PDE-constrained optimization using physics-informed neural networks

Background

Partial differential equation (PDE) constrained optimization problems appear in a wide variety of applications, for example climate modeling, geophysics and signal processing. Most solution approaches rely on traditional methods for solving PDEs, such as finite elements or finite differences. However, these methods become increasingly complex in the presence of general domains and boundary conditions. Recent work have shown that neural networks can be used to solve PDEs by incorporating the physics into the training of the network, resulting in so called physics-informed neural networks (PINNs). Additionally, with small modifications they can be used to optimize for unknown parameters in the PDE (model discovery). See [1, 2, 3] for examples.

Project description

The main focus of this project is to investigate PINNs as a tool for solving the inverse problem

$$\underset{a(x)}{\text{minimize}} \quad \frac{1}{N}\sum_{i=1}^{N} |u(t_i, \hat{x}) - \hat{u}_i|^2 \tag{1}$$

constrained by the second order wave equation:

$$\begin{aligned} u_{tt} &= (a(x)u_x)_x, && 0 \le x \le 1, && t > 0, \\ u(t,x) &= u_0(x), && 0 \le x \le 1, && t = 0, \\ u_t(t,x) &= 0, && 0 \le x \le 1, && t = 0, \\ u(t,x) &= 0, && x = 0, 1, && t > 0, \end{aligned} \tag{2}$$

where $\hat{u}_i$, $i = 1, 2, ..., N$, is N measurement data points (exact solution) at $x = \hat{x}$ and $t = t_i$, and $a(x)$ is the sought after wave speed function. The project work should include:

- An implementation of a PINN to solve the model problem.
- An investigation of the importance of various implementation choices. For example network hyperparameters, optimizer and generation of collocation points.
- A comparison of computational costs to traditional methods (if there is time).

Suggested programming framework: TensorFlow with Python

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References

[1] Jens Berg and Kaj Nyström. Data-driven discovery of pdes in complex datasets. *Journal of Computational Physics*, 384:239–252, 2019.

[2] M. Raissi, P. Perdikaris, and G.E. Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, 378:686–707, 2019.

[3] Yibo Yang and Paris Perdikaris. Adversarial uncertainty quantification in physics-informed neural networks. *Journal of Computational Physics*, 394:136–152, 2019.