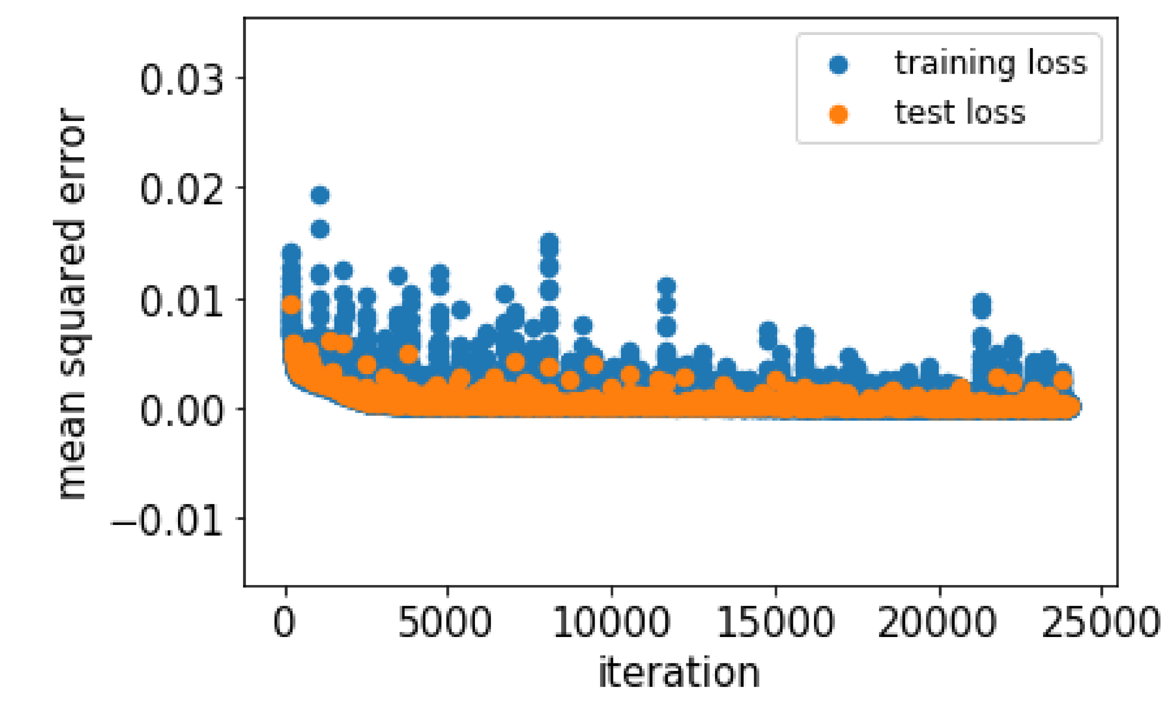
Training the neural network

ANN

The numerical solver is then used to generate steady state solutions given more complex bottom topographies. The complete dataset contains 2700 different bottoms and corresponding steady states. Different initial conditions are used in each simulation.



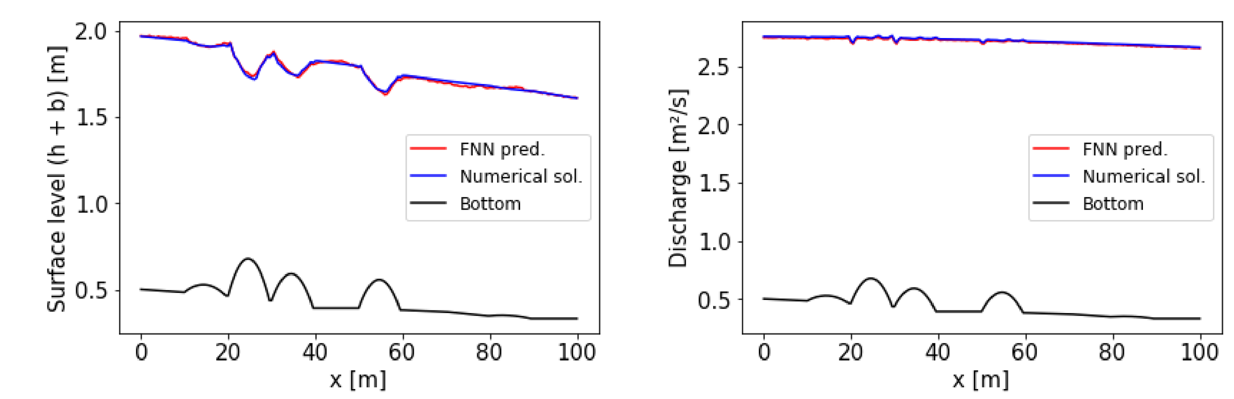
The network takes the bottom slope, initial discharge and water depth as input and returns a prediction of the steady state water depth and discharge. The grid uses 201 datapoints which is why the input dimension is 603 and the output dimension is 402.



The loss of choice is the mean squared error and the total training time is 8 minutes using the dataset containing 2700 different topographies.

Results

The network predicts the steady state with good accuracy. The steady state solution consists of the water depth and the discharge. Predictions by the neural network are instant while the numerical solver has to iterate for some time in order to reach a steady state.



Conclusions

The result shows that the feedforward neural network is successful in reproducing the solutions of a standard numerical solver. This implies that there is a potential time-save to be made in avoiding long simulations and instead predicting the steady state solutions instantly, with just information about the bottom topography, boundary conditions and initial conditions. However, this is provided that enough training data is available. In this project we have focused on a very specific type of setting and in order to generalize the problem a tremendous amount of data would be needed. The amount of time needed to generate such data is a major drawback to this approach.

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