Scheduling theory, part 1

Aperiodic jobs

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2018-09-14

Real-time systems?

Not necessarily fast…

…but always predictable

Real-time systems: theory & practice

Practice

Building systems

Software, middleware, RTOS…

Hardware design

⋮

Real-time systems: theory & practice

Theory

Verification

Scheduling theory

Control theory

⋮

Practice

Building systems

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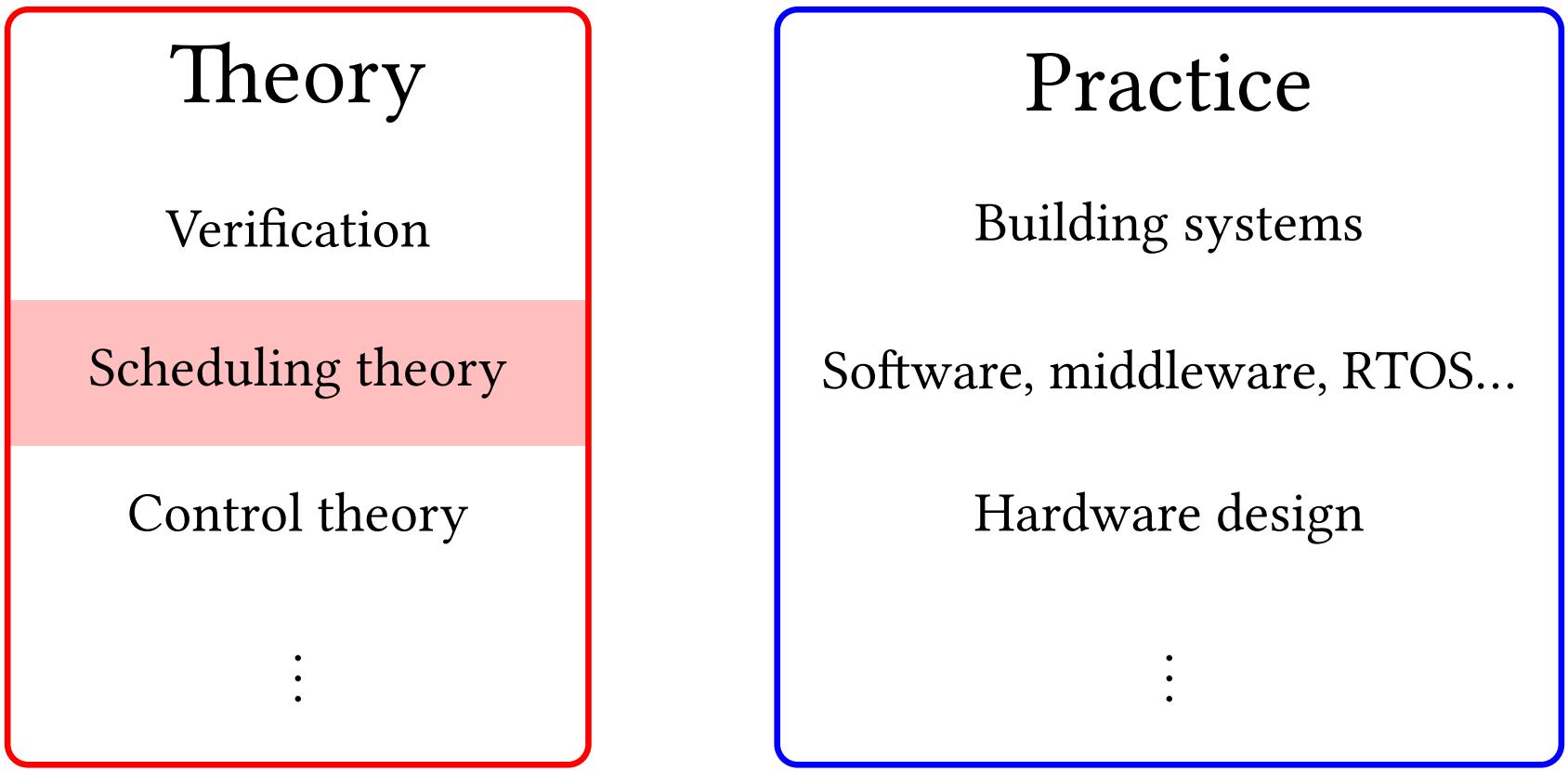
⋮

Real-time systems: theory & practice



Scheduling theory

Classical scheduling theory (e.g., in operations research)

generally deals with *finite* processes (job-shop, flow-shop &c.)

to *optimize* some metric.

Real-time scheduling theory

generally deals with *infinite* processes (control loops &c.)

to *guarantee* a safety specification.

The components of real-time scheduling theory

Task models:

Formalisms to specify workload and timing constraints

Scheduling algorithms:

Run-time strategies for scheduling workload

Analysis:

Offline methods for proving timing safety

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Modeling: the art of abstraction

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Edsger W. Dijkstra, 1972

"*[...] the purpose of abstracting is* **not** *to be vague, but to create a new semantic level in which one can be absolutely precise.*"

The job: a unit of work

A job j_i is given by a triple $(A_i, C_i, D_i) \in \mathbb{N}^3$, where

- A_i is the arrival time (or release time).
- C_i is the worst-case execution time (WCET), and
- D_i is the deadline.

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$$j_i = (A_i, C_i, D_i) = (3, 6, 21)$$

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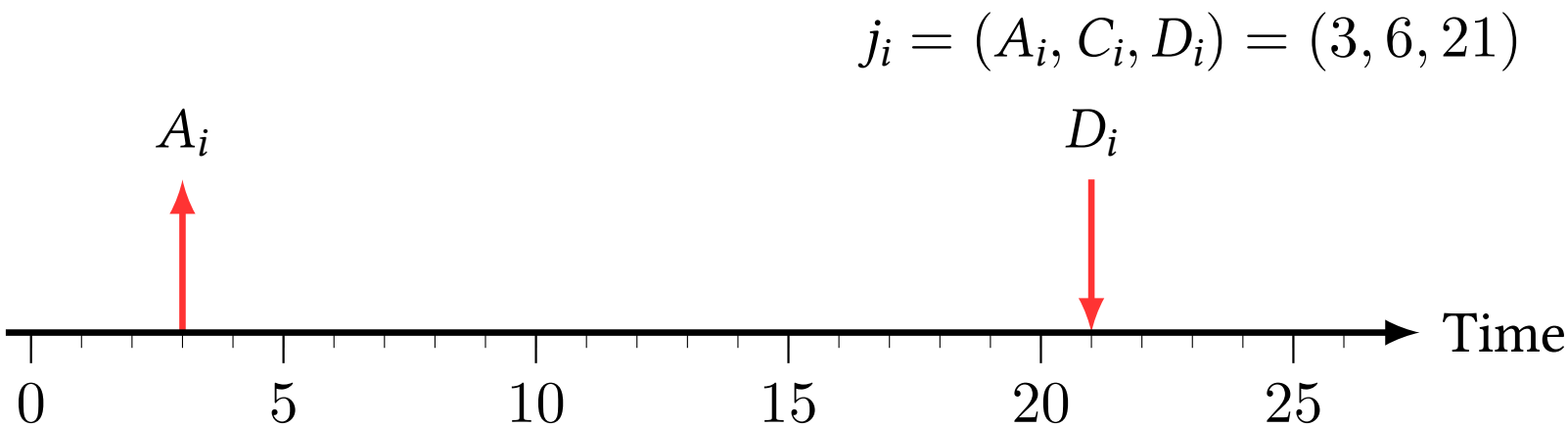

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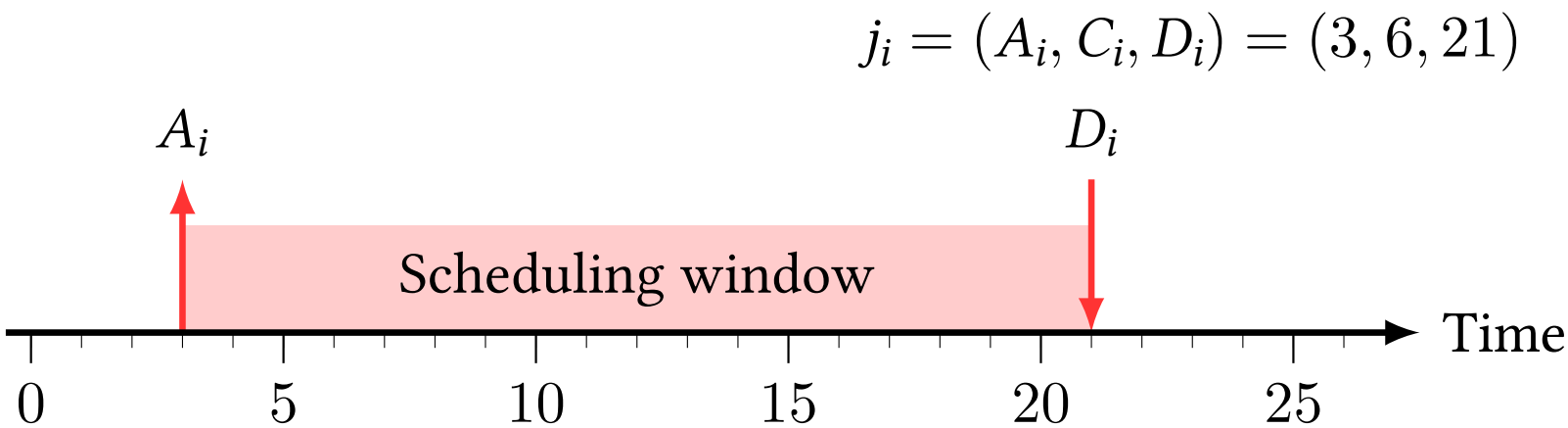

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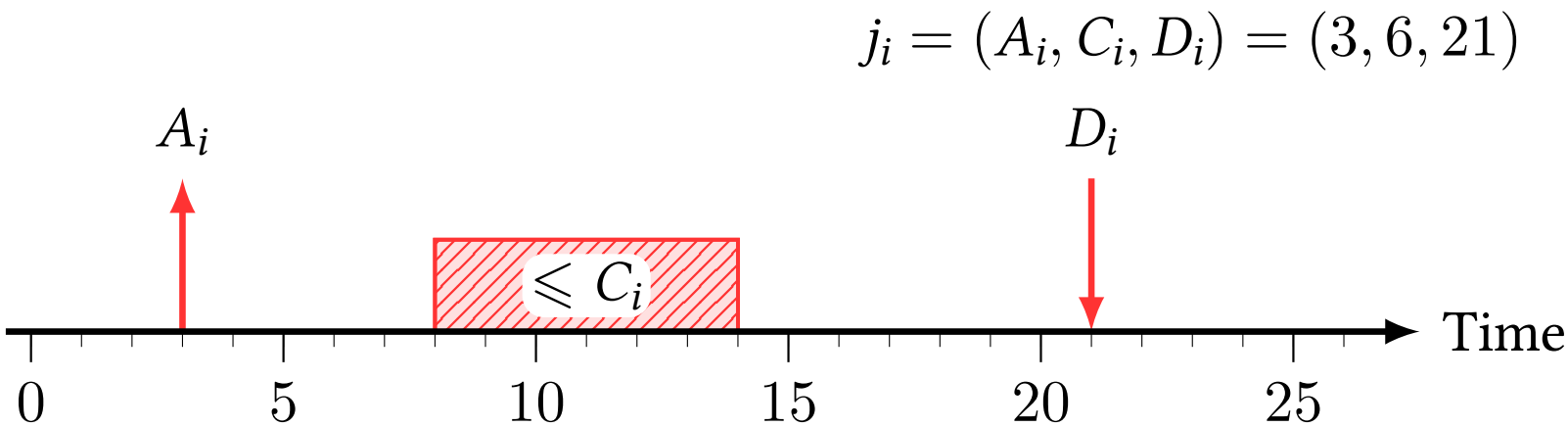

Scheduling a collection of independent jobs

The problem

Given a (multi-)set $\mathcal{J} = \{j_1, \ldots, j_n\}$ of n jobs, find a schedule where all jobs meet their deadlines.

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- All jobs are independent
- A single processor
- Fully preemptive *-or-* non-preemptive scheduling

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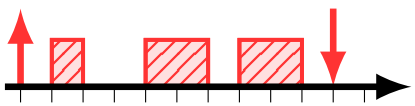
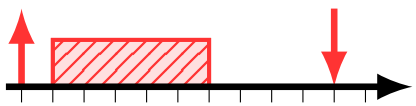
Preemptive: 

Non-preemptive: 

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Challenge

Schedule these jobs

$$\mathcal{J} = \left\{ \begin{array}{lll} (0,2,6), & (0,2,14), & (0,2,3), \\ (0,7,13), & (0,1,15), & (0,1,2) \end{array} \right\}$$

Note: All jobs in $\mathcal{J}$ have the same arrival time. Such jobs are called *synchronous*.

A solution

Earliest Deadline First (EDF)

Scheduling rule: *Choose among the ready jobs to execute the job with the earliest deadline (ties broken arbitrarily).*

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Note 2: In this setting, EDF is also called *Earliest Due Date (EDD)* or *Jackson's algorithm*.

Is this a good general strategy?

Question

Is EDF a good strategy for all sets of synchronous jobs?

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Theorem (Jackson, 1955)

If it is possible to schedule a set $\mathcal{J}$ of synchronous jobs, then $\mathcal{J}$ can also be scheduled by EDF.

Proof on black board!

Some important definitions

Schedulability

$\mathcal{J}$ is *$\mathcal{A}$-schedulable* iff scheduling algorithm $\mathcal{A}$ always generates a schedule without deadline misses for $\mathcal{J}$.

Feasibility

$\mathcal{J}$ is *feasible* iff there exists a scheduling algorithm $\mathcal{A}$ such that $\mathcal{J}$ is $\mathcal{A}$-schedulable.

Optimality

$\mathcal{A}$ is *optimal* iff all feasible $\mathcal{J}$ are also $\mathcal{A}$-schedulable.

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Some simple analysis

How do we know if a set $\mathcal{J}$ of synchronous jobs is EDF-schedulable?

Schedulability test (Jackson, 1955)

Without loss of generality, let the indices of the jobs in $\mathcal{J} = \{j_1, \ldots, j_n\}$ be ordered by non-decreasing deadlines, and let all the arrival times be zero. Then, $\mathcal{J}$ is EDF-schedulable iff

$$\forall i, 1 \leqslant i \leqslant n : \quad \sum_{k=1}^{i} C_k \leqslant D_i.$$

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For a set $\mathcal{J}$ of jobs with *asynchronous* arrival times, does it matter if we allow preemptions?

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YES!

The preemptive case

Question

Is EDF still optimal for preemptive scheduling of job sets with asynchronous arrival times?

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Theorem (Dertouzos, 1973)

EDF is optimal for scheduling *any* set of independent jobs on a single preemptive processor.

Proof of the optimality of EDF

First, let's define an important function.

The demand bound function

For a job j_i and time instants t_1 and t_2, where $0 \leqslant t_1 \leqslant t_2$, let the *demand bound function* $\mathrm{dbf}(j_i, t_1, t_2)$ be defined as

$$\mathrm{dbf}(j_i, t_1, t_2) = \begin{cases} C_i, & \text{if } t_1 \leqslant A_i \text{ and } D_i \leqslant t_2 \\ 0, & \text{otherwise.} \end{cases}$$

For a job set $\mathcal{J}$, let $\mathrm{dbf}(\mathcal{J}, t_1, t_2)$ be defined as

$$\mathrm{dbf}(\mathcal{J}, t_1, t_2) = \sum_{j_i \in \mathcal{J}} \mathrm{dbf}(j_i, t_1, t_2).$$

Proof of the optimality of EDF

Feasibility test / EDF-schedulability test

A job set $\mathcal{J}$ is feasible on a single preemptive processor iff

$$\forall t_1, t_2 \text{ such that } 0 \leqslant t_1 \leqslant t_2 : \quad \text{dbf}(\mathcal{J}, t_1, t_2) \leqslant t_2 - t_1.$$

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Conclusion: The test is valid *and* EDF is optimal. (Why?)

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Proofs on the blackboard!

The preemptive case: conclusions

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$$\forall t_1, t_2 \text{ such that } 0 \leqslant t_1 \leqslant t_2 : \quad \text{dbf}(\mathcal{J}, t_1, t_2) \leqslant t_2 - t_1.$$

(It is enough to consider values of t_1 picked from the arrival times and values of t_2 picked from the deadlines.)

The non-preemptive case

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Is EDF still optimal for non-preemptive scheduling of job sets with asynchronous arrival times?

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NO!

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(But it is still optimal if idling is forbidden! Proof omitted.)

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Step 1: Assume the execution time of all jobs is the WCET.
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Step 2: Try all possible orderings of executing the jobs.

Good news: This works!

Bad news: There are $n!$ orderings of n jobs.

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Probably not: This problem is strongly NP-hard.
(There is a simple reduction from 3-PARTITION.)

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In practice, various heuristic search techniques could work well.