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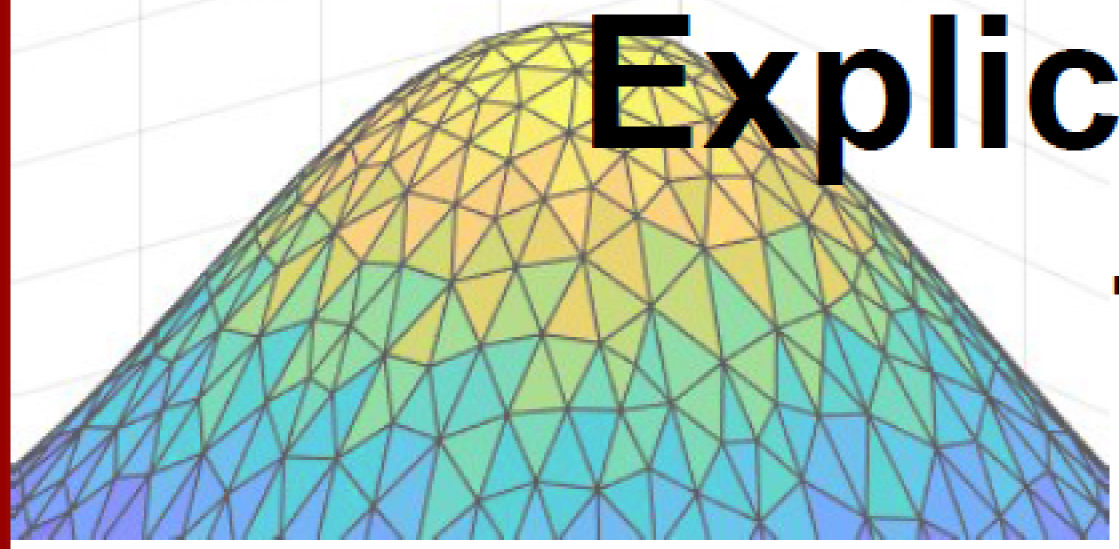

Explicit Time-step Analysis of Methods for the Advection Equation

Project in Computational Science
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Summary:
Stability analysis of higher order finite element methods and finite difference methods with central scheme is used to solve the advection equation in 1D. An investigation of stabilization techniques and discontinuous Galerkin are done.

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0, & x \in [0,1], t > 0, \\ u(x+1,t) = u(x,t), & x \in [0,1), t \geq 0, \\ u(x,0) = u_0(x), & x \in [0,1) \end{cases}$$

Advection equation in 1D

Aim:

- Investigation on how the maximum stable time-step depends on choosing the polynomial interpolation points.
- Examining the impact on the maximum stable time-step when the order/degree of the methods is increased.
- Understanding the effect of using stabilization techniques on the time-step.

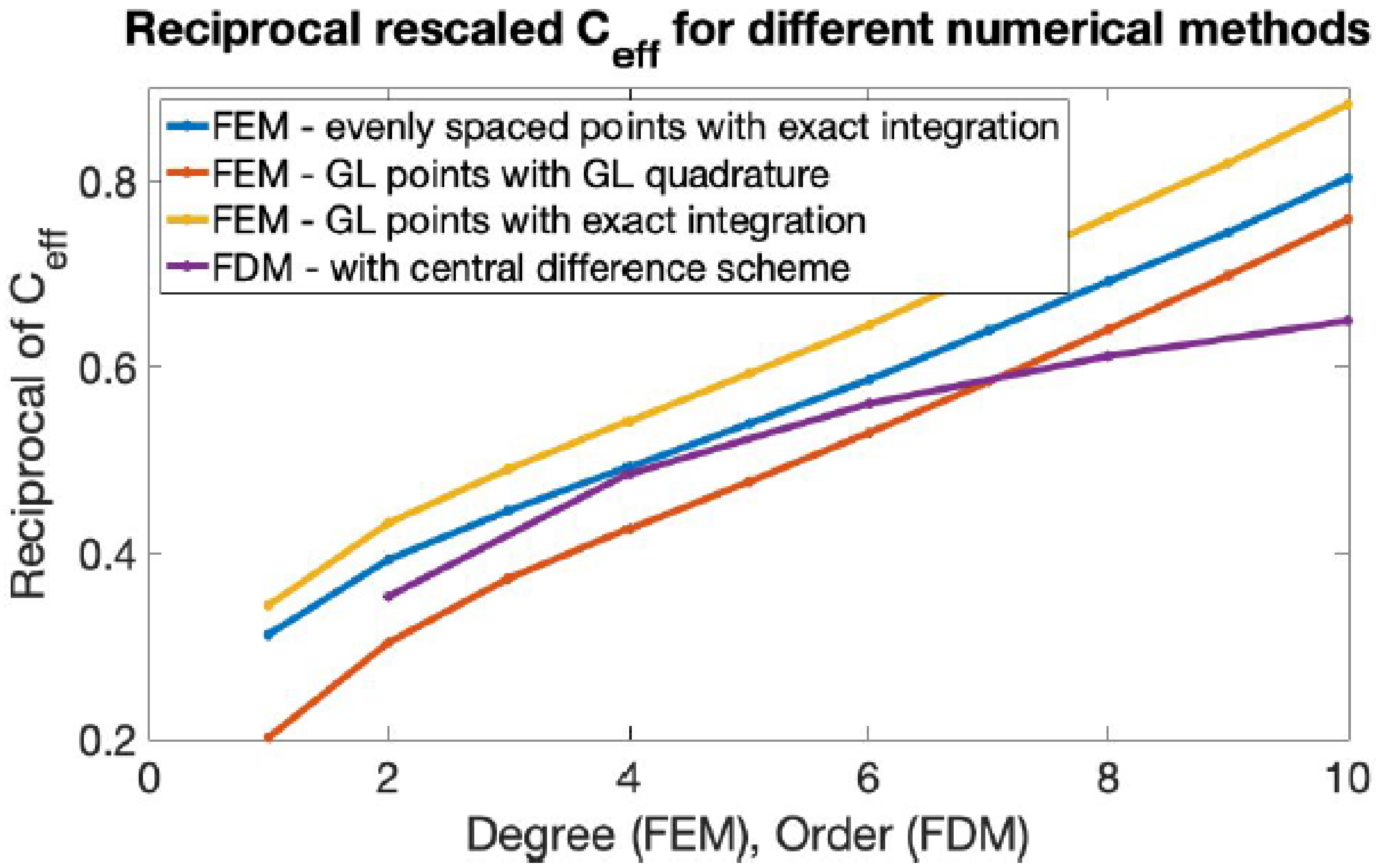

Figure 1: The C_eff number for different methods solving the advection equation without stabilization.

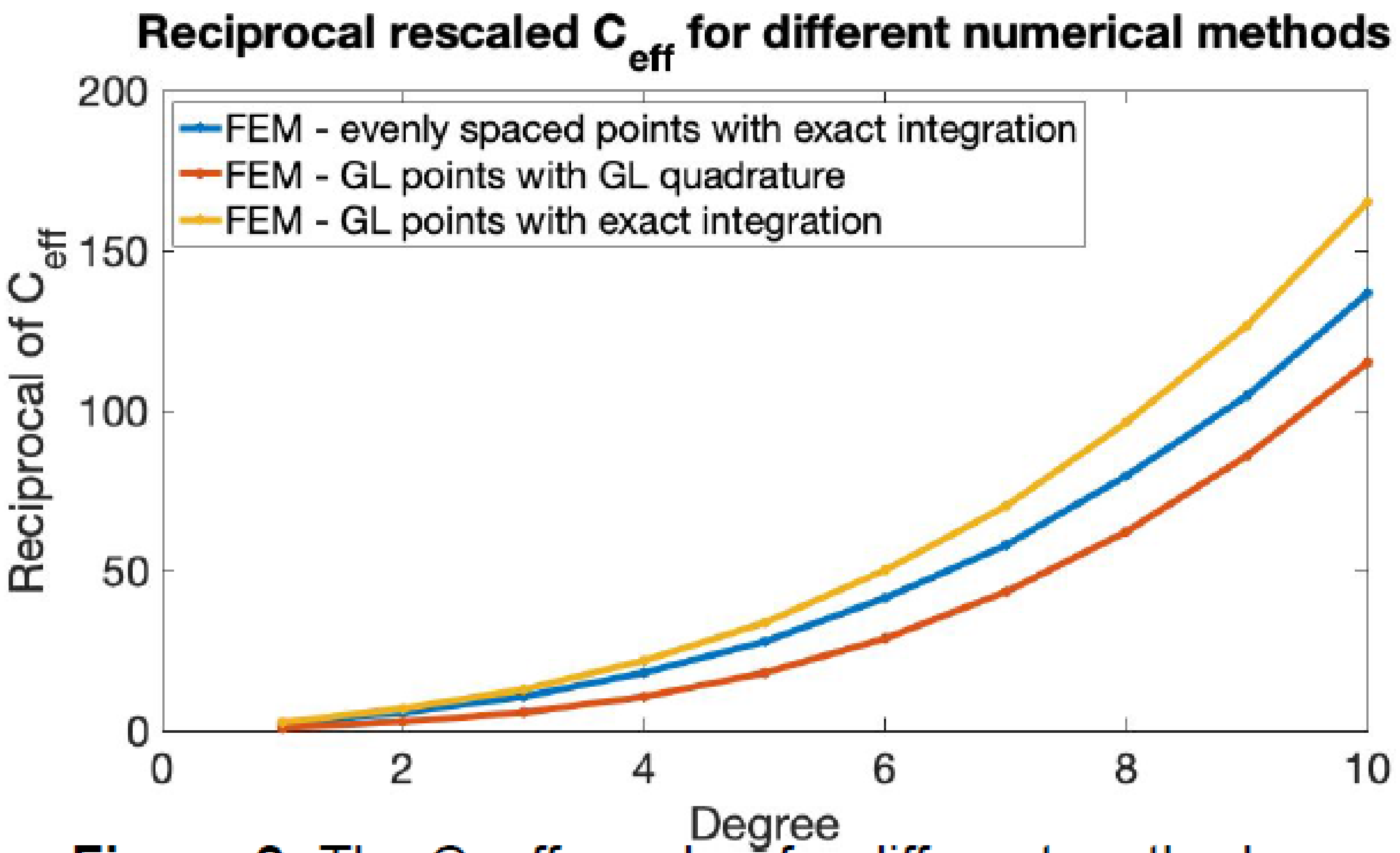

Figure 2: The C_eff number for different methods solving the advection equation with stabilization.

Conclusion:

- A linear behaviour appeared when no stabilisation technique was used, while a quadratic relation is obtained when using a stability technique (artificial viscosity method).
- The restrictions on the time step increase with the order/degree and there are more restrictions on the time step for a problem with a stability term.

Future work:

- Applying the methods for the advection equation in multiple dimensions.
- Further increasing the order/degree of the methods.