UPPSALA UNIVERSITET

Divergence Cleaning Methods in MHD

Magnetohydrodynamics

The MHD equations describe the behavior of conducting fluids and stems from combining the Euler and Maxwell's equations. The equations can be used for modelling, for example, plasma in fusion reactors.

AIM: reduce divergence errors

Numerical methods introduce errors to the divergence-free condition in the magnetic field B.

$$\nabla \cdot B = 0$$

Divergence cleaning methods

Projection

$$\Delta\psi + \nabla \cdot B = 0$$

Pseudo-time-stepping

$$\partial_\tau \psi - \Delta\psi + \nabla \cdot B = 0$$

$$\partial_t \boldsymbol{B} + \nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{B} - \boldsymbol{B} \otimes \boldsymbol{u}) + \nabla \cdot \psi = 0$$

Generalized Lagrange Multiplier

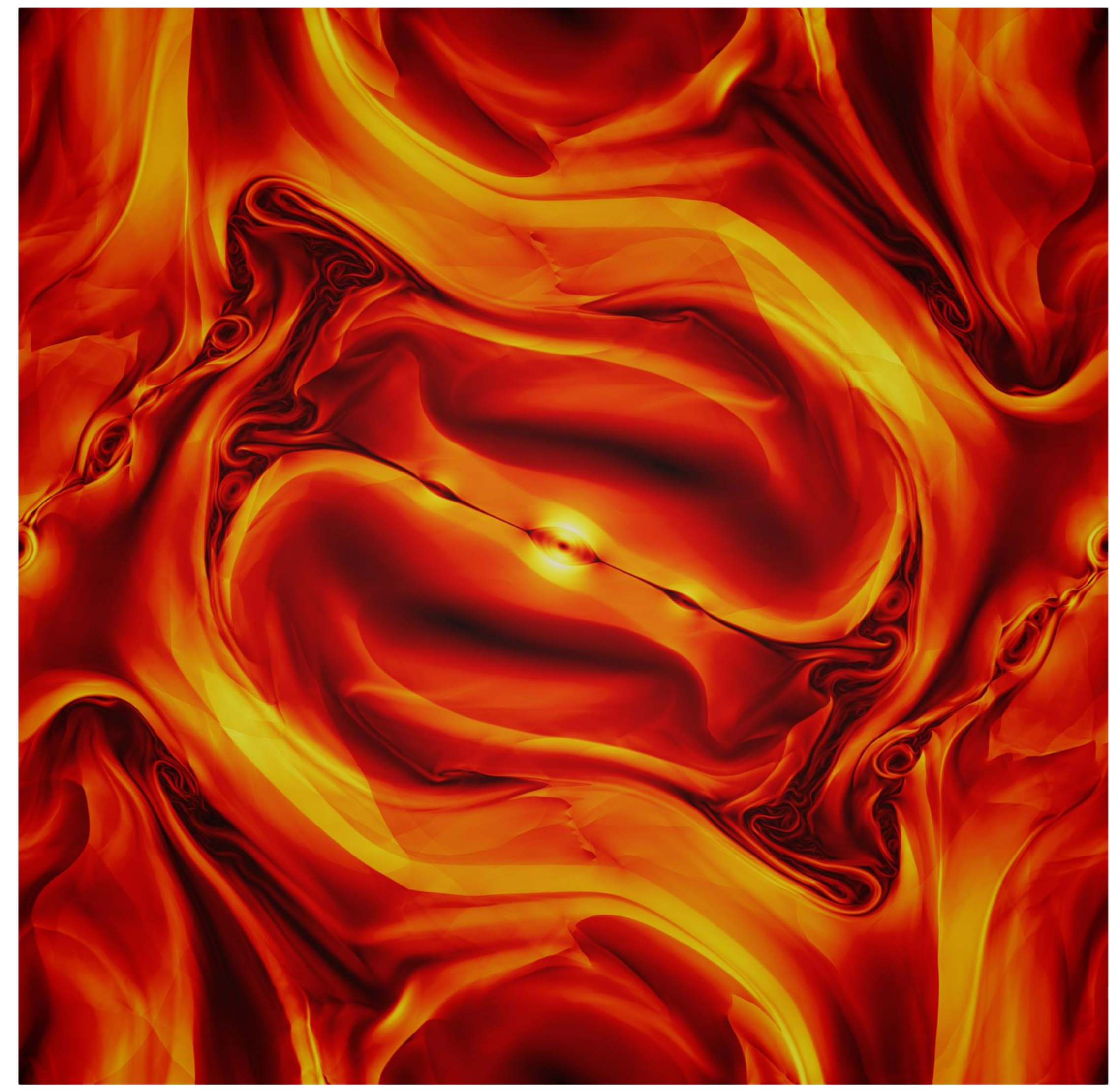
$$\partial_t \psi + c_h \nabla \cdot \boldsymbol{B} = -\frac{c_r c_h}{h_h} \psi$$

Artificial Compressibility

$$\partial_t \psi + \lambda \nabla \cdot \boldsymbol{B} = 0$$

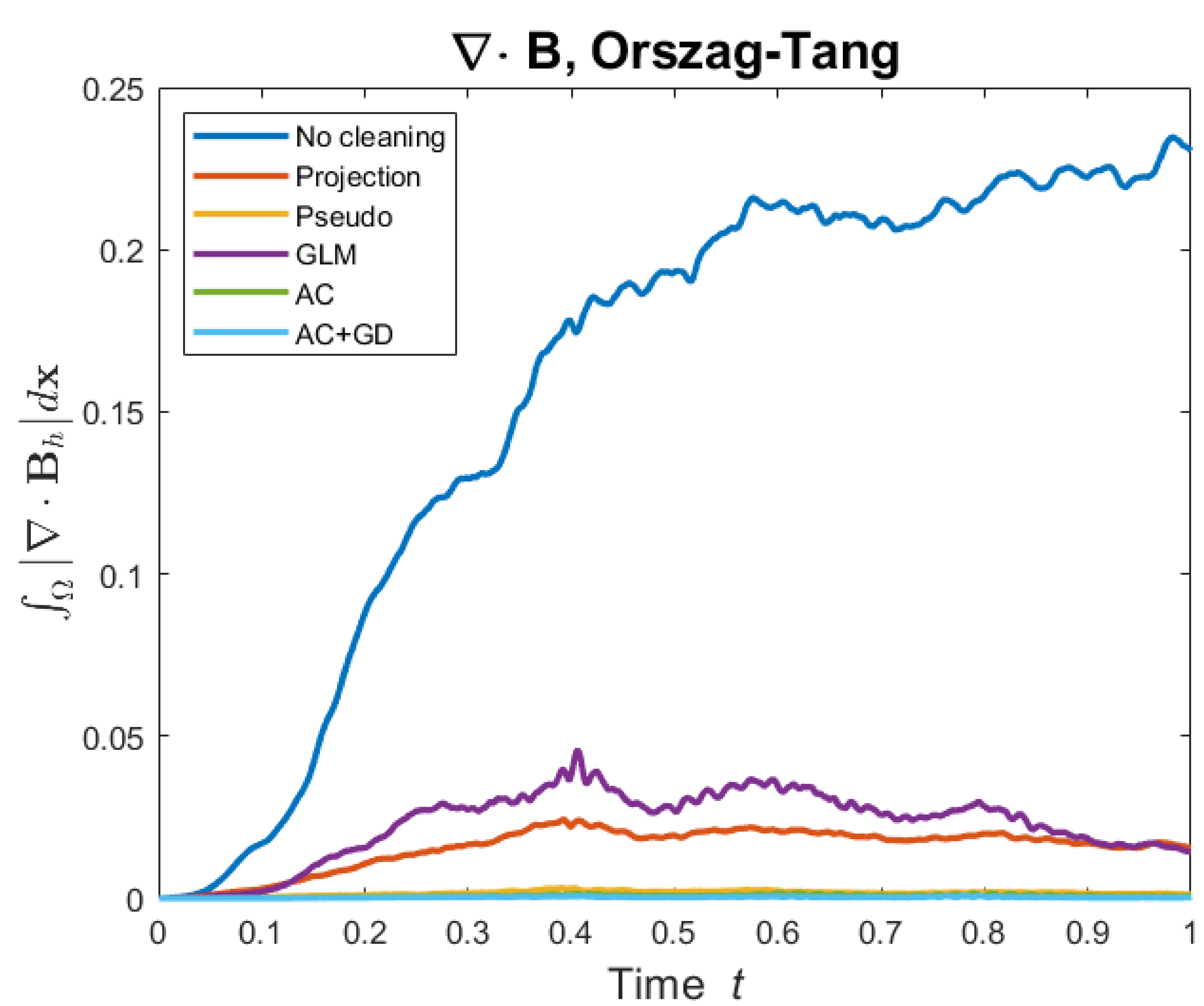
Grad-Div (GD) stabilization

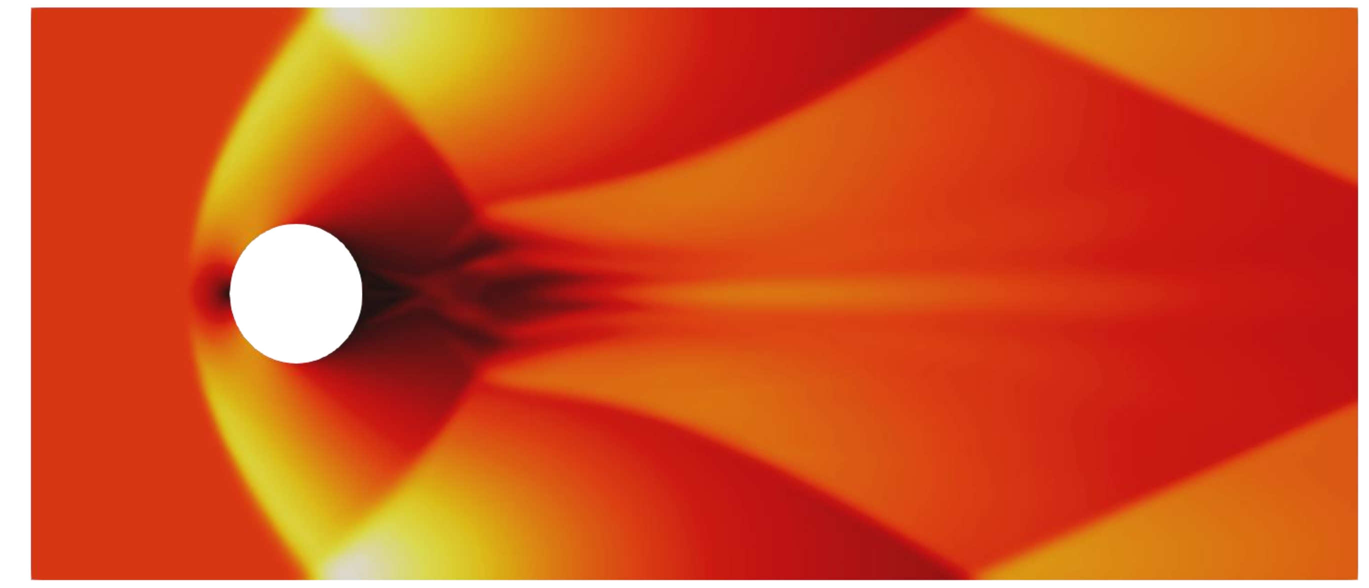
$$-\epsilon \nabla (\nabla \cdot \boldsymbol{B})$$



Orszag-Tang Benchmark

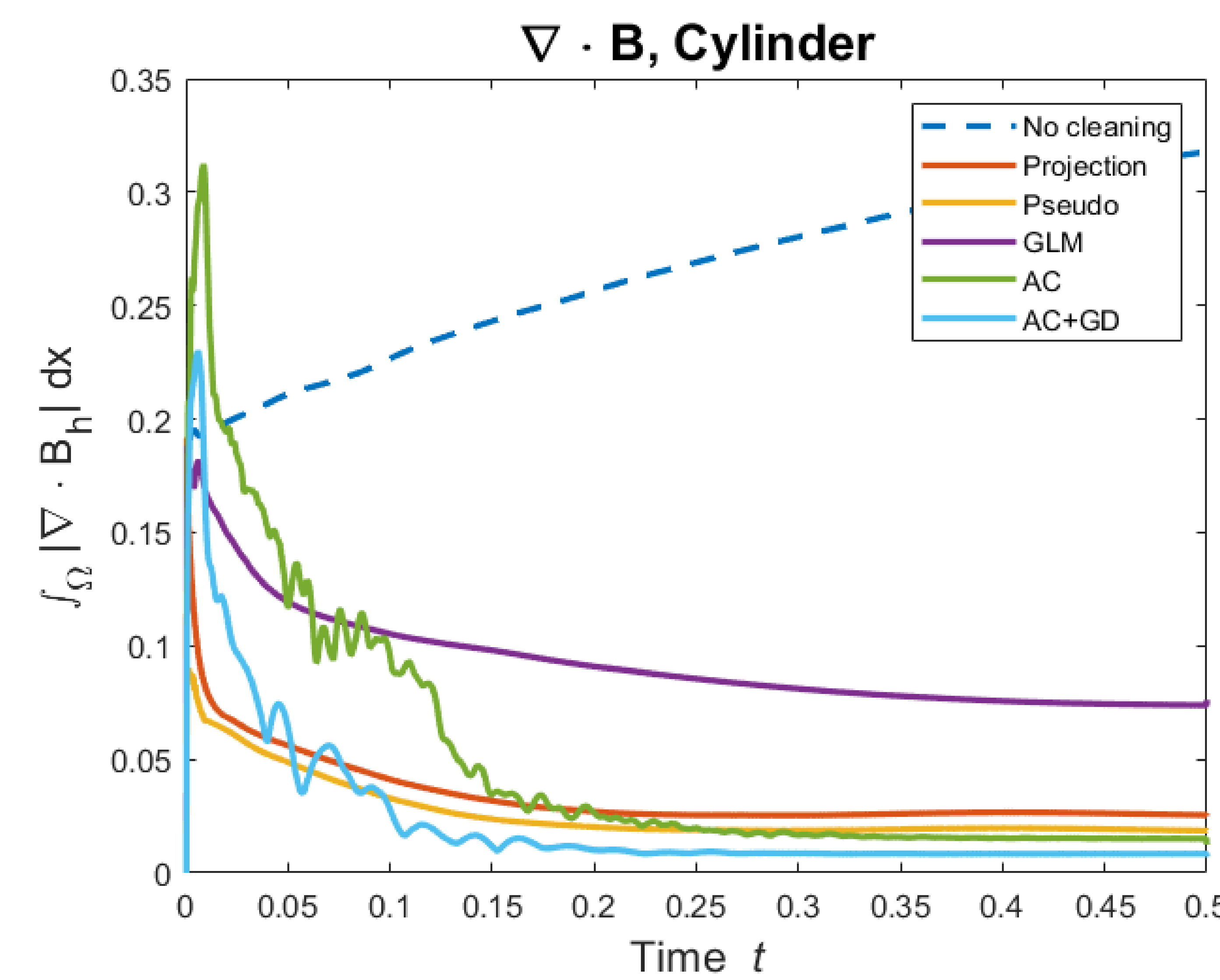
Results from trials on the 2D Orszag-Tang showed all cleaning methods succeed in reducing the divergence error, with AC+GD performing the best. However, AC and AC+GD requires some parameter tuning of λ for stability.





Cylinder Benchmark

All described cleaning methods succeed in lowering the magnetic divergence. The best performing method is AC+GD. One interesting difference with the cylinder benchmark to Orszag-Tang is the applied boundary conditions that requires handling by the different methods.



Please ask questions! :)

Project in Scientific Computing

2021

Authors:

David Niemelä
david.niemela@hotmail.com

Matilda Tiberg
matilda.tiberg@gmail.com

Anton Waltgård
anton.waltgard@outlook.com

Supervisors:

Tuan Anh Dao
tuananh.dao@it.uu.se

Lukas Lundgren
lukas.lundgren@it.uu.se