Magnetic divergence cleaning methods in Magnetohydrodynamics

(Project in Computational Science, 15 ECTS)

Outline

The system of magnetohydrodynamics (MHD) equations is a system of conservation laws describing the governing hydrodynamics and electromagnetism of conducting fluids. The MHD equations can be derived by combining the compressible Euler equtions and the Maxwell equations. The equations were first derived by Hannes Alfvén who was awarded the Nobel prize in 1970. Compared to the Euler equations or the Navier-Stokes equations, the additional difficulties in solving the MHD equations include: (i) developing accurate shock and turbulence capturing techniques, and (ii) numerically satisfying the divergence-free condition $\nabla \cdot \overrightarrow{\mathbf{B}} = 0$, where $\overrightarrow{\mathbf{B}}$ is the magnetic field.

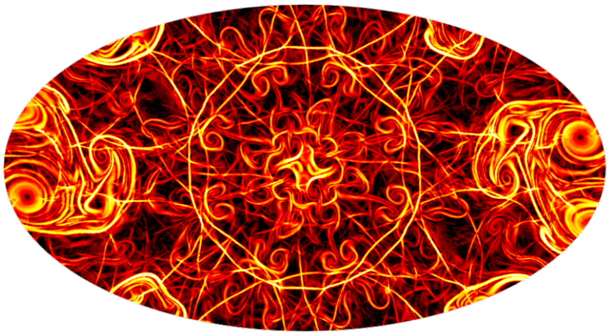

Figure 1: Finite element simulation of shock waves in a compressible flow model.

Our research group has an ongoing collaboration with ITER (https://iter.org) on numerical simulation of high energy plasma inside a tokamak nuclear fusion reactor.

Project description

The students are expected to implement and numerically compare several divergence cleaning techniques for the magnetic field in the finite element method setting. The suggested techniques are: projection method [1], generalized Lagrange Multiplier [3], parabolic cleaning, artificial compressibility, grad-div stabilized Taylor-Hood elements [2] and Nédélec edge elements [4].

The students will be given an MHD solver written in C++ using FEniCS (https://fenicsproject.org/).

Time plan

1. Install the necessary libraries and get familiar with the solver (2 weeks)
2. Understand the mechanisms behind the divergence cleaning methods (1 week)
3. Implement the divergence cleaning methods (3 weeks)
4. Collect numerical results and finalize the report (4 weeks)

Relevant courses

Applied Finite Element Methods (1TD056) or equivalent is required for this project. Advanced Numerical Methods (1TD050) is also recommended.

Supervisors

Lukas Lundgren, Department of Information Technology, Division of Scientific Computing, Uppsala University, email: lukas.lundgren@it.uu.se
Tuan Anh Dao, Department of Information Technology, Division of Scientific Computing, Uppsala University, email: tuananh.dao@it.uu.se

References

[1] J. U. Brackbill and D. C. Barnes. The effect of nonzero $\nabla \cdot \mathbf{B}$ on the numerical solution of the magnetohydrodynamic equations. *J. Comput. Phys.*, 35(3):426–430, 1980.

[2] Michael A. Case, Vincent J. Ervin, Alexander Linke, and Leo G. Rebholz. A connection between Scott-Vogelius and grad-div stabilized Taylor-Hood FE approximations of the Navier-Stokes equations. *SIAM J. Numer. Anal.*, 49(4):1461–1481, 2011.

[3] A. Dedner, F. Kemm, D. Kröner, C.-D. Munz, T. Schnitzer, and M. Wesenberg. Hyperbolic divergence cleaning for the MHD equations. *J. Comput. Phys.*, 175(2):645–673, 2002.

[4] Kaibo Hu, Yicong Ma, and Jinchao Xu. Stable finite element methods preserving $\nabla \cdot B = 0$ exactly for MHD models. *Numer. Math.*, 135(2):371–396, 2017.