



Goal-Oriented Adaptive FEM for Option Pricing with Duality-Based A Posteriori Estimates

Introduction

To have the right to buy or sell one unit of a stock at a fixed price, that is prespecified within a particular period is what defines an option.

To price these options, the Black-Scholes (BS) PDE is solved, in our case the 2D spread option.

Important requirement:
Since the price may change quickly the pricing must be both fast and accurate.

Method

Finite element method, combined with goal-oriented mesh refinement.

The area of interest is around point $(s_1, s_2) = (100, 100)$, which is the current value of the stocks.

Solution

The solution is similar to the terminal condition, but where the discontinuity is diffused. The top right corner is more diffused compared to the bottom left.

$$\frac{\partial u}{\partial t} + \sum_{i,j=1}^2 \sigma_i \sigma_j \rho_{i,j} s_i s_j \frac{\partial^2 u}{\partial s_i \partial s_j} + r \sum_{i=1}^2 s_i \frac{\partial u}{\partial s_i} = ru$$
$$u(s, T) = \max(s_1 - s_2 - K, 0)$$

The coefficients:

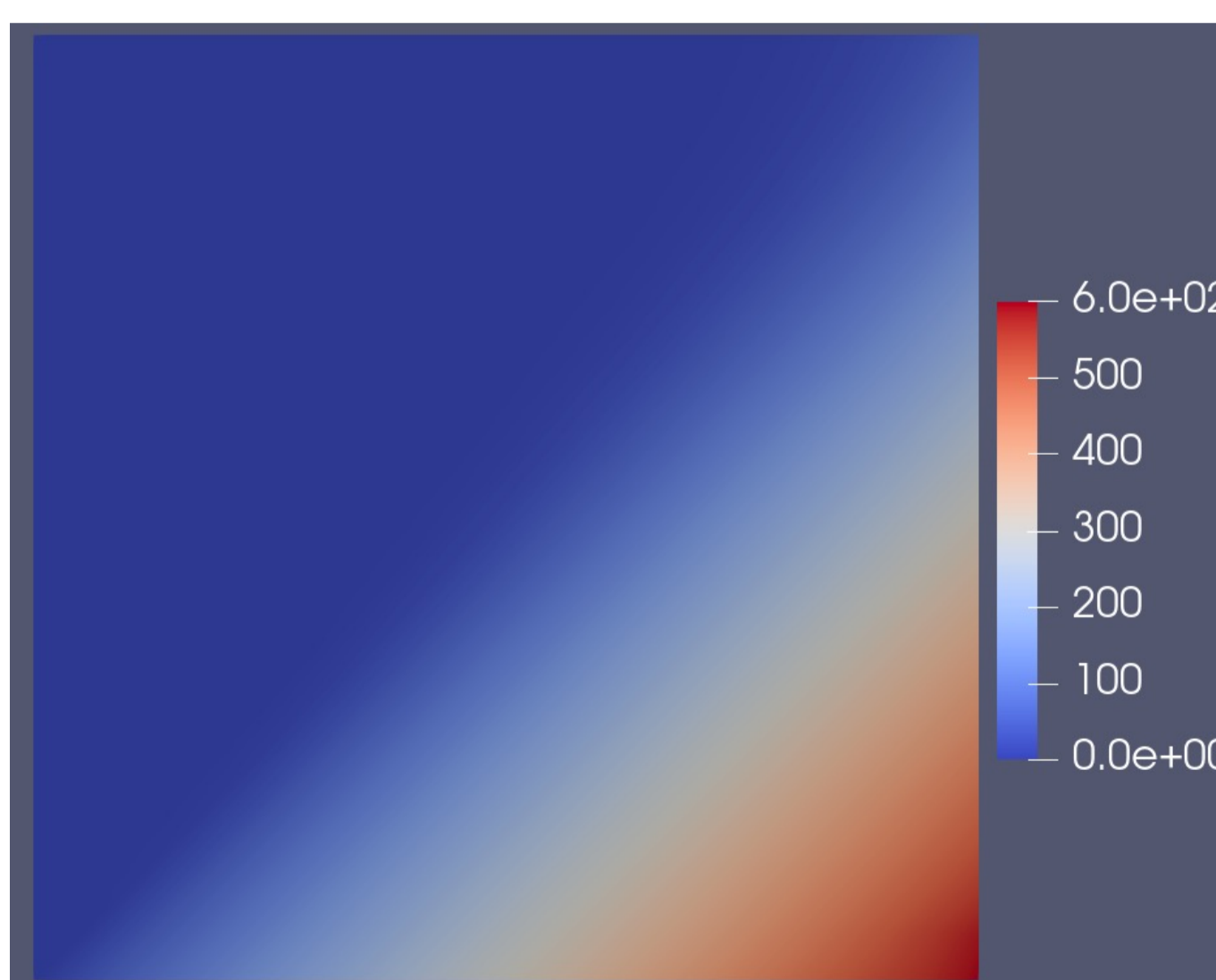
- u - Option price
- σ_i - Volatility of stock i
- $\rho_{i,j}$ - Correlation of stock i and j
- r - Interest rate
- K - Strike price
- s_i - Price of stock i
- T - Maturity time

Mesh-Refinement

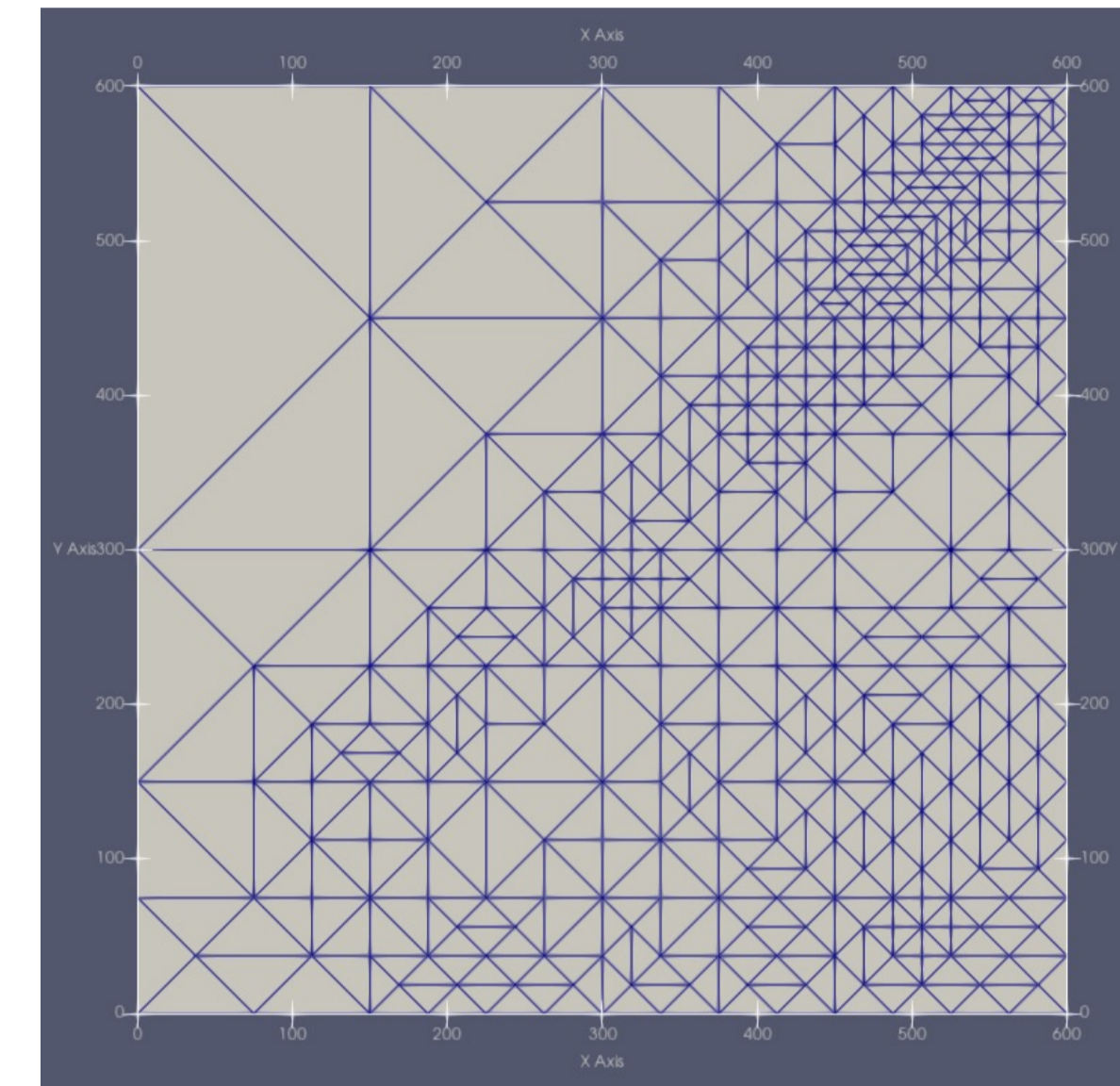
Four mesh refinement techniques were tested:

- Residual-based refinement (RR)
- Uniform refinement (UR)
- Local refinement (LR)
- Goal-oriented refinement (GR)

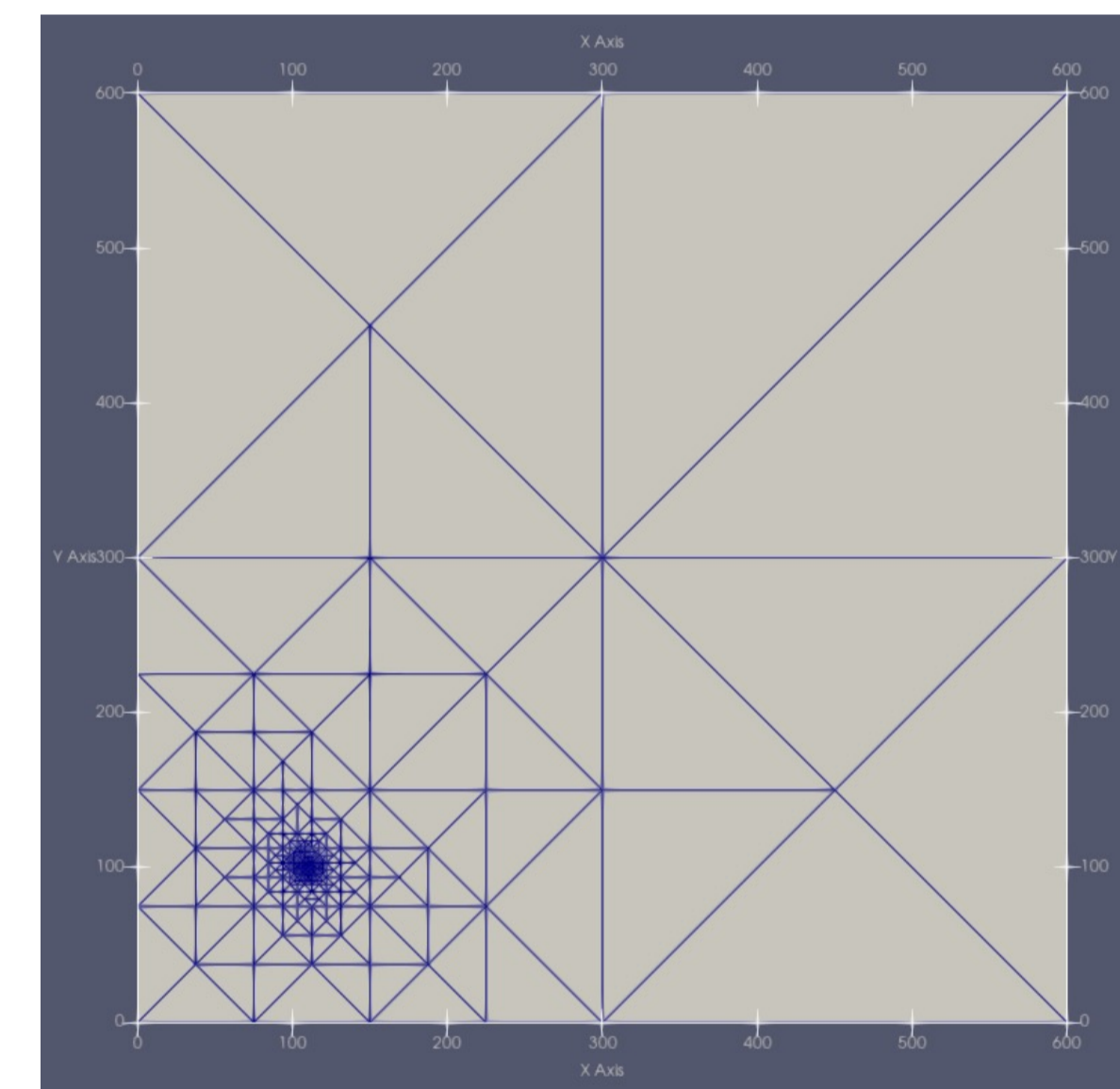
The local refinement only refines around the interesting area.



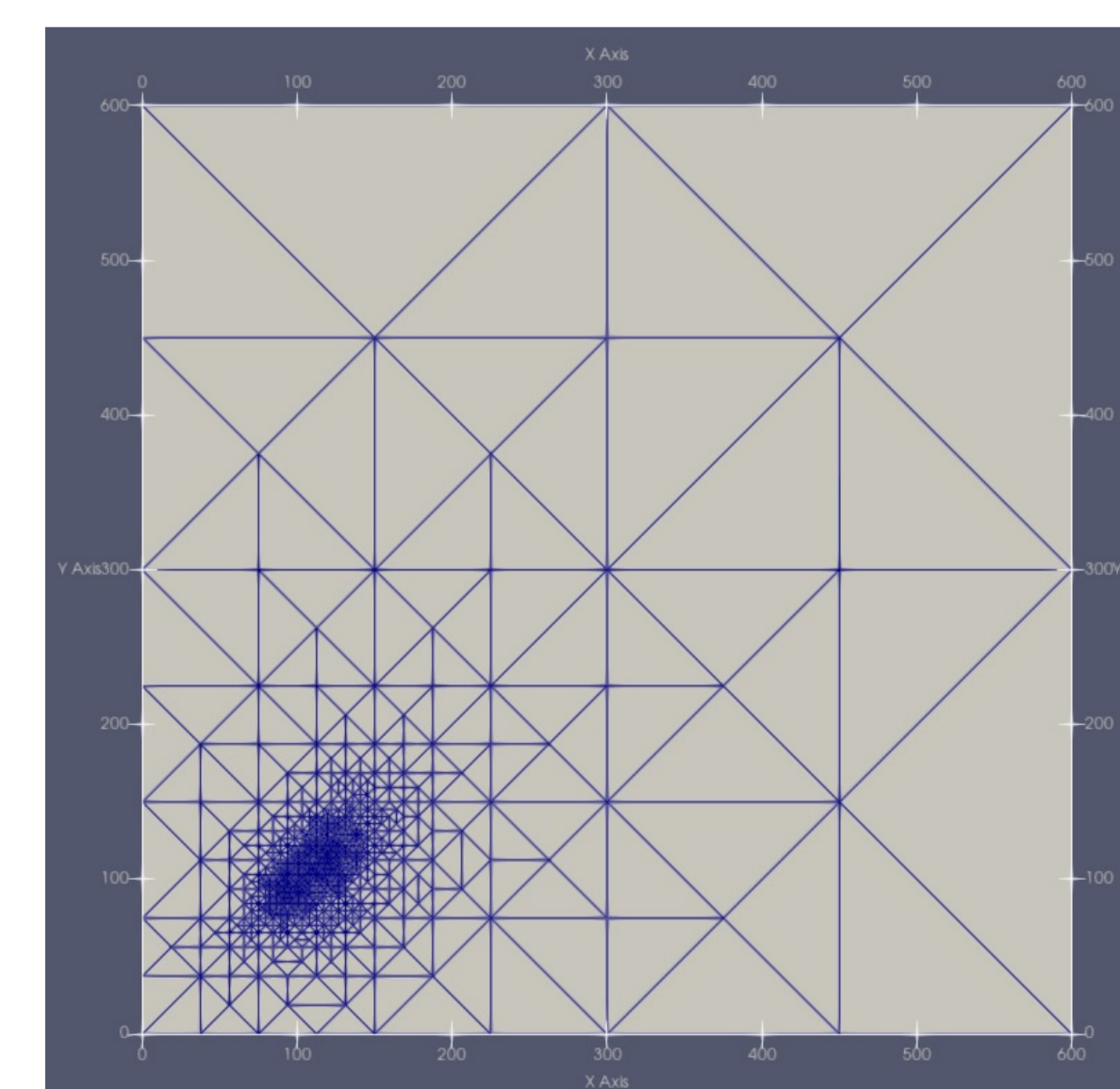
The solution, $u(s_1, s_2)$



Residual-based refinement



Local-based refinement

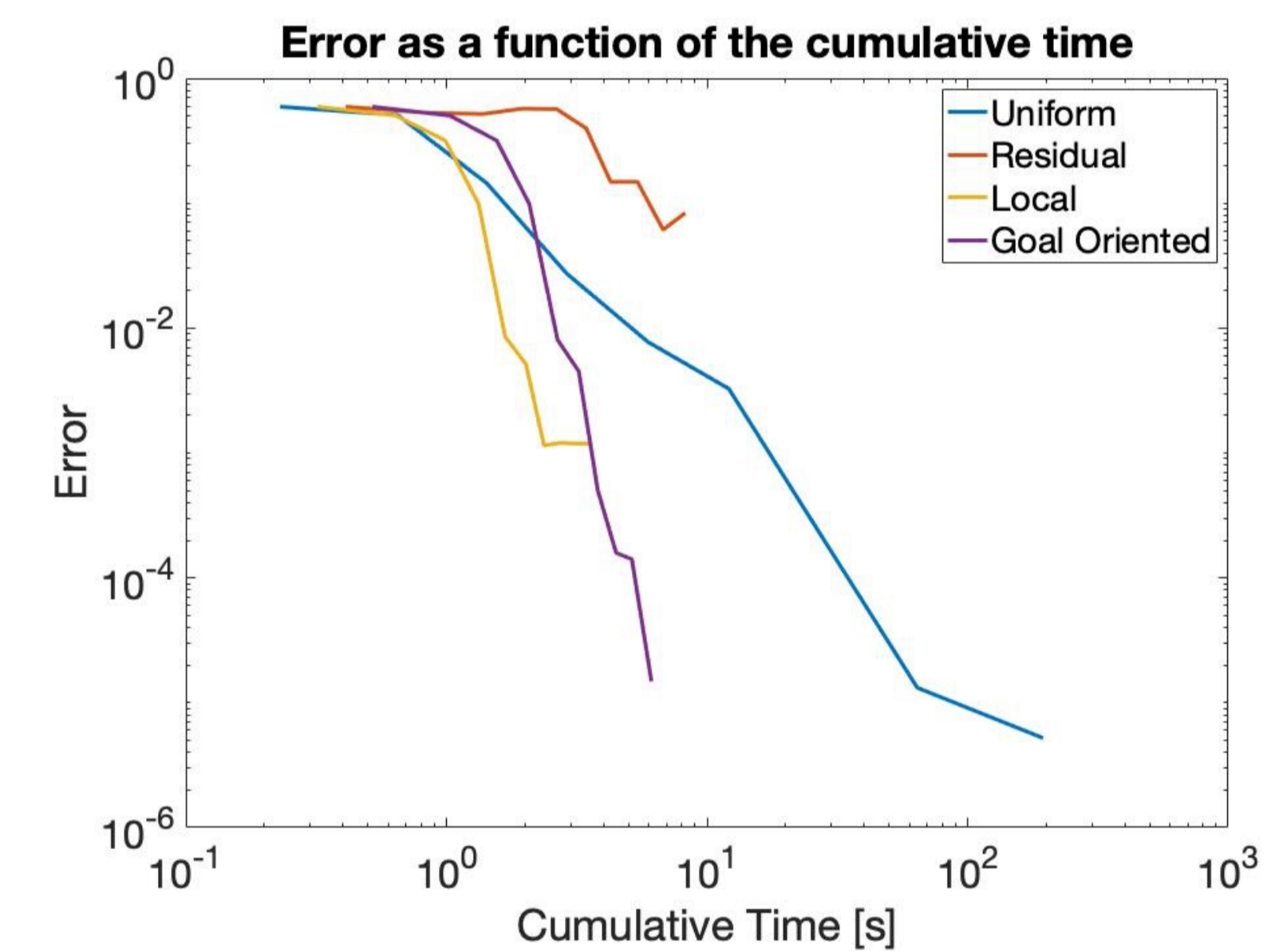


Goal-oriented refinement

Accuracy vs. Time

- GR: Fastest convergence
- LR and RR: No convergence
- UR: Converges, but slow

The residual is not large at the interesting area and thus will not be refined enough for RR. GR is 10x faster than UR.



Error vs. time

Additional Investigations

More investigations in the report

- 3D
- Non-linear BS
- Basket options
- Different correlations
- Proof of well-posedness
- Comparison to other papers