Multiprocessor scheduling, part 2

Scheduling theory

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Platform assumptions

Assumptions

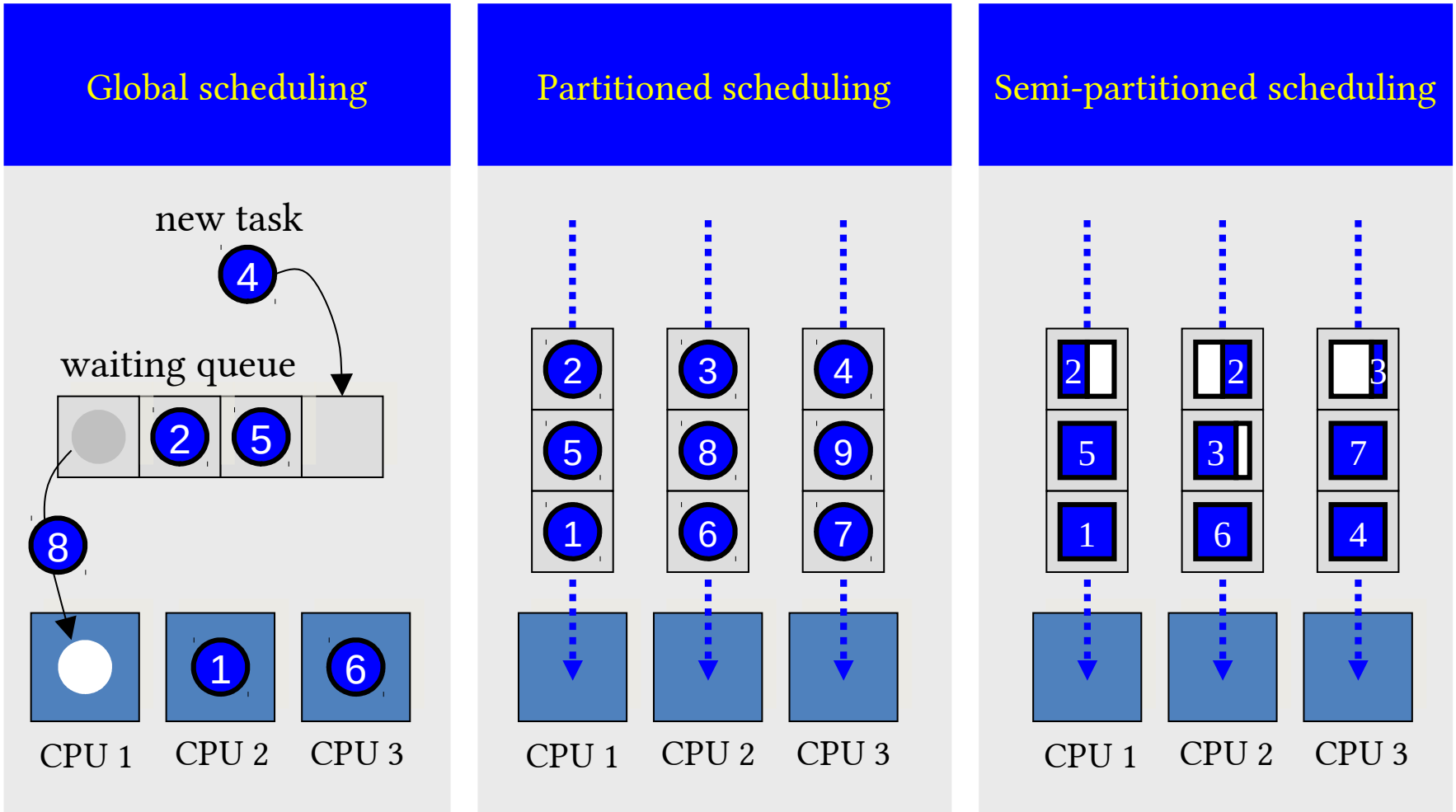
- We have m *independent* processors, where $m > 1$
  - No interference between processors/cores!
  - In principle achievable by carefully selecting hardware and middleware
  - Otherwise it is hopefully a good approximation
- Processors are *identical*
  - Any job can be executed on any processor, with the same WCET
  - Other possible models are *uniform* and *heterogeneous*

Task model assumptions

Assumptions

- Code is *sequential*
  - No parallelism allowed inside the individual jobs
  - *But:* more expressive task models do permit intra-task parallelism
- We will consider ordinary *sporadic tasks*
  - That are preemptive
  - Don't share mutually exclusive resources
  - *But:* some techniques generalize nicely to other task models

Main classification of multiprocessor schedulers



Partitioned scheduling

Partitioned scheduling

=

Partitioning strategy

+

Uniprocessor scheduling

Partitioned scheduling

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Uniprocessor scheduling

Well understood!

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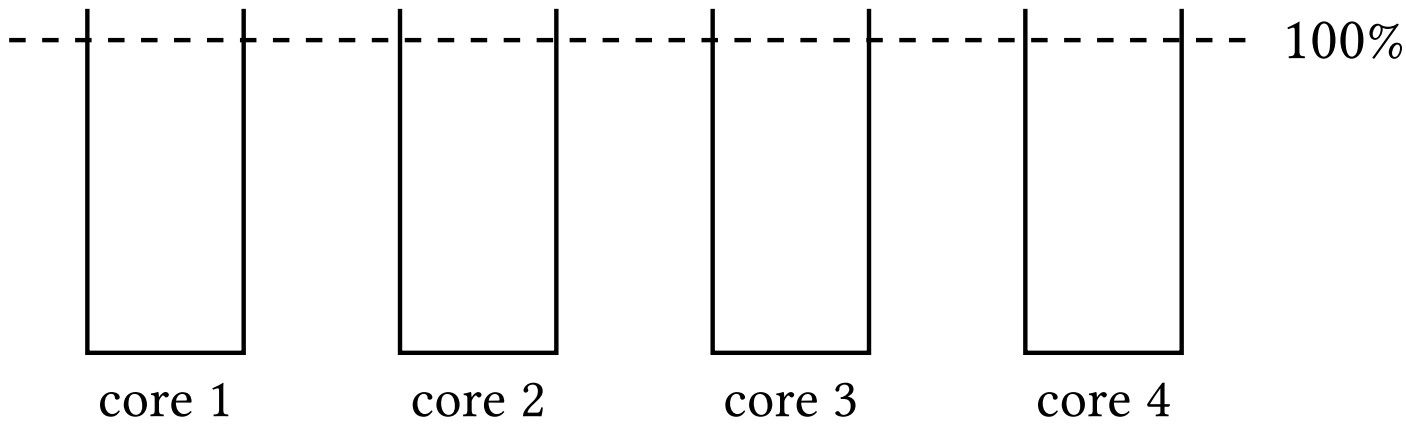

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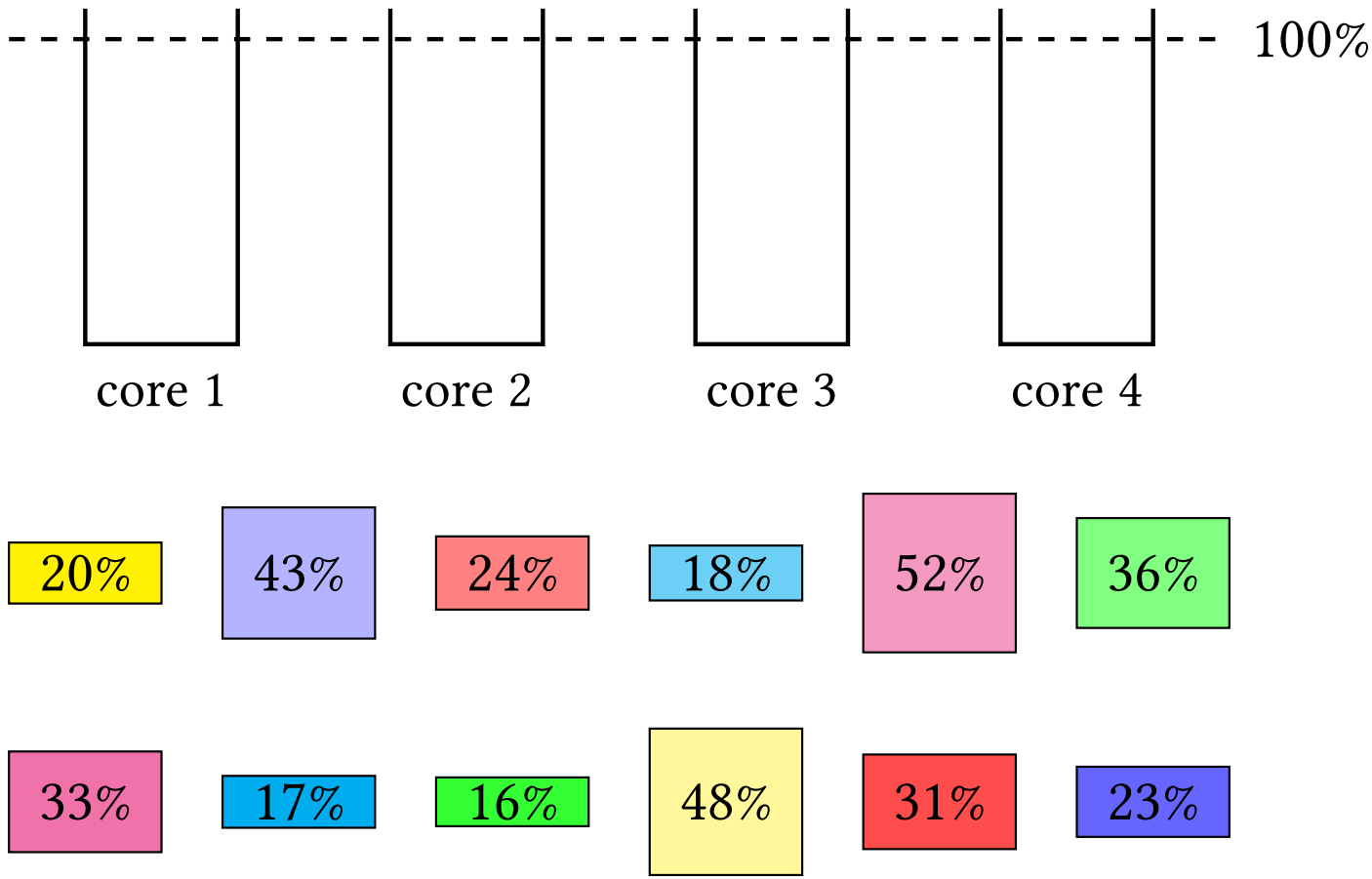

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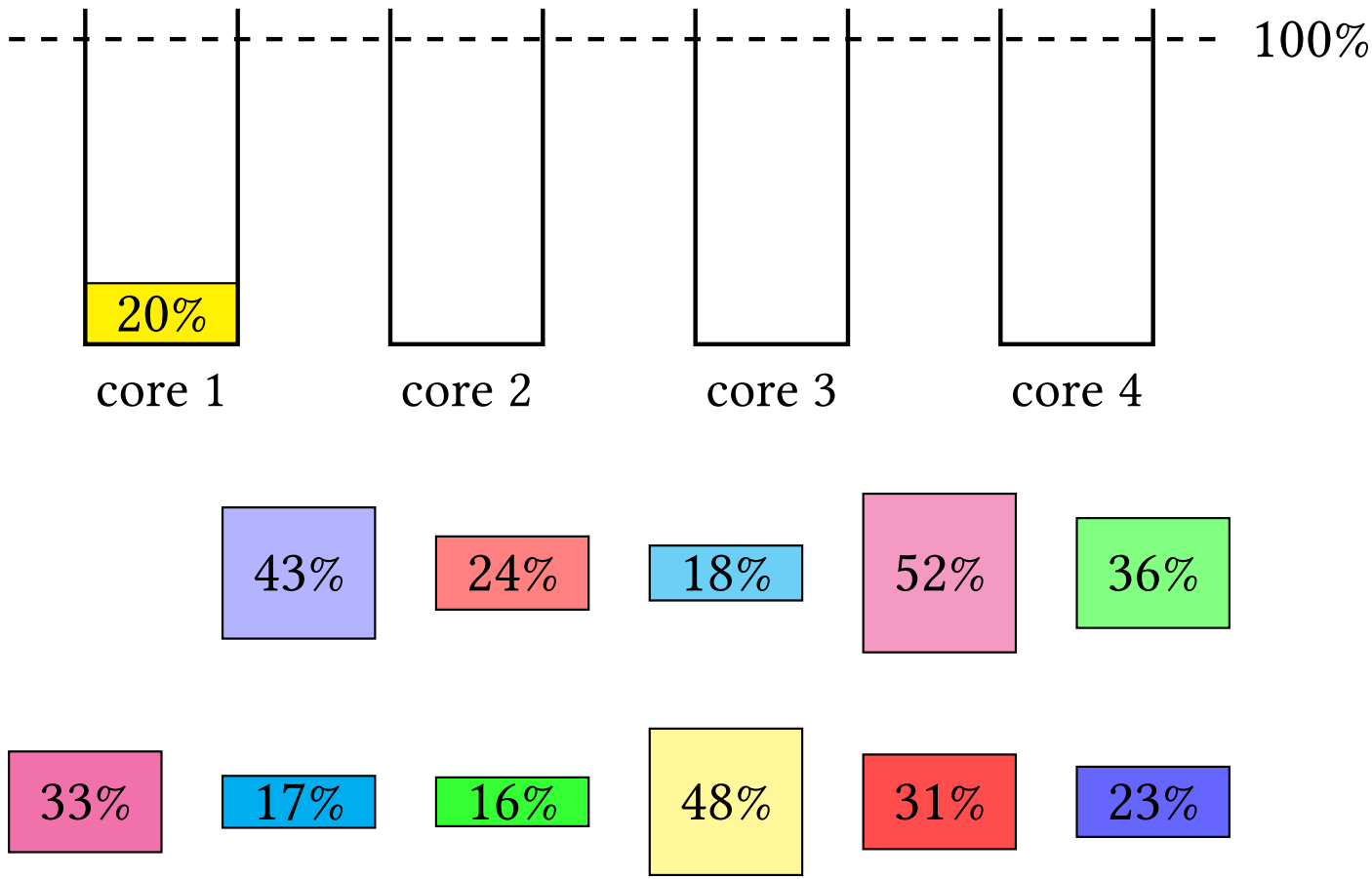

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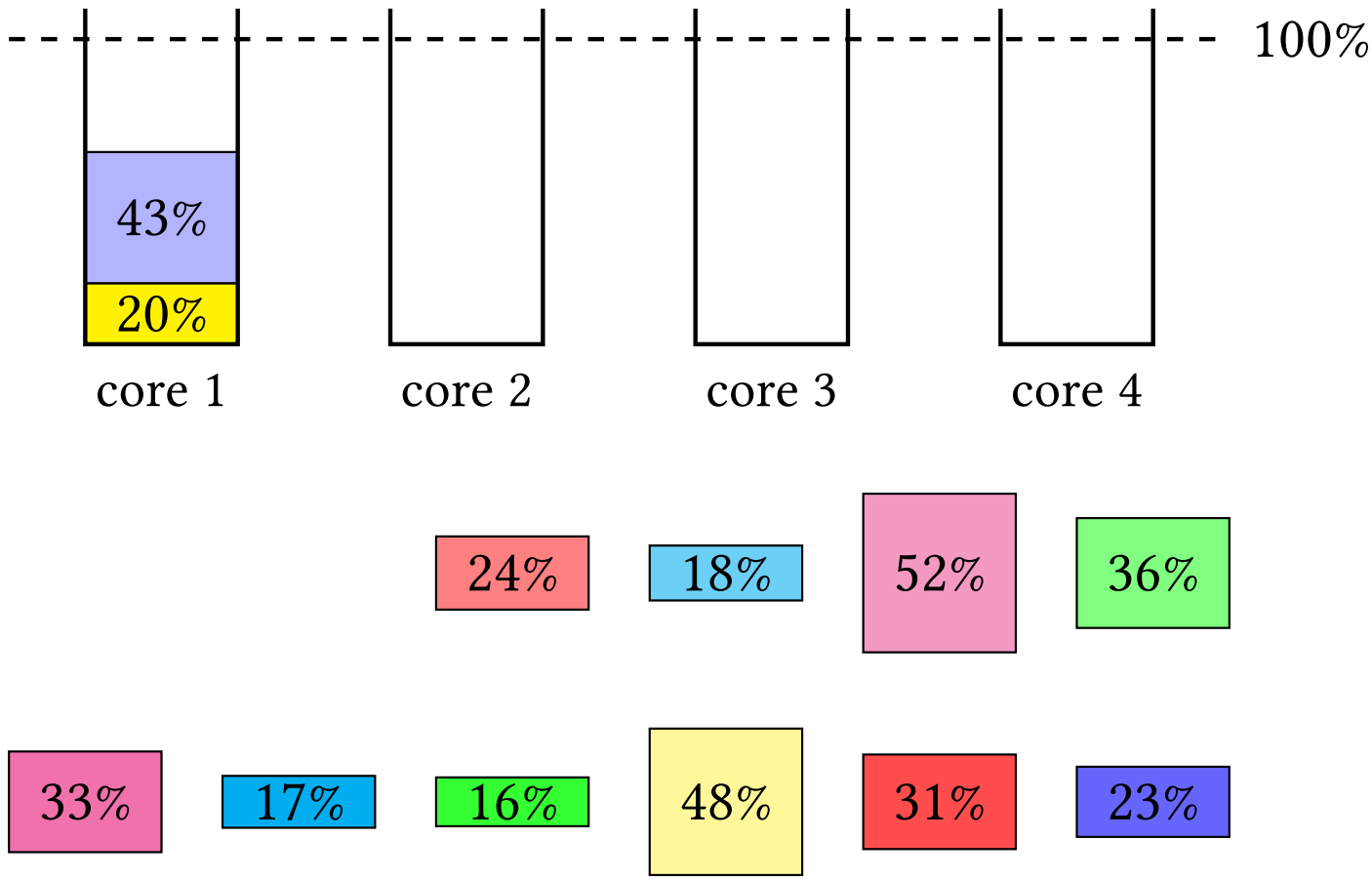

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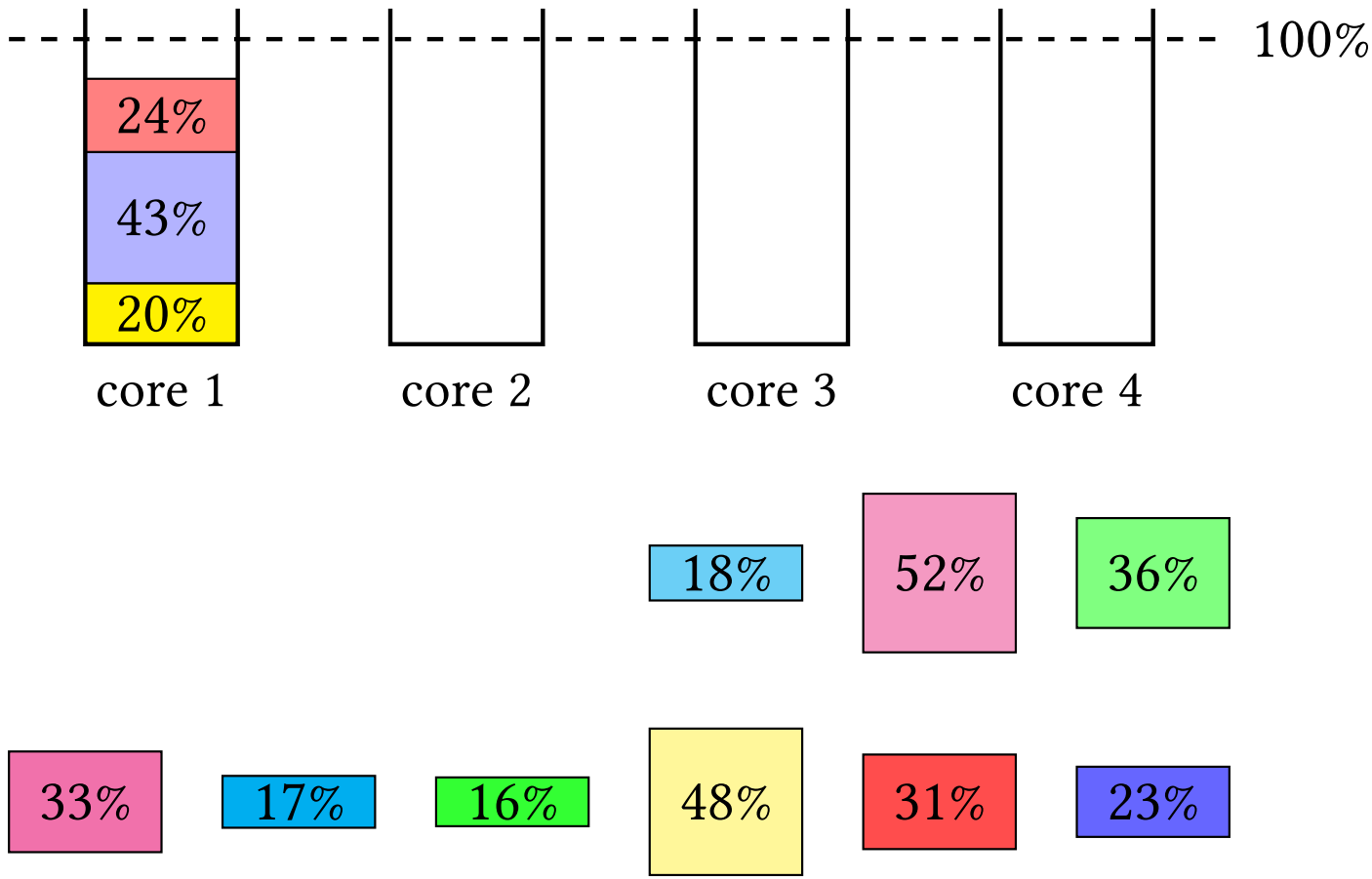

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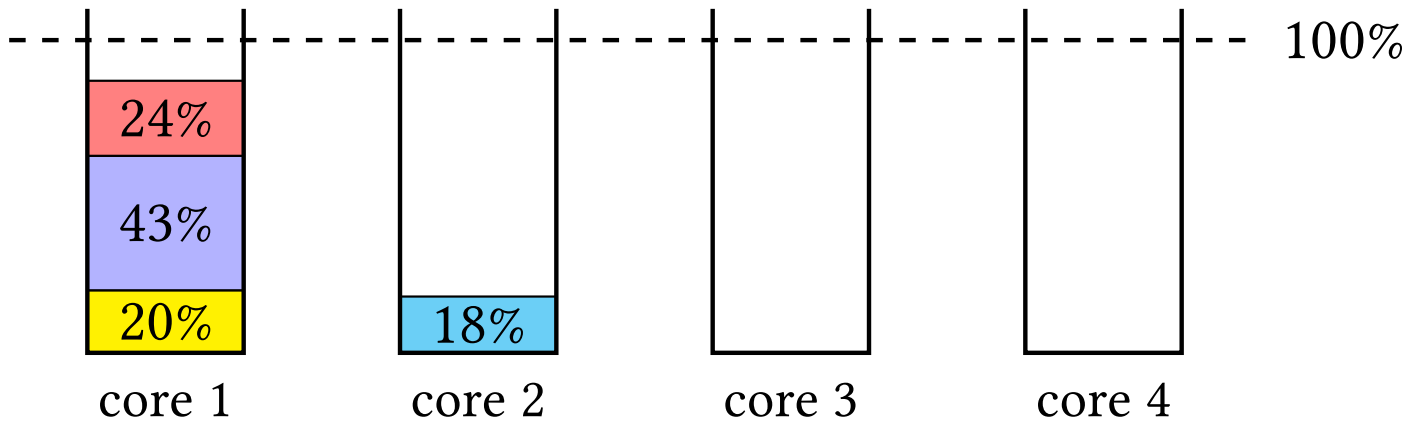

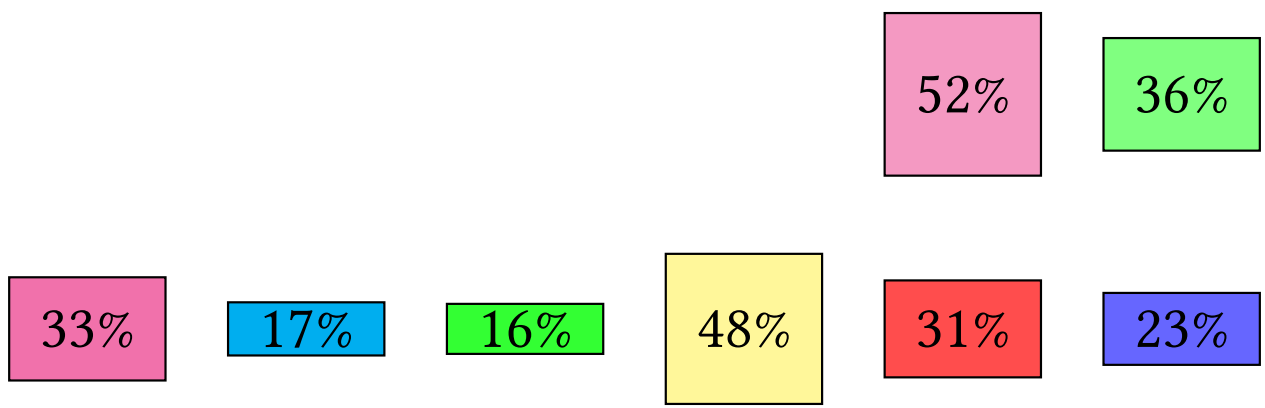

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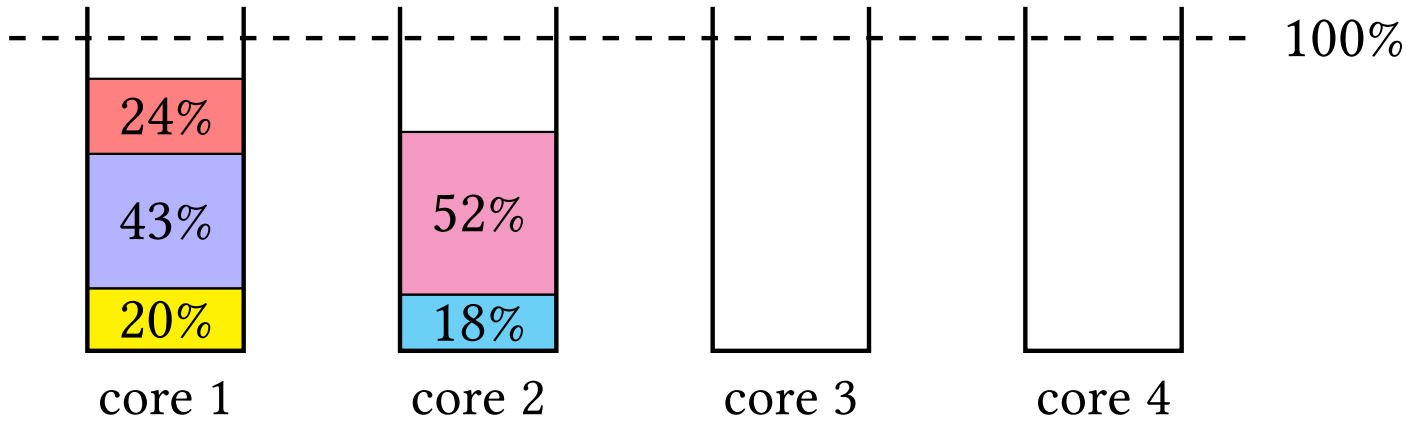

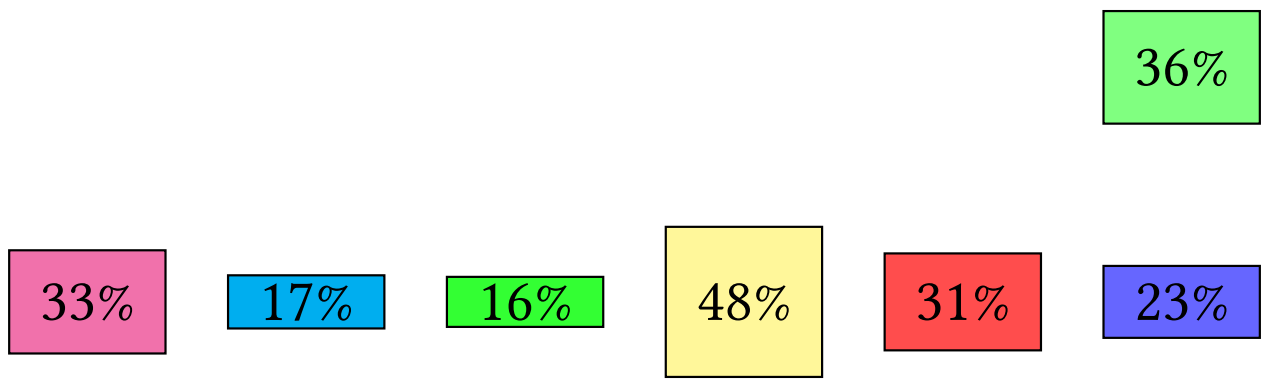

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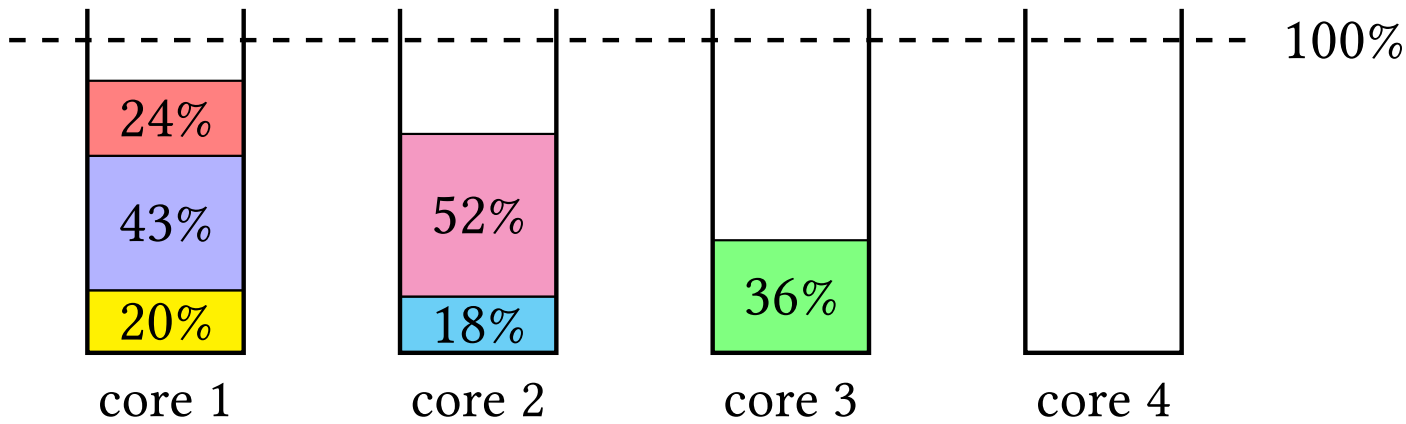

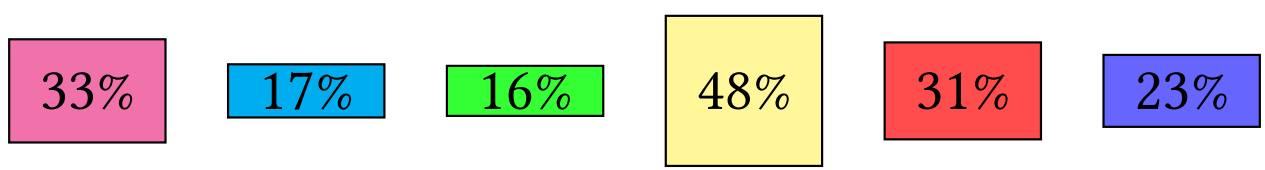

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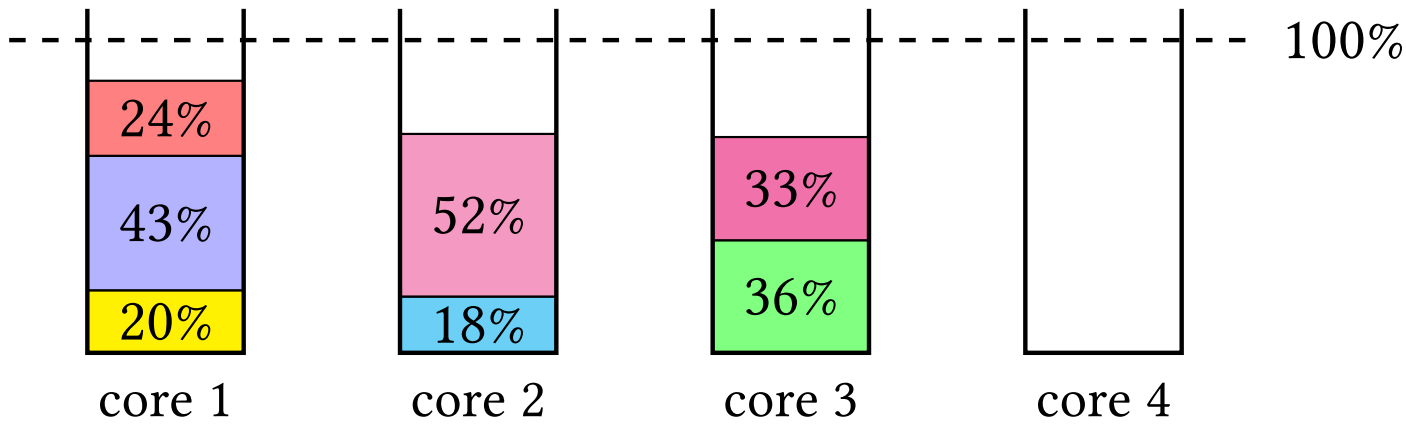

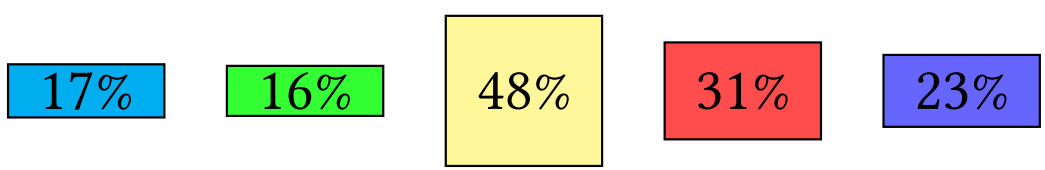

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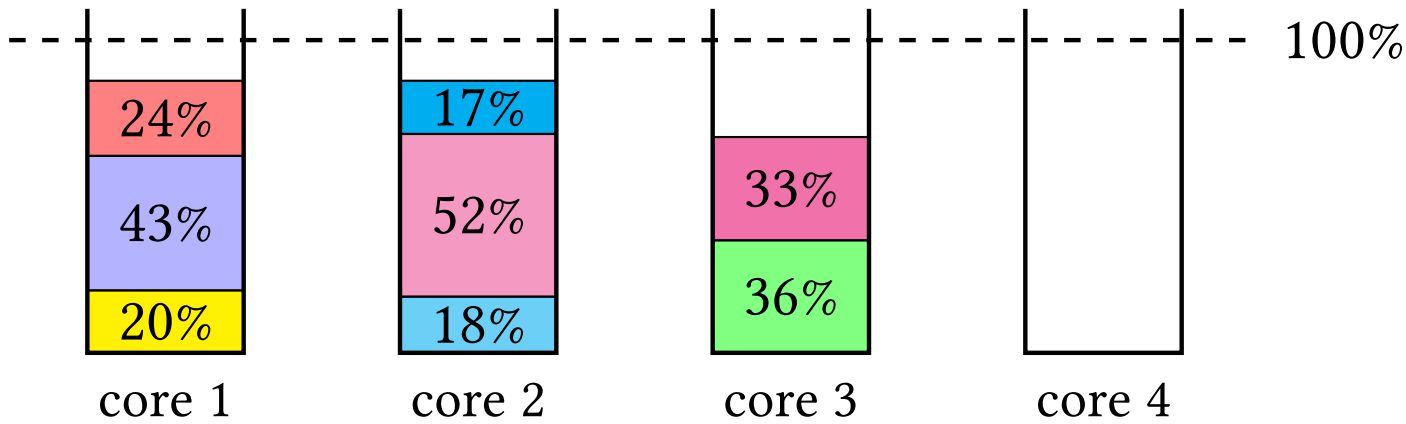

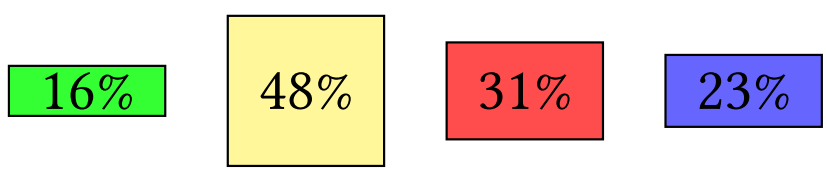

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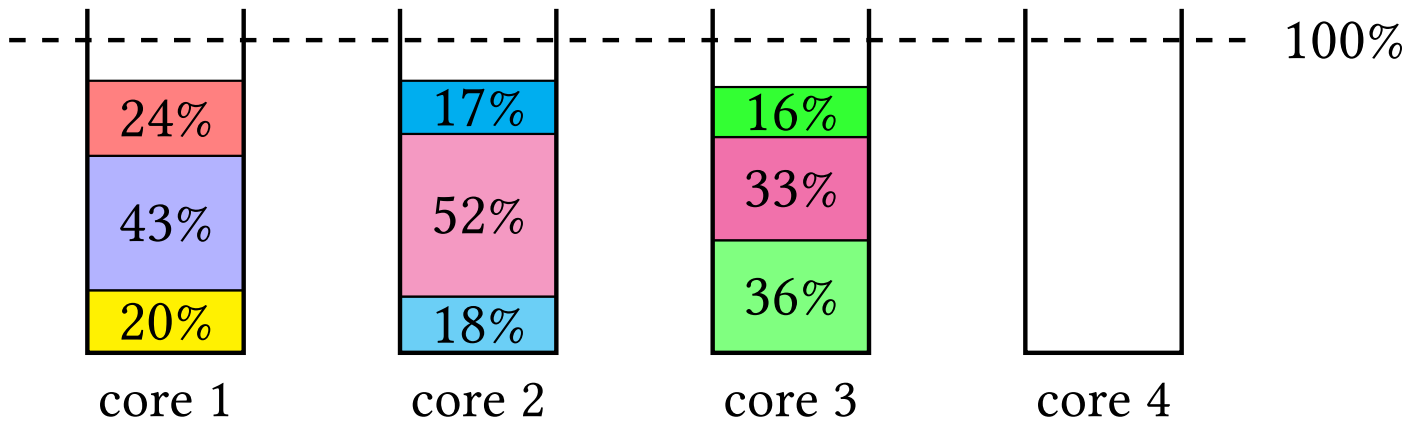

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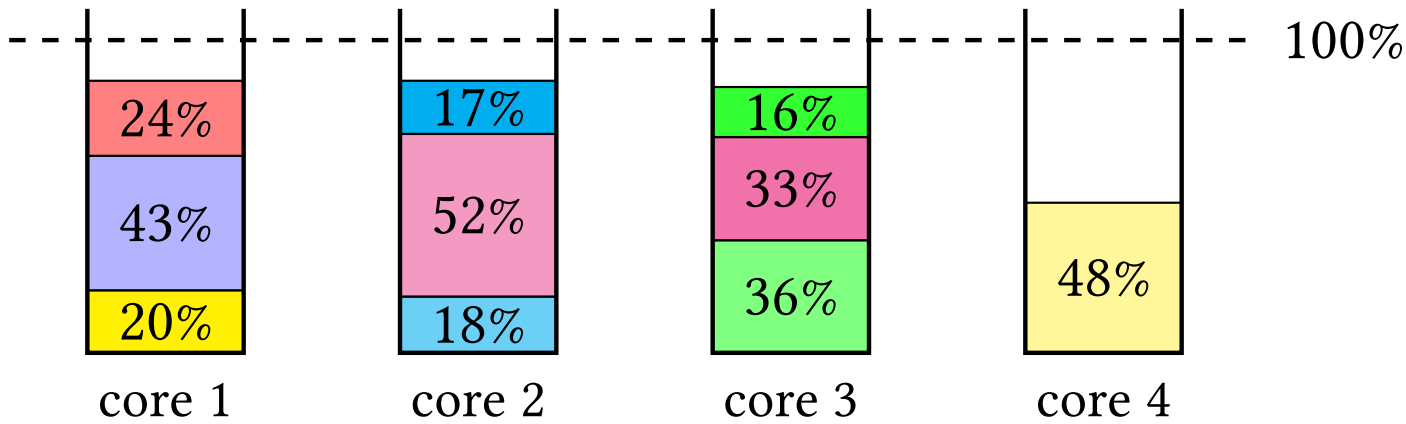

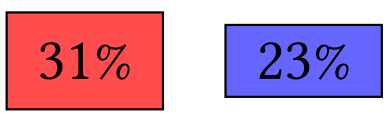

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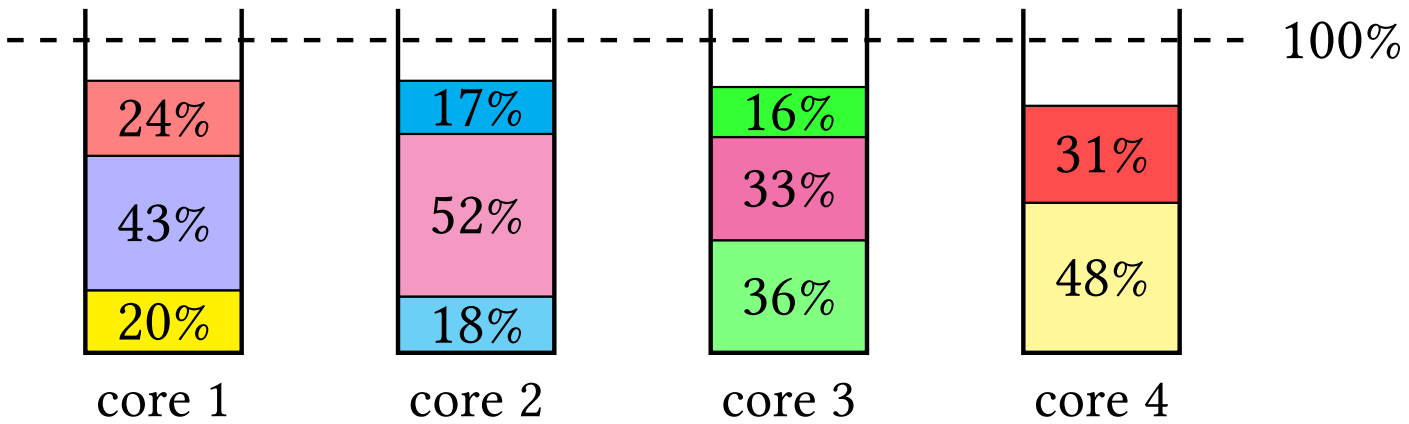

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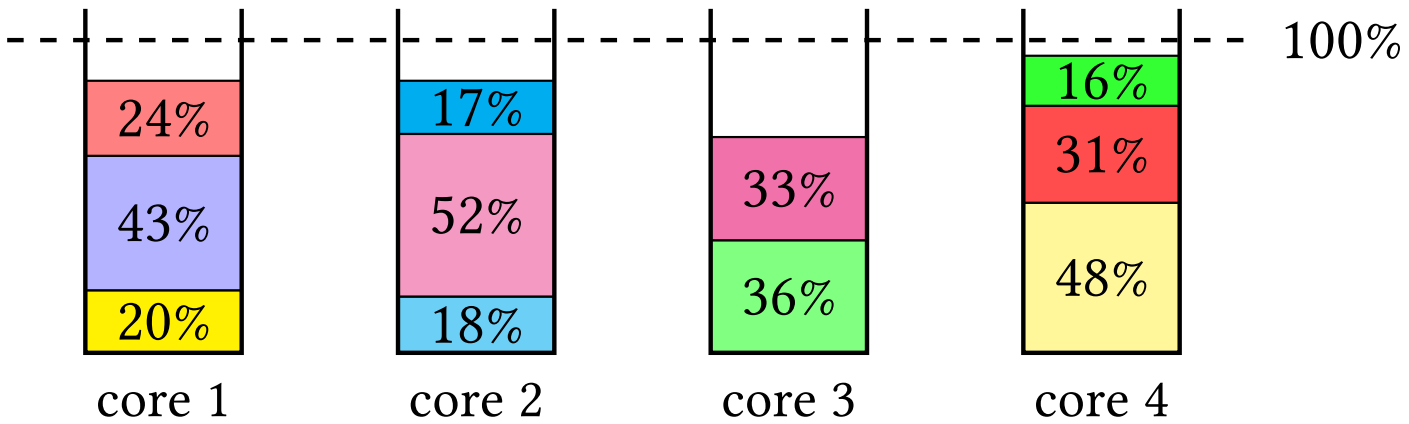

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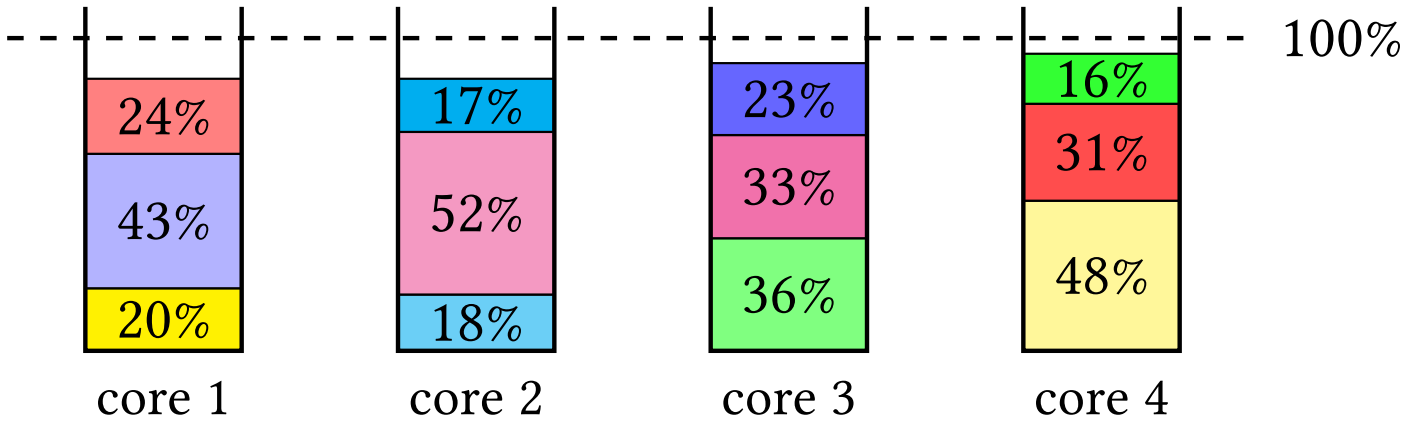

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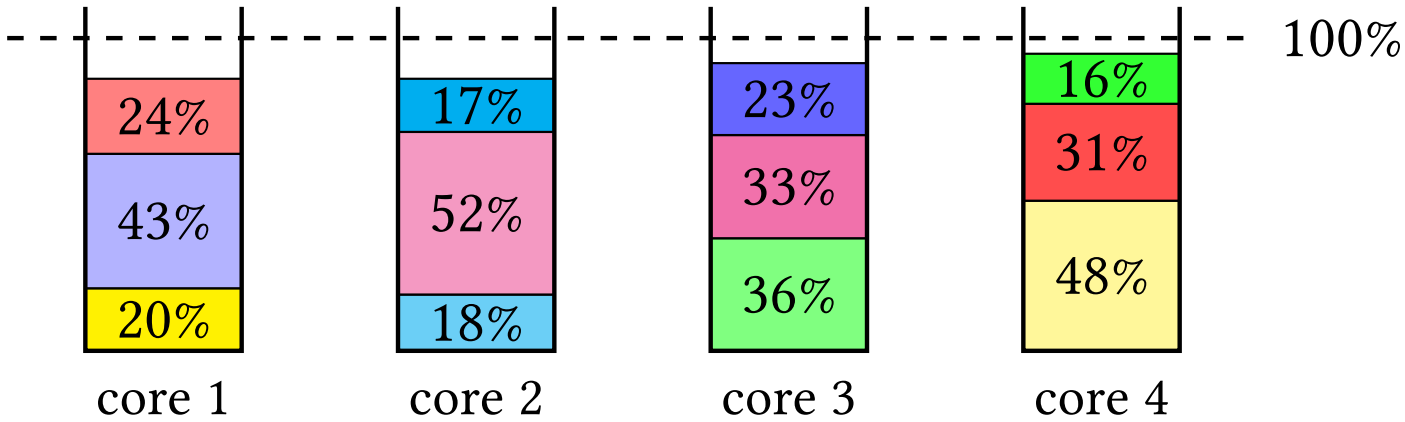

Bin Packing!

(Famously NP-hard)

How good is partitioned scheduling?

Fact

A set $\mathcal{T}$ of implicit-deadline tasks is schedulable by EDF on 1 preemptive processor iff $U(\mathcal{T}) \leqslant 1$.

Question

Is a set $\mathcal{T}$ of implicit-deadline tasks schedulable by partitioned EDF on m preemptive processors iff $U(\mathcal{T}) \leqslant m$?

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No!

A utilization bound

Theorem (López et al., 2000)

A set $\mathcal{T}$ of implicit-deadline sporadic tasks is schedulable by partitioned EDF on m preemptive processors if

$$\mathrm{U}(\mathcal{T}) \leqslant \frac{m+1}{2}.$$

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- There exists no better utilization bound. (Proof on blackboard)
- If the condition holds, we can always find a valid partitioning with a simple heuristic (for example, *First-Fit* as tried on previous slide).

Improvements still possible?

Definition

Let $U_{max}(\mathcal{T})$ be the largest utilization of any task in $\mathcal{T}$, and let

$$\beta = \left\lfloor \frac{1}{U_{max}(\mathcal{T})} \right\rfloor .$$

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$$U(\mathcal{T}) \leqslant \frac{\beta m + 1}{\beta + 1}.$$

Partitioning in other settings

The same general idea applies for other task models and schedulers:

1. Make a partitioning
2. Evaluate the (uniprocessor) schedulability of each partition
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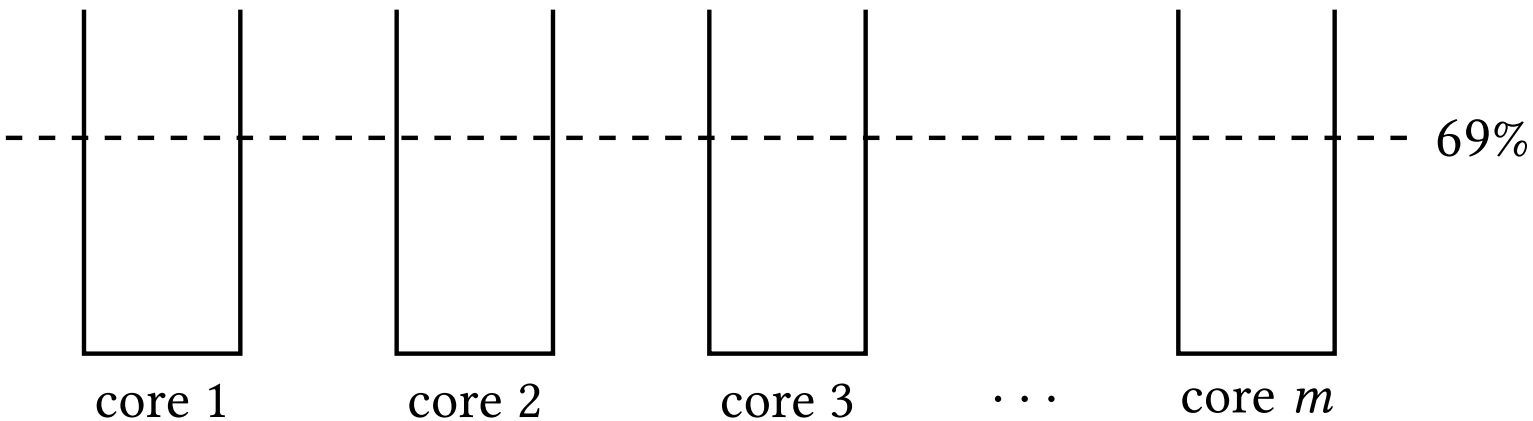

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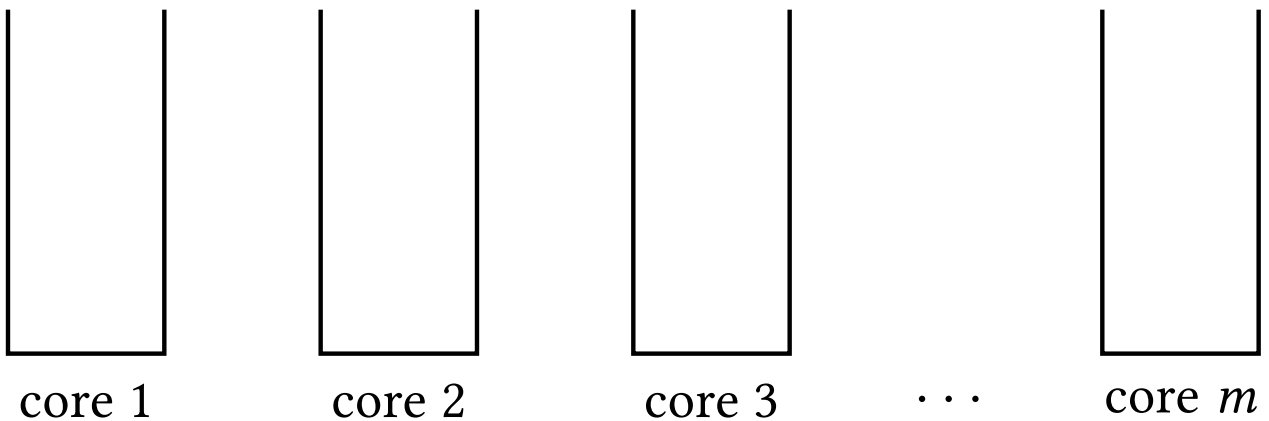

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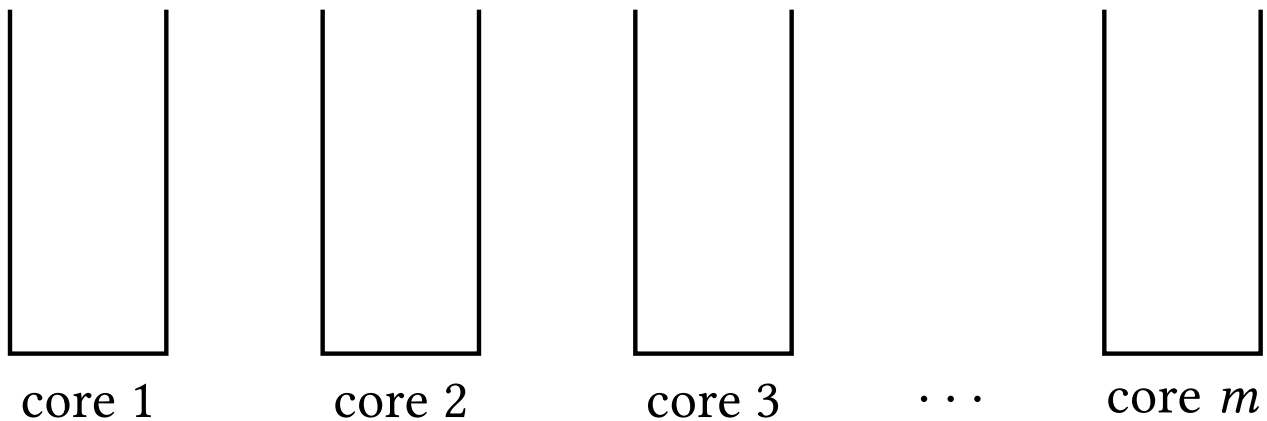
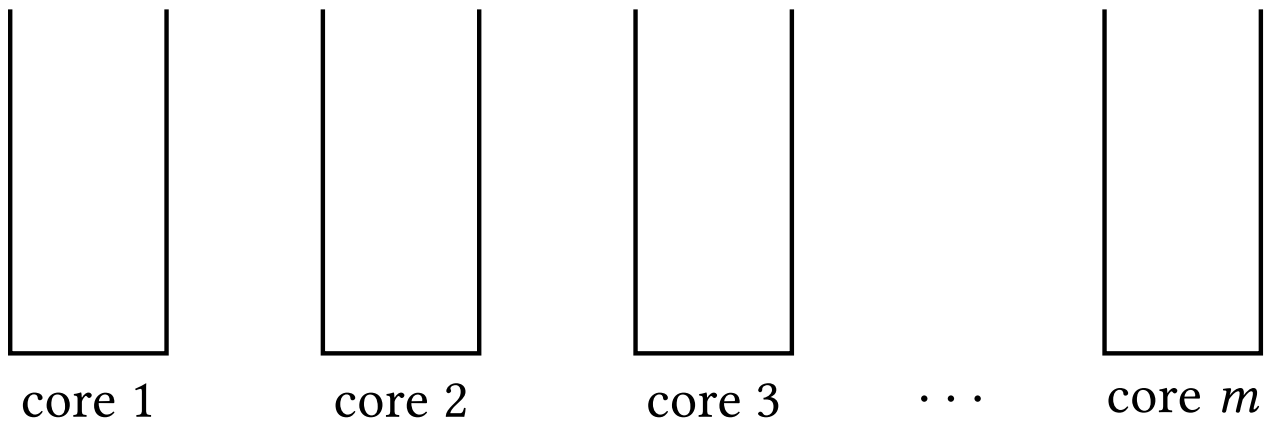


Note: We don't need to use the same scheduler on every processor.

Partitioning for existing subsystems

We could also assign different cores to existing (sub-)systems and control them via a *hypervisor*. For example:



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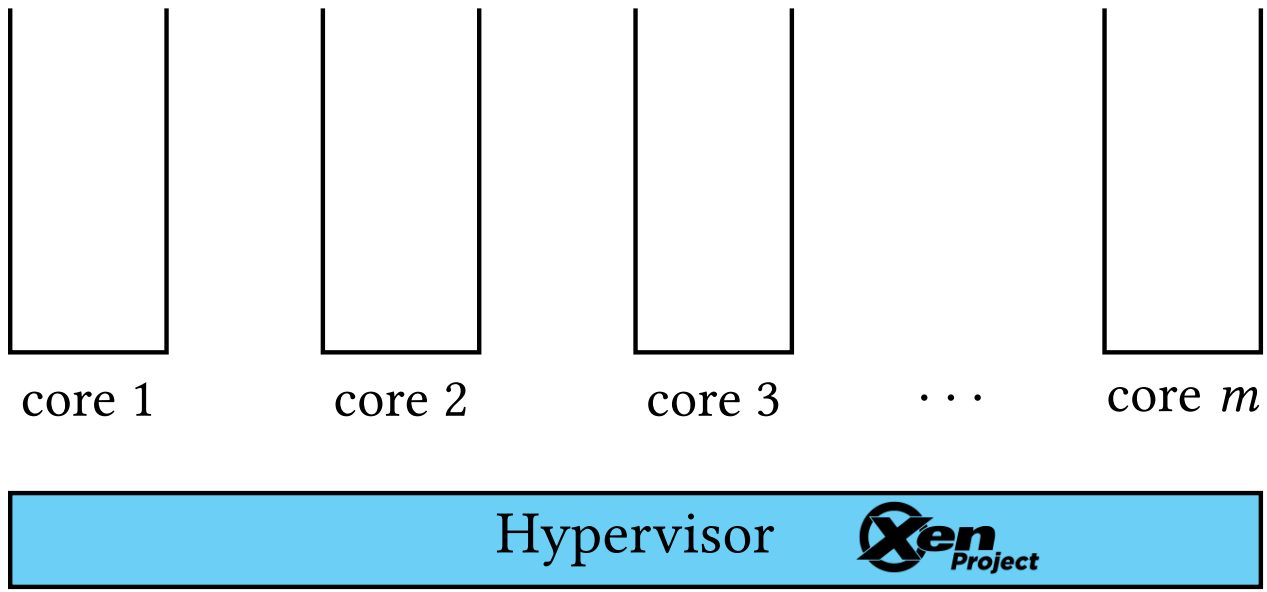

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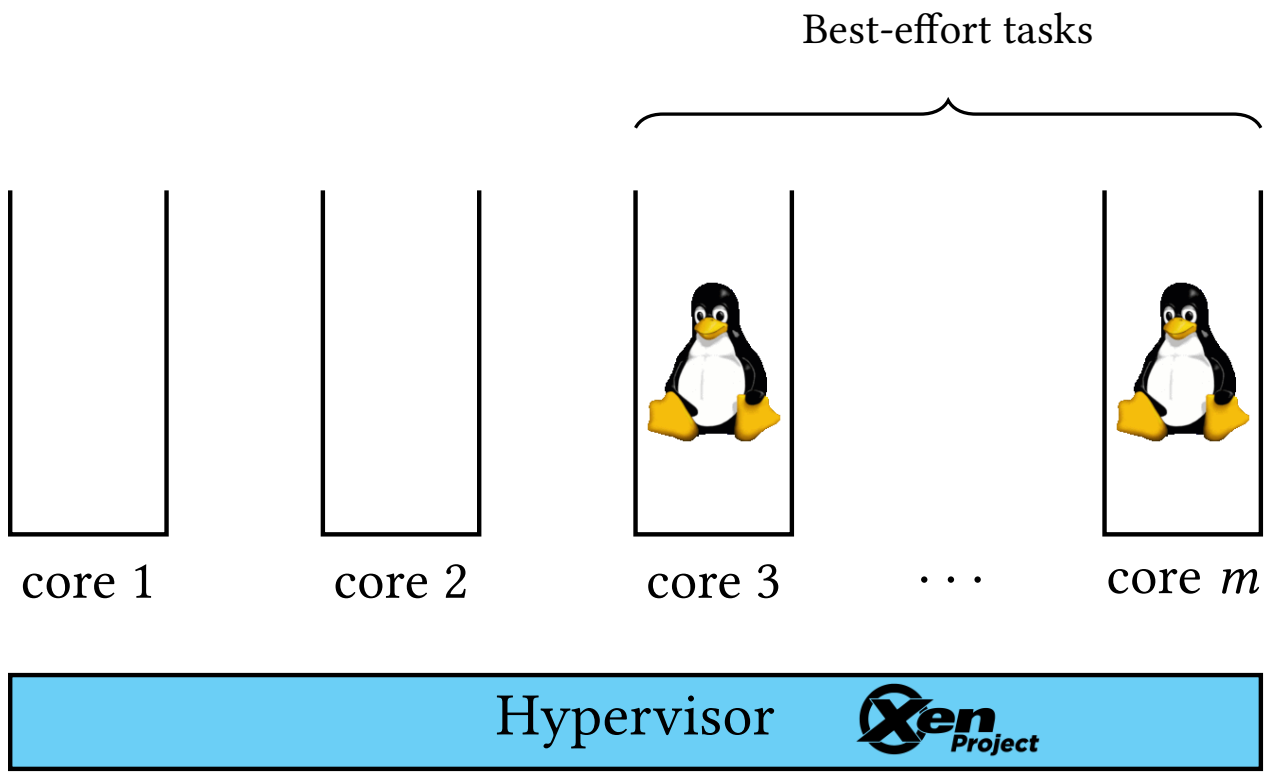

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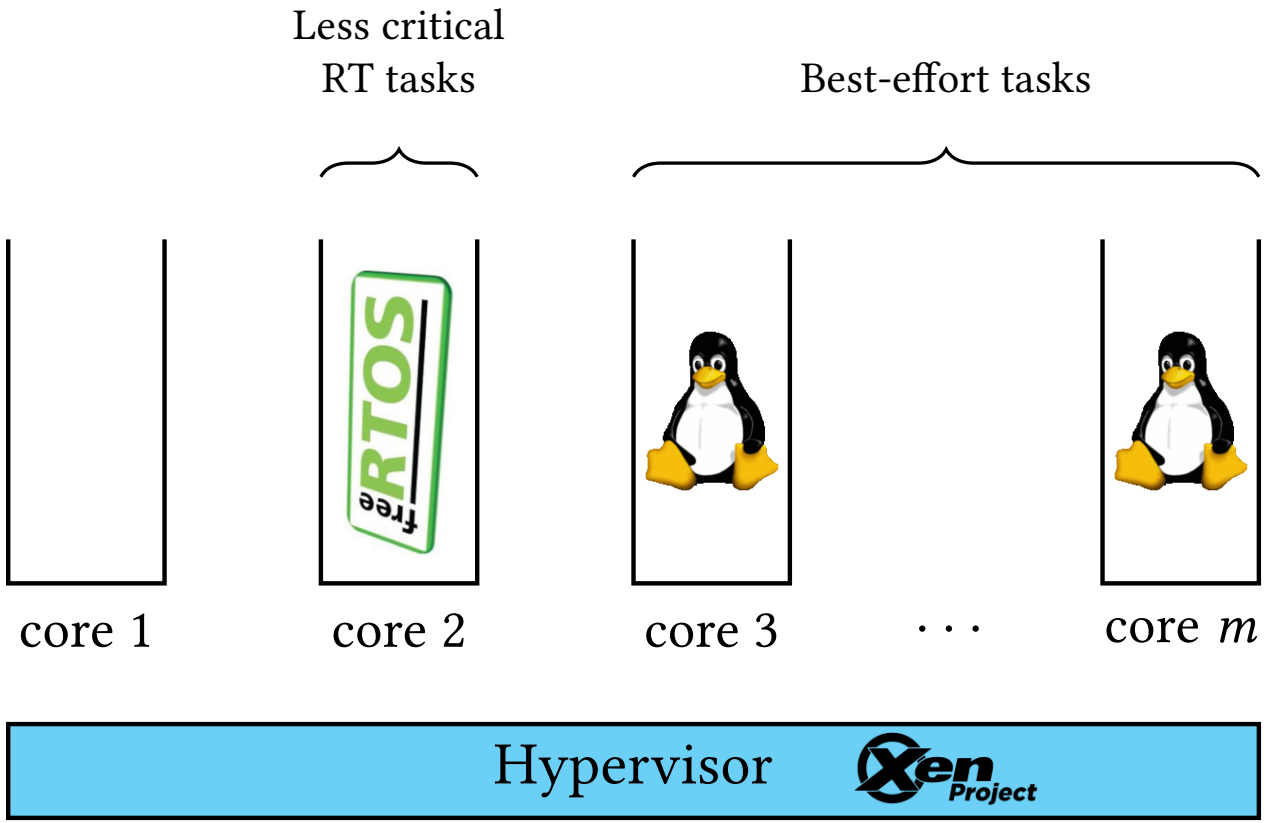

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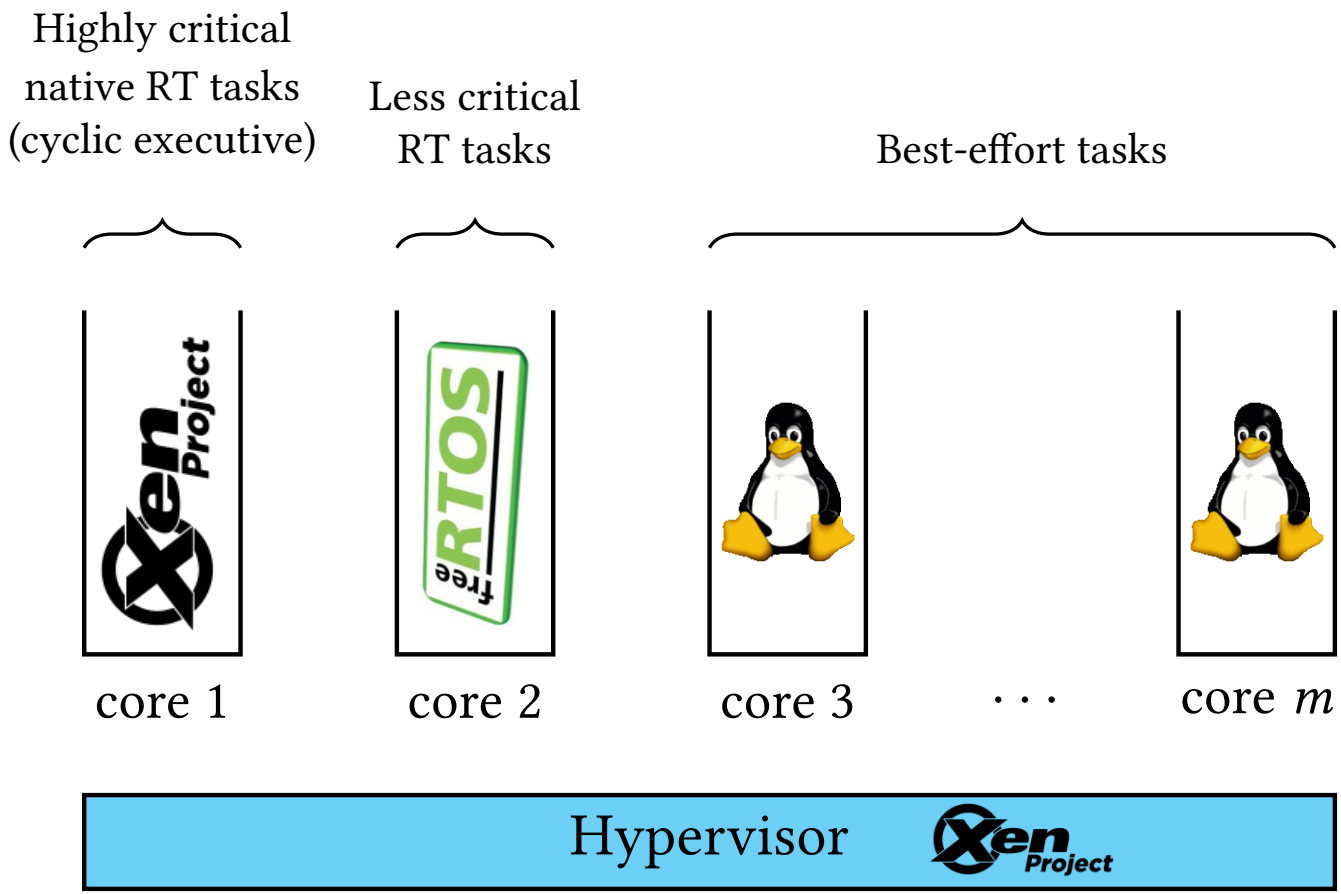

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  Global EDF (G-EDF): Run the m jobs with the earliest deadlines
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  Global EDF (G-EDF): Run the m jobs with the earliest deadlines
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- Exact schedulability tests are usually *highly intractable*.

Global EDF

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No better than with a single processor!

But this is only sufficient

No larger bound is valid (Proof on blackboard)

Global EDF—improved utilization bound

Utilization bound, improved

A set $\mathcal{T}$ of implicit-deadline sporadic tasks is schedulable by Global EDF on m preemptive processors if

$$\mathrm{U}(\mathcal{T}) \leqslant m - (m-1)\mathrm{U}_{\max}(\mathcal{T}).$$

Again, it is easier to schedule "light" tasks.

Global FP?

Global Fixed-Priority has similar (but slightly worse) utilization bounds. Details omitted here.

An optimal algorithm for implicit-deadline tasks!

Good news

It is possible to achieve *optimal* scheduling of implicit-deadline sporadic tasks on m processors, with the utilization bound

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Bad news

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Proportionate fairness

Rough idea

In every time interval of length t, execute task τ_i for a total of $t \cdot \mathrm{U}(\tau_i)$ time units.

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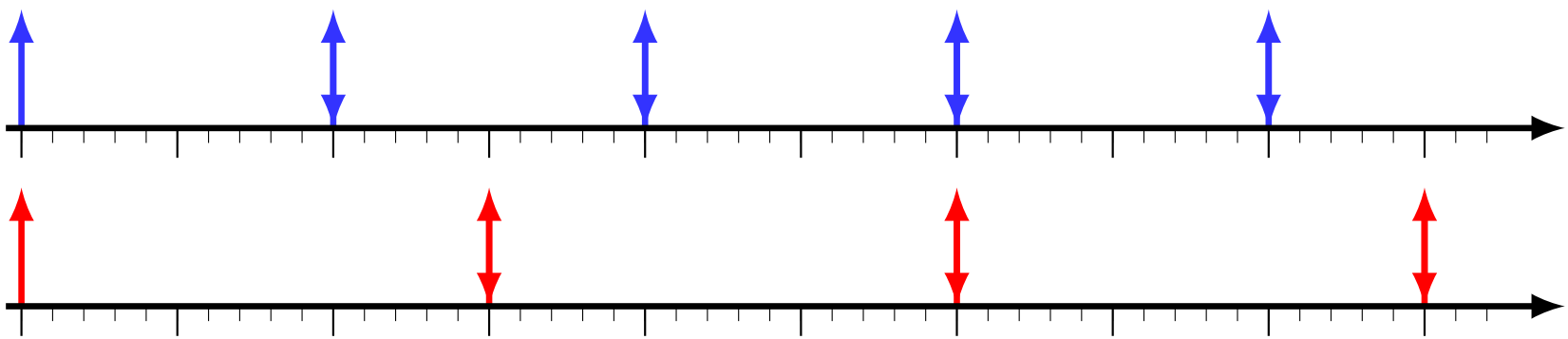

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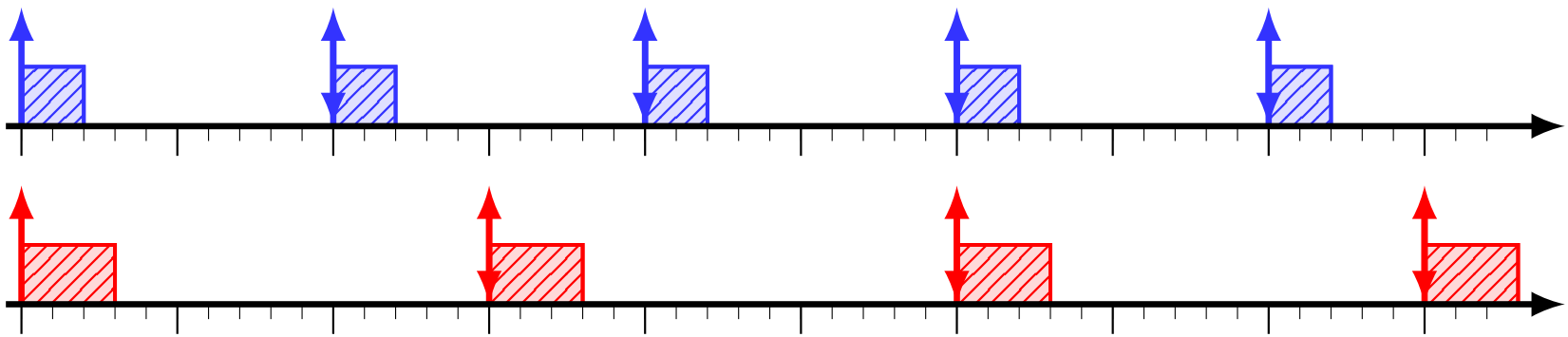

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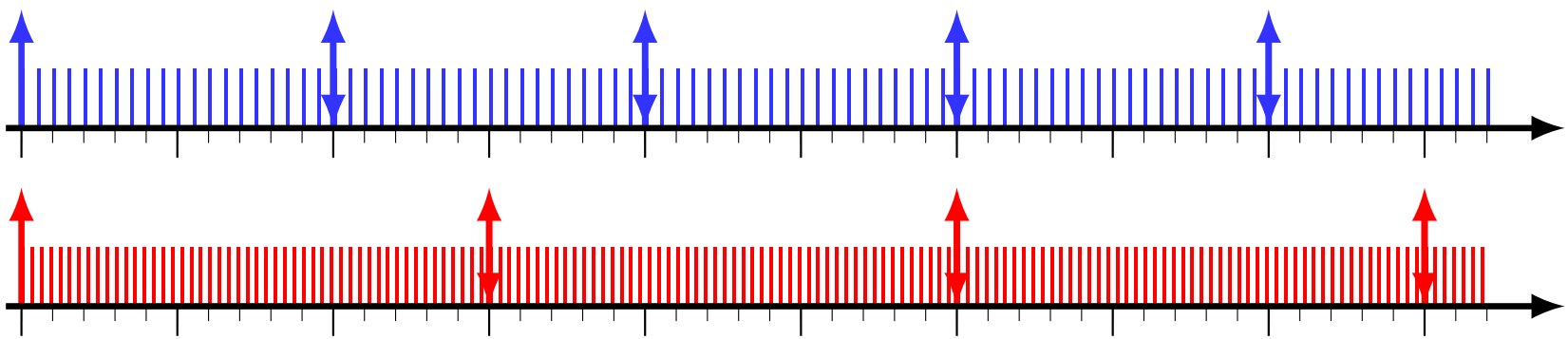

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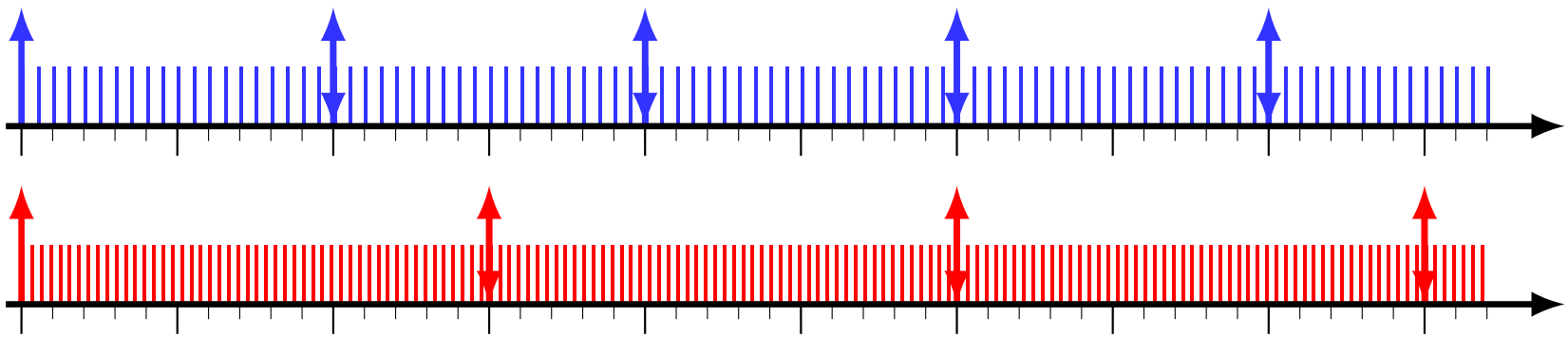

Now it is easy to construct a schedule if the overall rate does not exceed the processing capacity, i.e., if $\mathrm{U}(\mathcal{T}) \leqslant m$.

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Question

How many preemptions can EDF or FP create?

Why is it impractical?

This approach creates a *huge* number of preemptions and migrations. In reality, they are not for free.

Question

How many preemptions can EDF or FP create?

At most one per job. (Why?)

Some main points: Partitioned vs. Global

Partitioned:

- \+ KISS-compatible
- \+ Reuses mature results for single processors
- \+ No migrations
- − Non-optimal
- − Can utilize only 50% of the available resources in the worst case (but more on average)

Global:

- \+ In principle more powerful
- \+ Better at load-balancing and average latency
- − Simple algorithms (G-EDF, G-FP etc.) work poorly for meeting hard deadlines
- − Higher overheads
- − Exact tests are often intractable

Taking the best from both: semi-partitioning

Semi-partitioning

Semi-partitioned scheduling is roughly to

1. partition as many tasks as possible, then
2. *split* the remaining tasks onto several partitions.

Taking the best from both: semi-partitioning

Semi-partitioning

Semi-partitioned scheduling is roughly to

1. partition as many tasks as possible, then
2. *split* the remaining tasks onto several partitions.

We would like the splitting to be done in such a way that ordinary uniprocessor scheduling and analysis can be applied to each processor.

One way for EDF: The $C = D$ approach

To "partition" the task on processors $P_1, \ldots, P_m$ using the $C = D$ approach:

1. Set $k = 1$
2. Put as many tasks as possible on processor P_k
3. Take some task $\tau = (C, D, T)$ that did not fit, and split it into two "tasks"
   - $\tau^1 = (C', C', T)$
   - $\tau^2 = (C - C', D - C', T)$

   such that C' is maximized and τ^1 can fit on P_k.
4. Put τ^1 on P_k and τ^2 on P_{k+1}.
5. Set $k = k + 1$. Repeat from ?? unless $k = m$.
6. Try to put the remaining tasks on P_m.

One way for EDF: The $C = D$ approach

- Read more in *Partitioned EDF Scheduling for Multiprocessors using a C=D Scheme* (Burns et al., 2010)
- Check out this paper for great empirical results using this method (+ some extra tweaks): *Global Scheduling Not Required: Simple, Near-Optimal Multiprocessor Real-Time Scheduling with Semi-Partitioned Reservations* (Brandenburg and Gül, 2016)

Semi-partitioning with fixed-priority?

Recall

A task set $\mathcal{T}$ with implicit deadlines is schedulable with RM priority ordering on a single processor if

$$\mathrm{U}(\mathcal{T}) \leqslant \ln(2) \approx 0.69.$$

There exists a semi-partitioned FP algorithm for implicit-deadline sporadic tasks that achieves a perfect scaling to m processors:

$$\mathrm{U}(\mathcal{T}) \leqslant m \cdot \ln(2)$$

- Read more in *Fixed-Priority Multiprocessor Scheduling with Liu & Layland's Utilization Bound* (Guan et al., 2010)