Adjoint-based inverse modelling of an acoustic material (Project course 15 ECTS)

Project description

Inverse modelling is the process of calculating underlying model parameters from data measurements. This project deals with inverse modelling of an acoustic medium, where potential applications are acoustic source reconstruction, underwater acoustic exploration, and design optimization among others. The propagation of acoustic waves in a domain Ω can be modeled by the initial-boundary value problem (IBVP)

$$\begin{array}{lll} \frac{\partial^2 p}{\partial t^2} = K\nabla\cdot(\rho^{-1}\nabla p) + S, & \bar{x}\in\Omega, & t>0, \\ p = f, & \bar{x}\in\Omega, & t=0, \\ \mathcal{L}p = g, & \bar{x}\in\partial\Omega, & t>0, \end{array} \tag{1}$$

where p is the pressure perturbation, ρ and K the density and bulk modulus of the material, and S a source function. Initial data is specified by f, while $\mathcal{L}p = g$ specifies boundary conditions (e.g Dirichlet or Neumann conditions) on the boundary $\partial\Omega$ where g is the boundary data. In a typical simulation setting the material parameters are known and (1) is solved for p at some final time T. This setting is commonly referred to as the 'forward problem' to (1). In this project we are interested in a setting where also the material parameters are unknown, but where we have a time series of measured pressure data $p_{data}(t)$, recorded at receivers located in the domain. Using the measured data, we now seek to reconstruct the material parameters. We consider the case where the density is allowed to vary spatially, i.e. $\rho = \rho(\bar{x})$, but to simplify matters the bulk modulus K is constant in the domain. To reconstruct ρ from the measured data, consider the following optimization problem, referred to as 'the inverse problem' to (1):

$$\min_{\rho} J(p), \quad J(p) = \int_0^T \left(p(\bar{x},t) - p_{data}(t)\right)^2 dt. \tag{2}$$

To solve the inverse problem, the cost function $J(p)$ is evaluated by solving the forward problem numerically for a sequence of ρ until a minima of (2) within accepted error tolerance is obtained. In the present project we will make use of the fact that (2) can be solved efficiently using gradient-based optimization methods, where the gradient $\frac{dJ}{dp}$ is derived and computed by solving the so-called 'adjoint problem' to (1).

To solve the forward- and adjoint problem, a method-of-lines approach will be employed, where high-order finite difference methods are used to discretize (1) in space. Higher-order methods (as compared to first- and second-order accurate methods) capture transient phenomena more efficiently since they allow a considerable reduction in the degrees of freedom, for a given error tolerance. In particular, high-order finite difference methods are ideally suited for hyperbolic problems of this type. In this project, we will utilize a well-proven and stable finite difference method for well-posed IBVPs, where summation-by-parts (SBP) operators [2, 3] are combined with imposition of boundary conditions through the simultaneous approximation term (SAT) method. Examples of the SBP-SAT approach can be found in [1, 2, 3]. What remains is then to choose a suitable time integration method, such that the gradient can be evaluated efficiently. Evaluating a suitable time integrator is a part of the present project. Once the numerical solvers to the forward- and adjoint problem are in place, the optimization problem (2) is ready to be solved. To do this, a suitable gradient-based optimization method is required. Evaluating which method to use is also a part of the present project.

Project topics

The project topics described in the previous section are summarized below.

1. Analyze stability of the forward problem to (1) discretized using the SBP-SAT method.
2. Derive, and analyze stability of the adjoint problem to (1) discretized using the SBP-SAT method.
3. Evaluate and implement a suitable time integrator to solve the forward- and adjoint problems.
4. Evaluate and implement a suitable optimization algorithm to solve the inverse problem (2).
5. Perform a numerical study where the inverse problem (2) is solved, and the efficiency of the method is evaluated.

The numerical solvers will be implemented using MATLAB. The SBP operators will be provided (see [3, 2] for details).

Relevant courses

The following course is required for the present project: Advanced numerical methods.

Supervisors

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References

[1] David C. Del Rey Fernández, Jason E. Hicken, and David W. Zingg. Review of summation-by-parts operators with simultaneous approximation terms for the numerical solution of partial differential equations. *Computers and Fluids*, 95:171–196, 2014.

[2] Ken Mattsson. Summation by Parts Operators for Finite Difference Approximations of Second-Derivatives with Variable Coefficients. *Journal of Scientific Computing*, 51(3):650–682, 2012.

[3] Ken Mattsson and Jan Nordström. Summation by parts operators for finite difference approximations of second derivatives. *Journal of Computational Physics*, 199(2):503–540, 2004.