Workload Models

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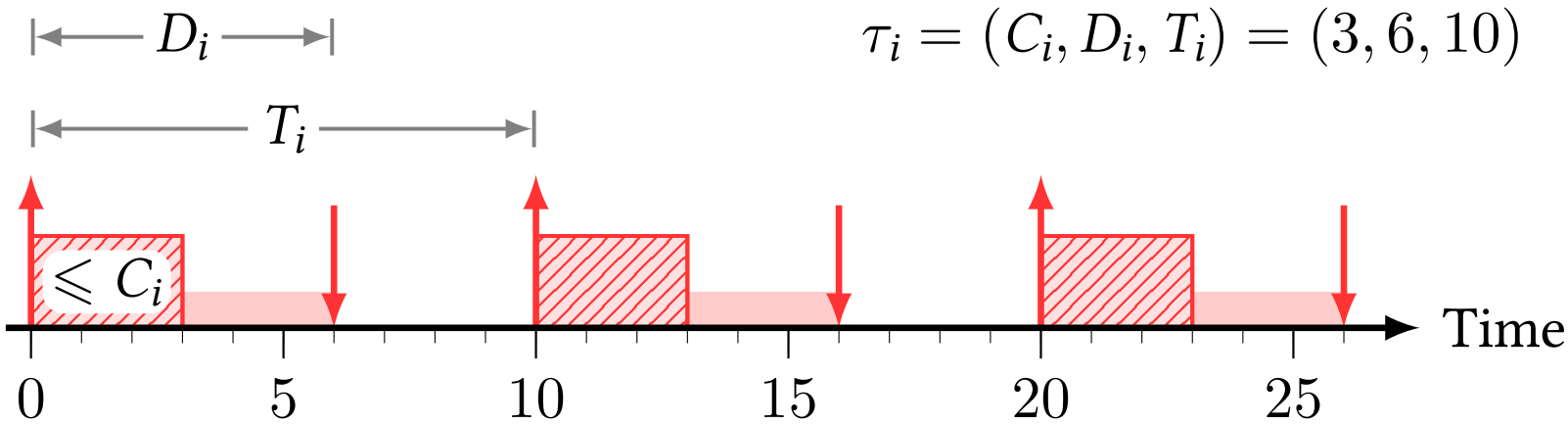
(With some slides borrowed from Martin Stigge)

Recap

A sporadic task τ_i is given by a triple $(C_i, D_i, T_i) \in \mathbb{N}^3$, where

- C_i is the worst-case execution time (WCET),
- D_i is the relative deadline, and
- T_i is the minimum inter-release separation time (or just *period*).



Today's topic

1. We will look at a few *generalizations* of the sporadic task model, which allow for more complicated patterns of job releases.
2. We will outline analysis techniques for feasibility (or EDF-schedulability) for one of these task models.

Recall the feasibility test for sporadic tasks

Feasibility test

A sporadic task set $\mathcal{T}$ is feasible on a single preemptive processor iff

$$\forall t, \text{ such that } t \geqslant 0 : \quad \text{dbf}(\mathcal{T}, t) \leqslant t.$$

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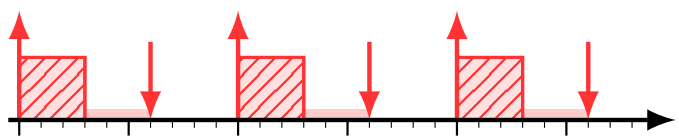

(C_i, D_i, T_i)

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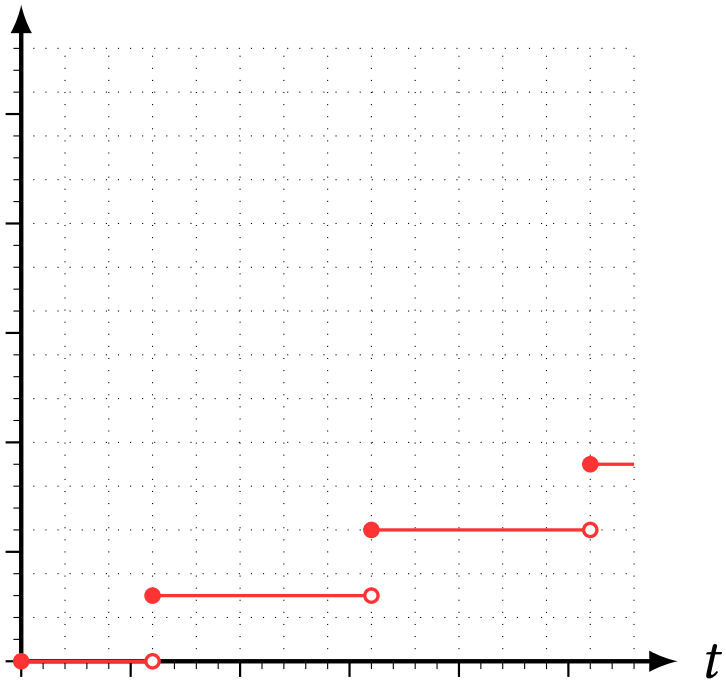

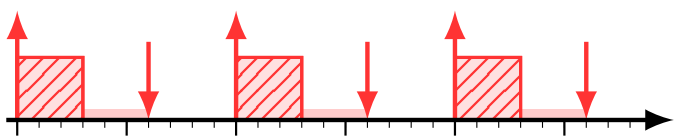

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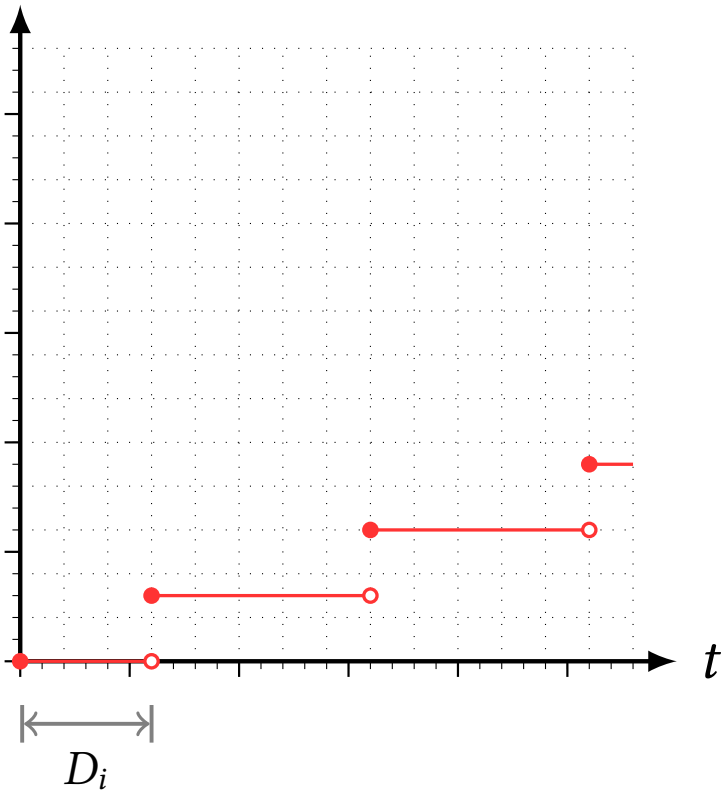

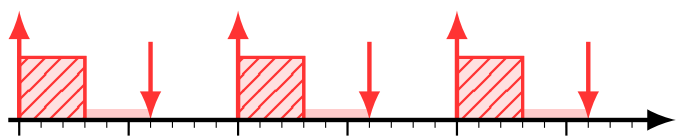

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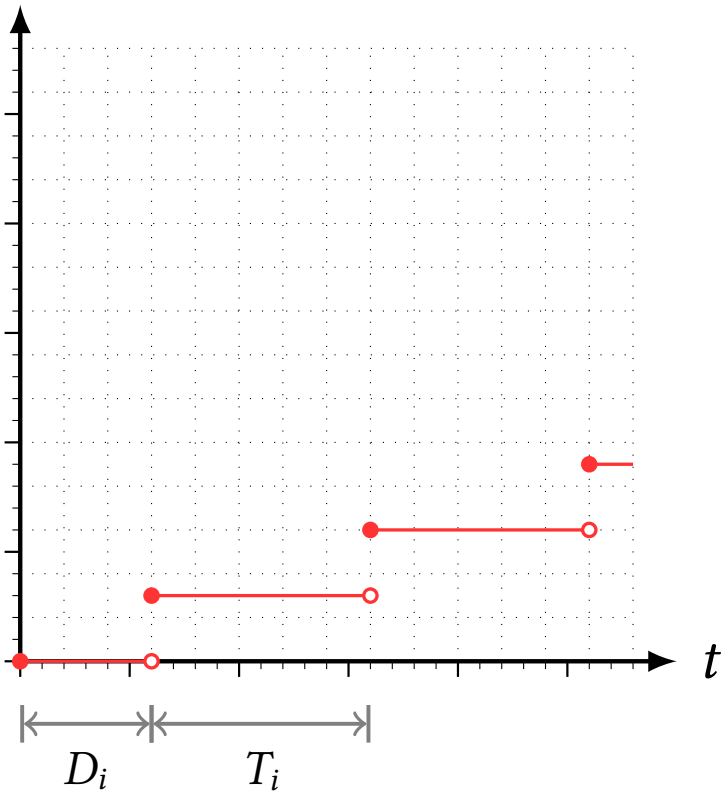

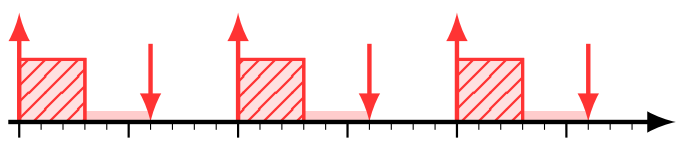

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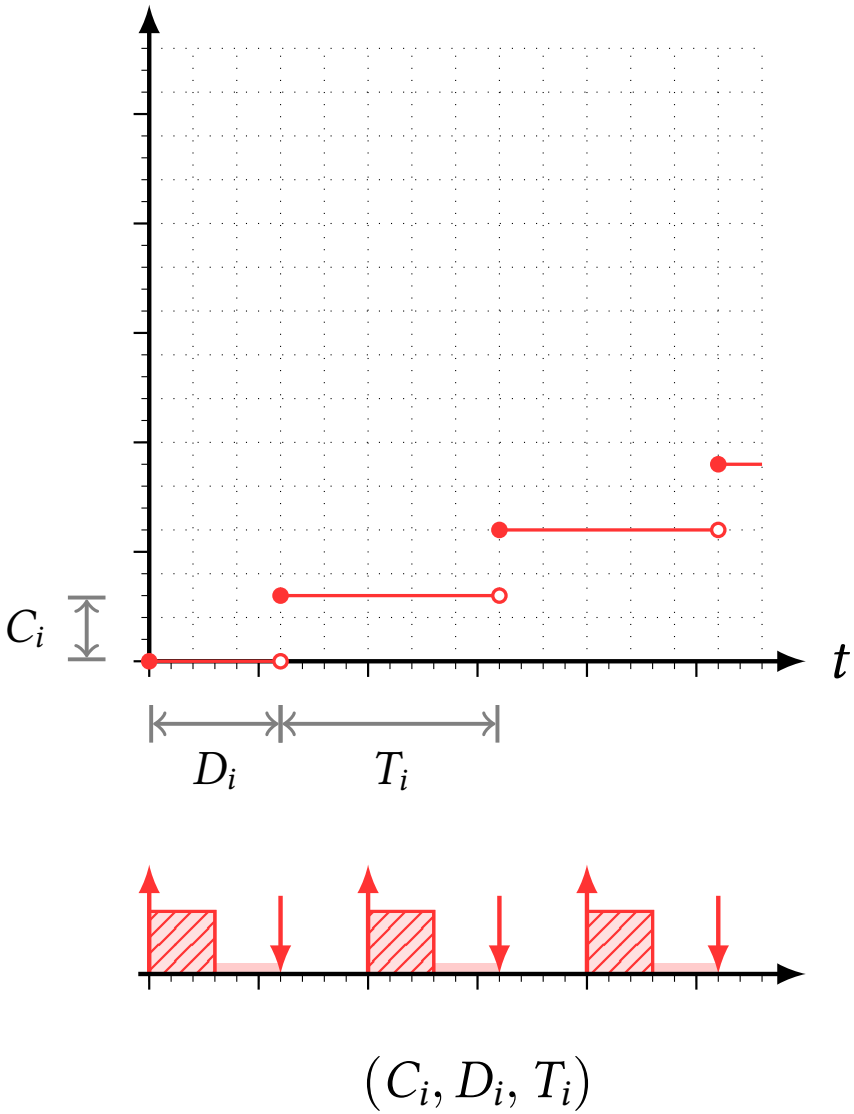

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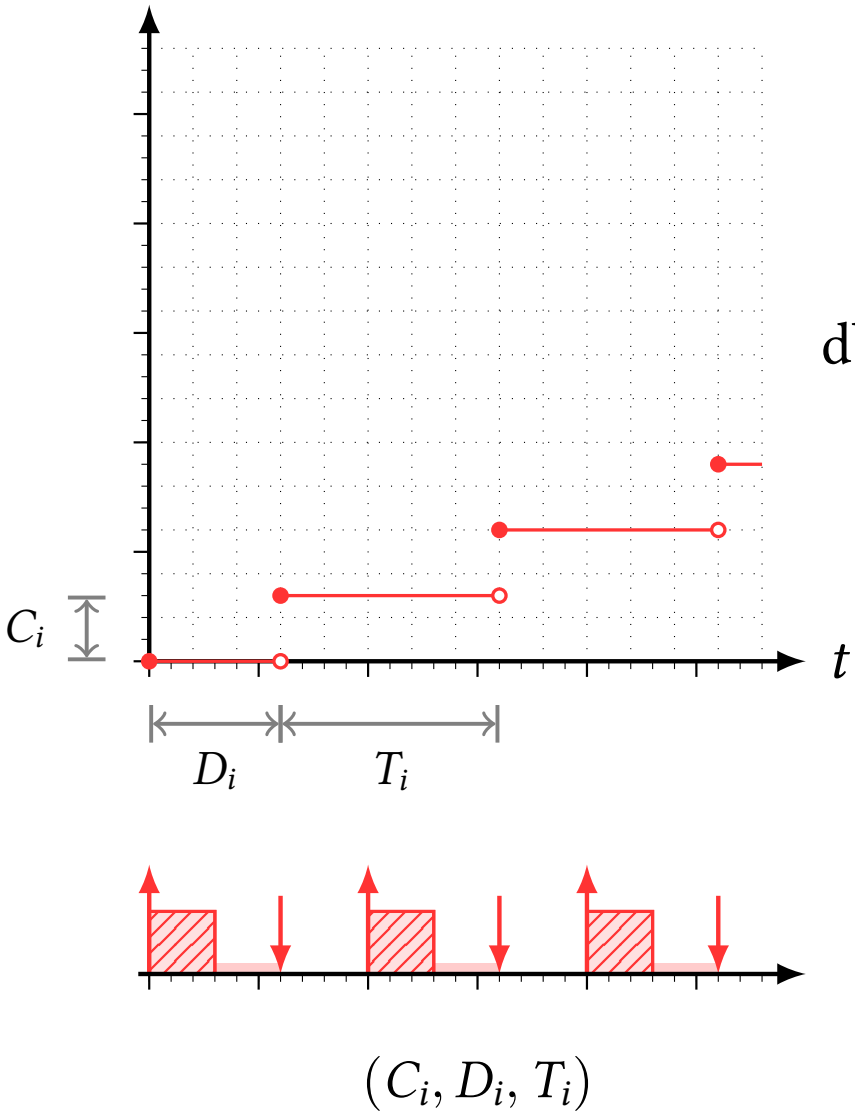

$$\mathrm{dbf}(\tau_i, t) = \max\left(0, \left\lfloor \frac{t - D_i}{T_i} \right\rfloor + 1\right) \cdot C_i$$

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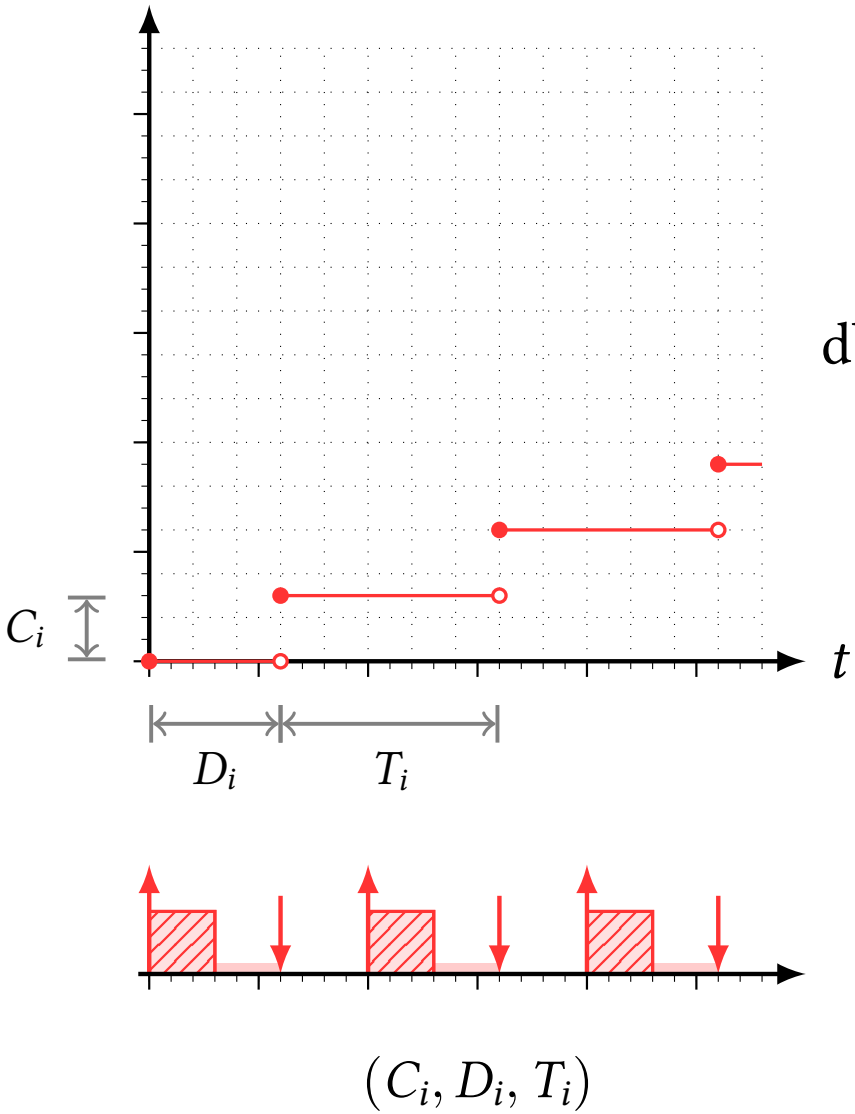

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$$\mathrm{dbf}(\mathcal{T}, t) = \sum_{\tau_i \in \mathcal{T}} \mathrm{dbf}(\tau_i, t)$$

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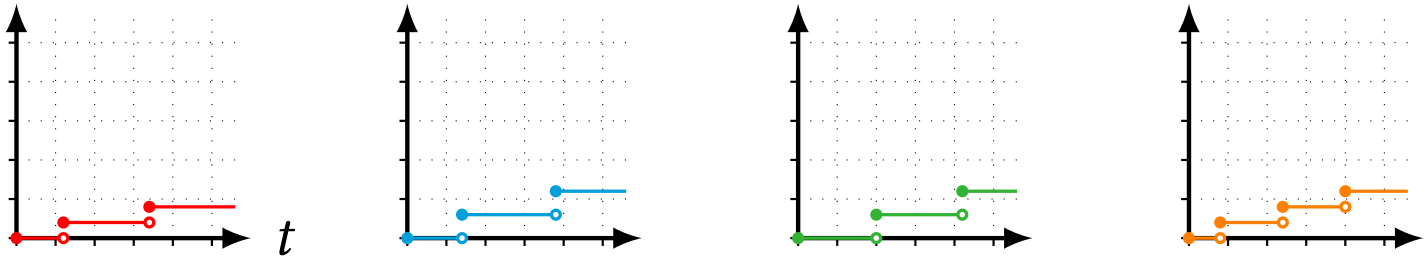

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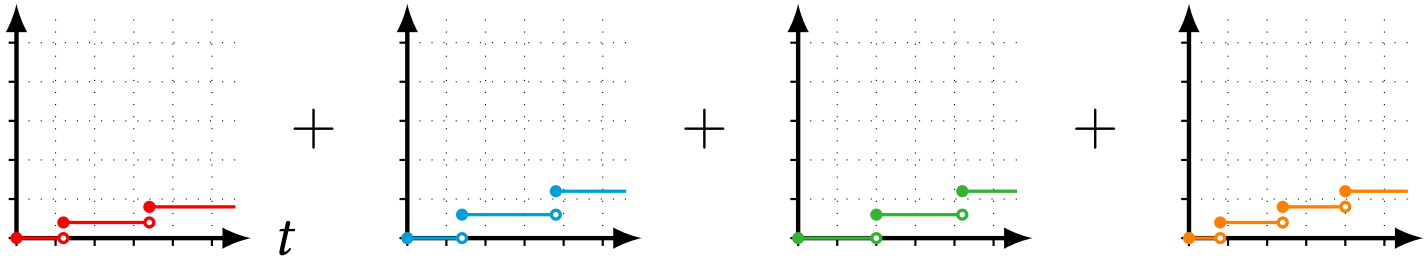

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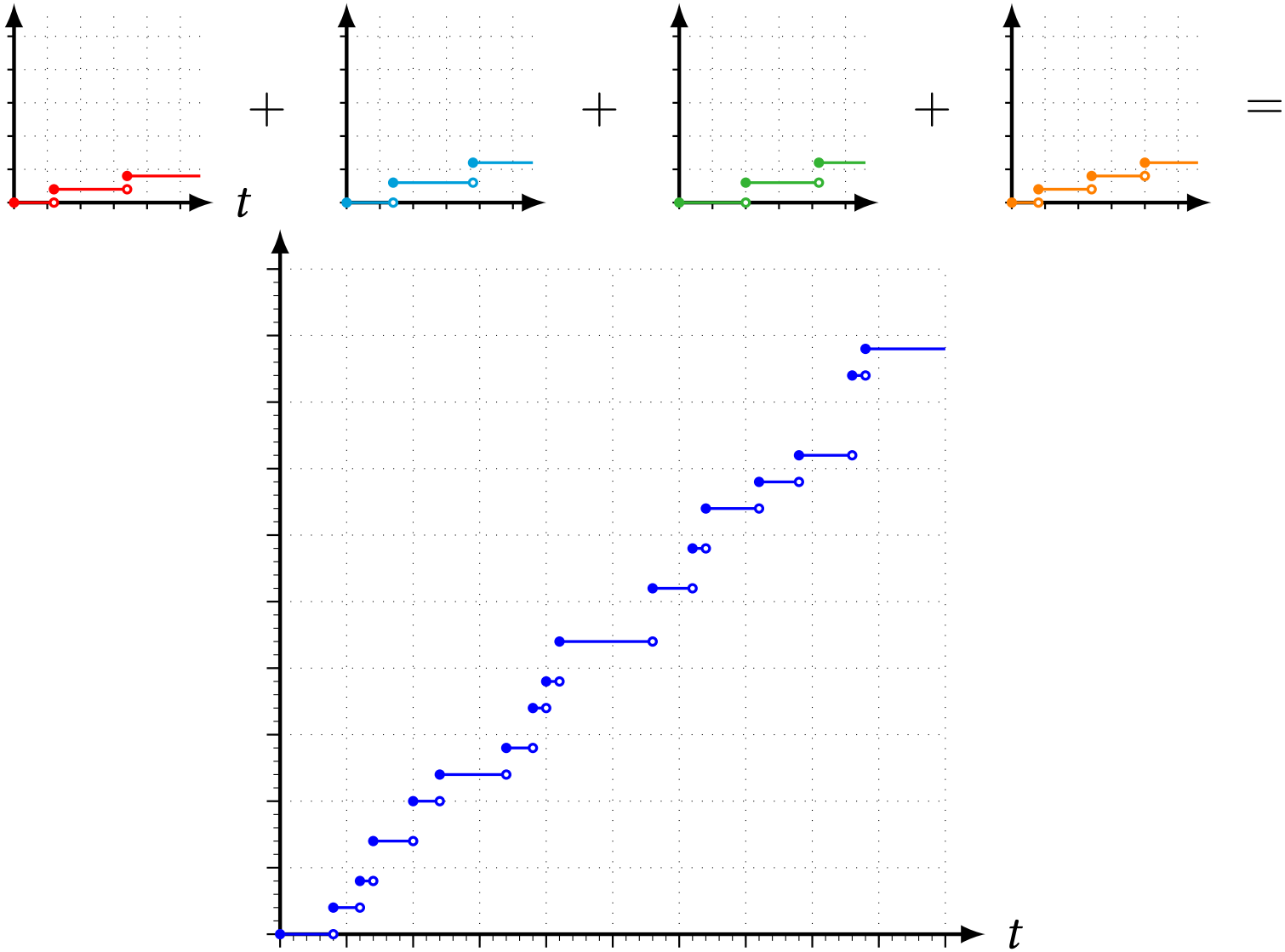

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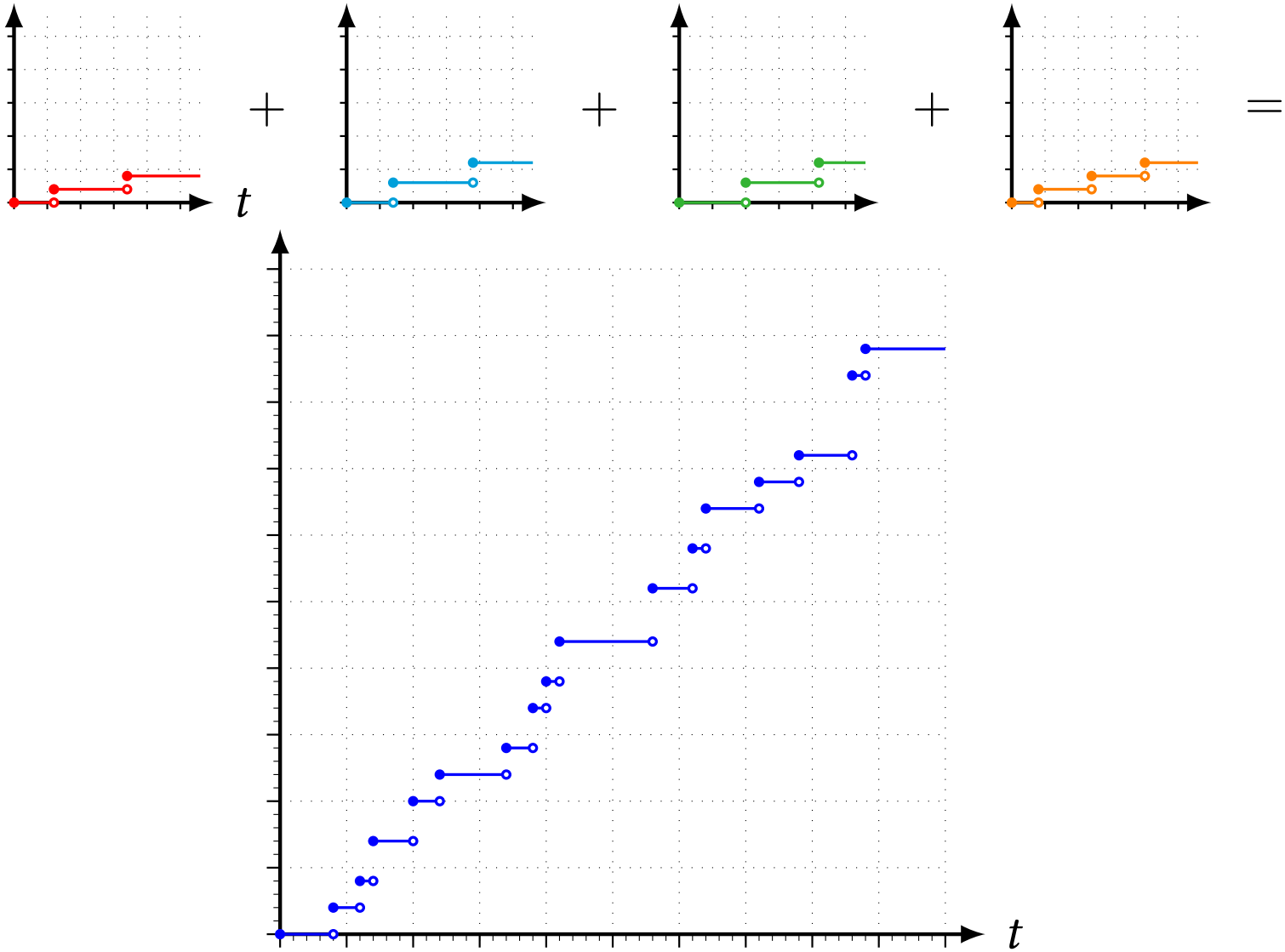

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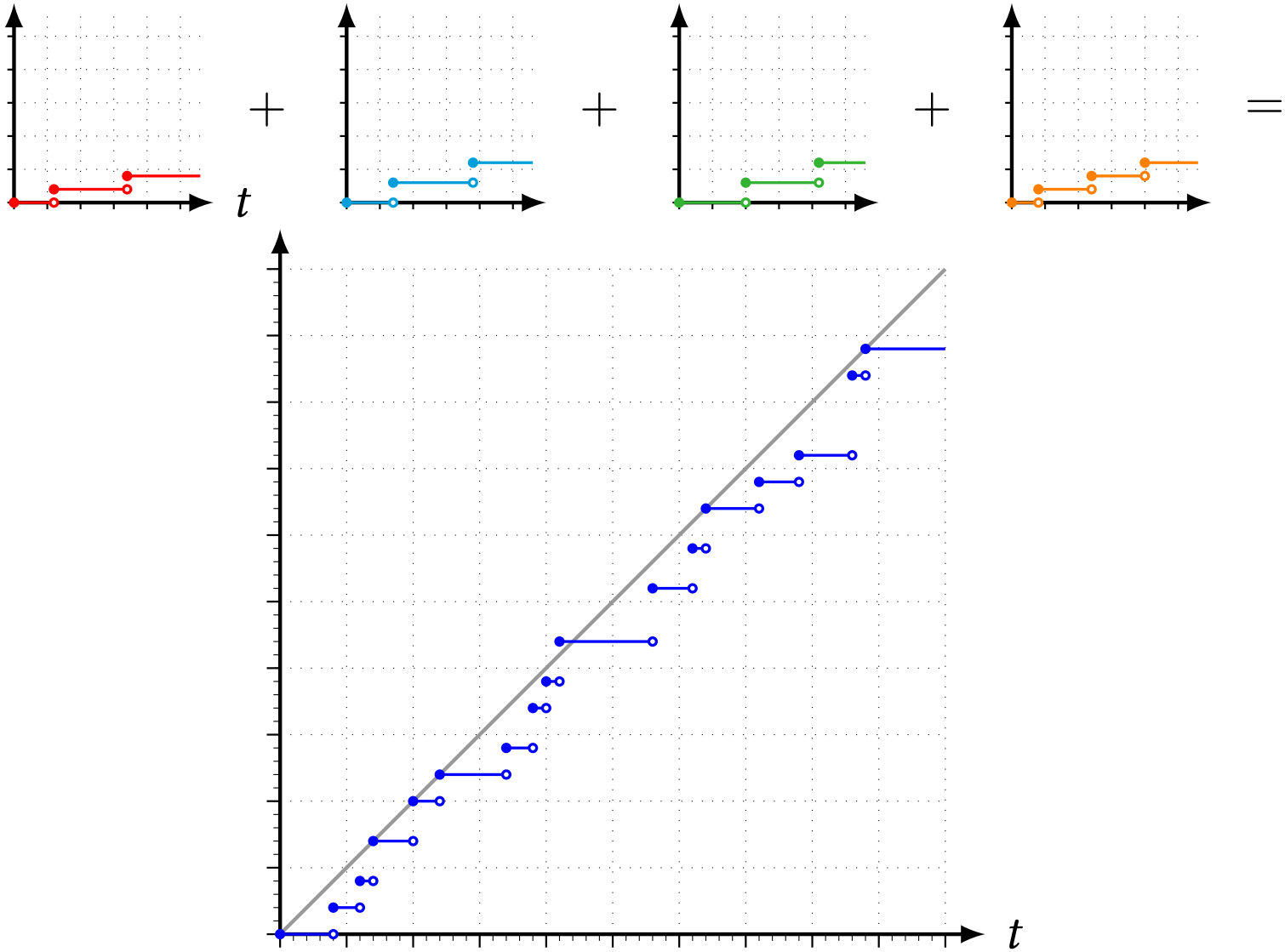

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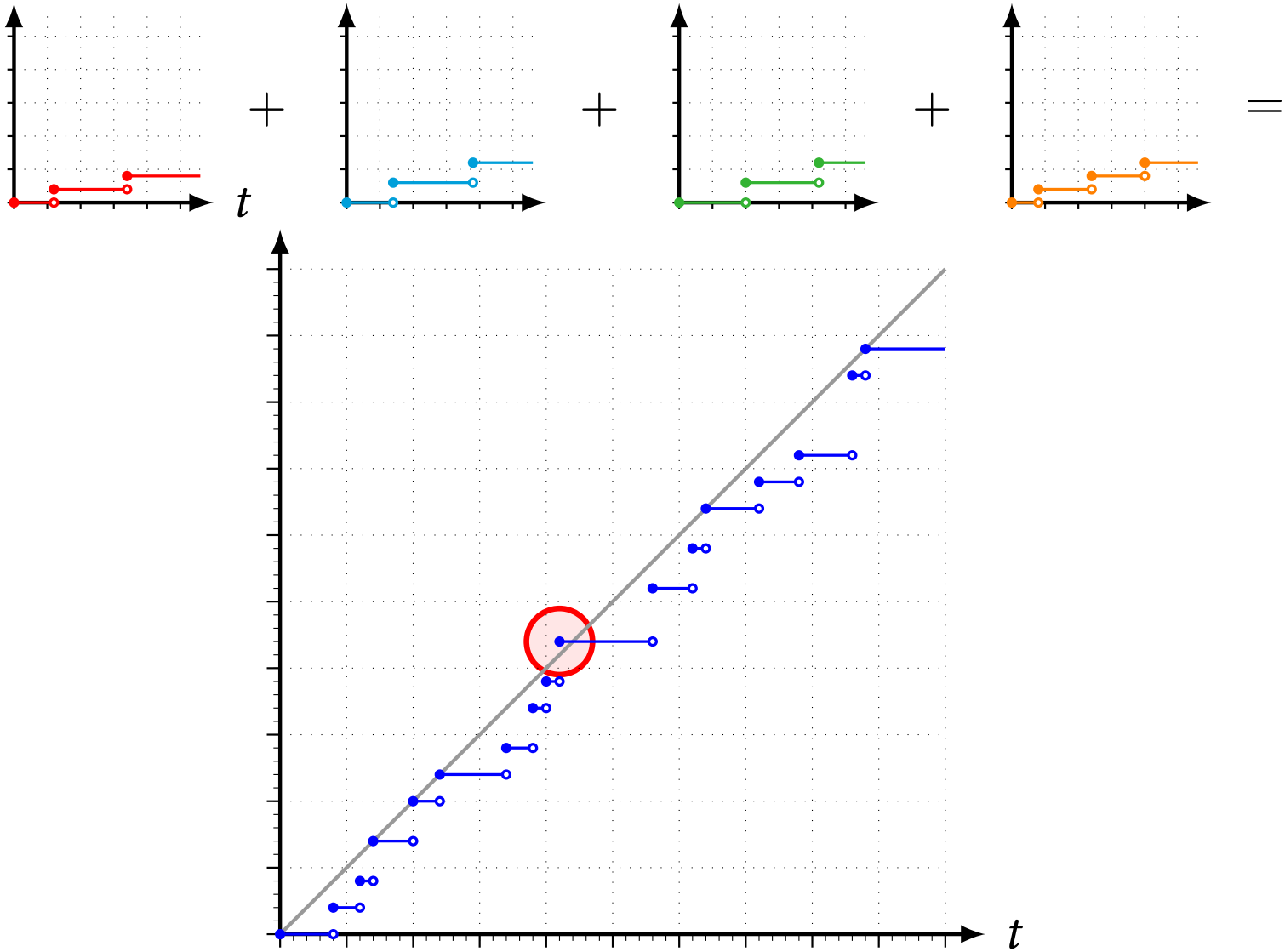

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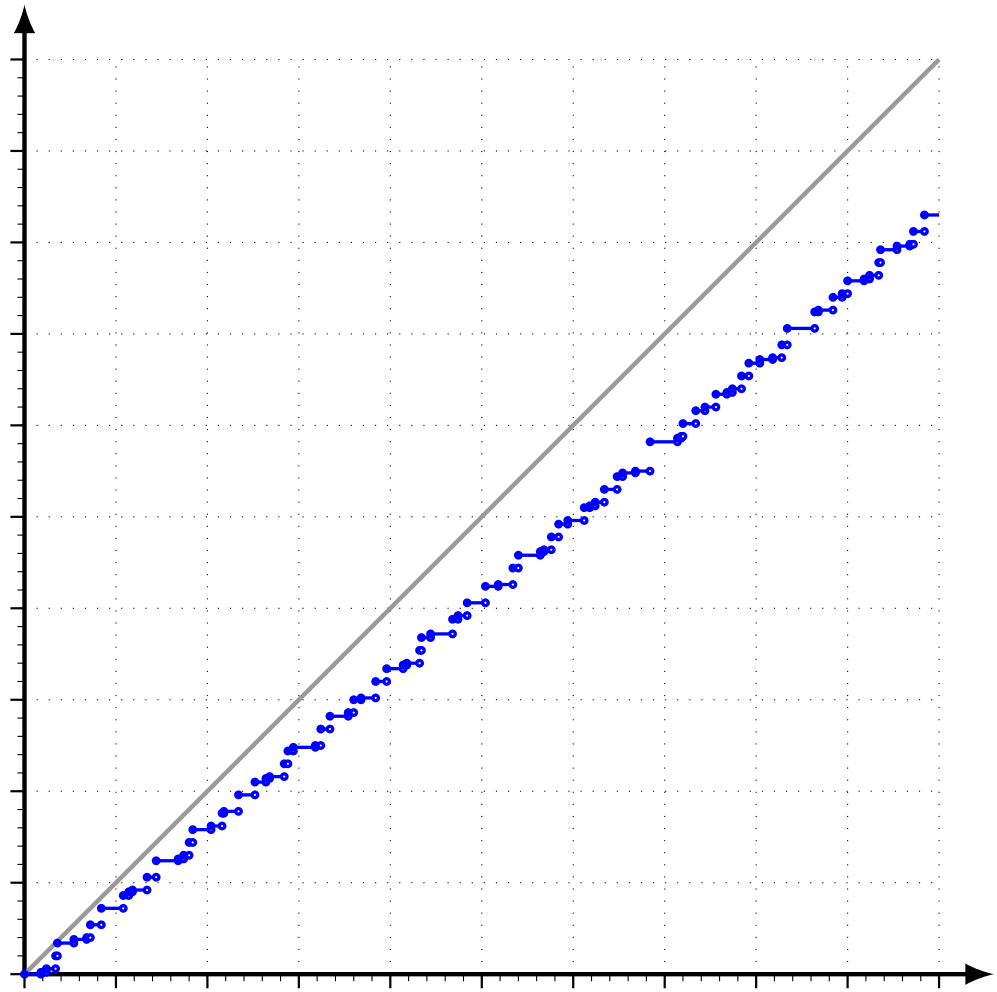

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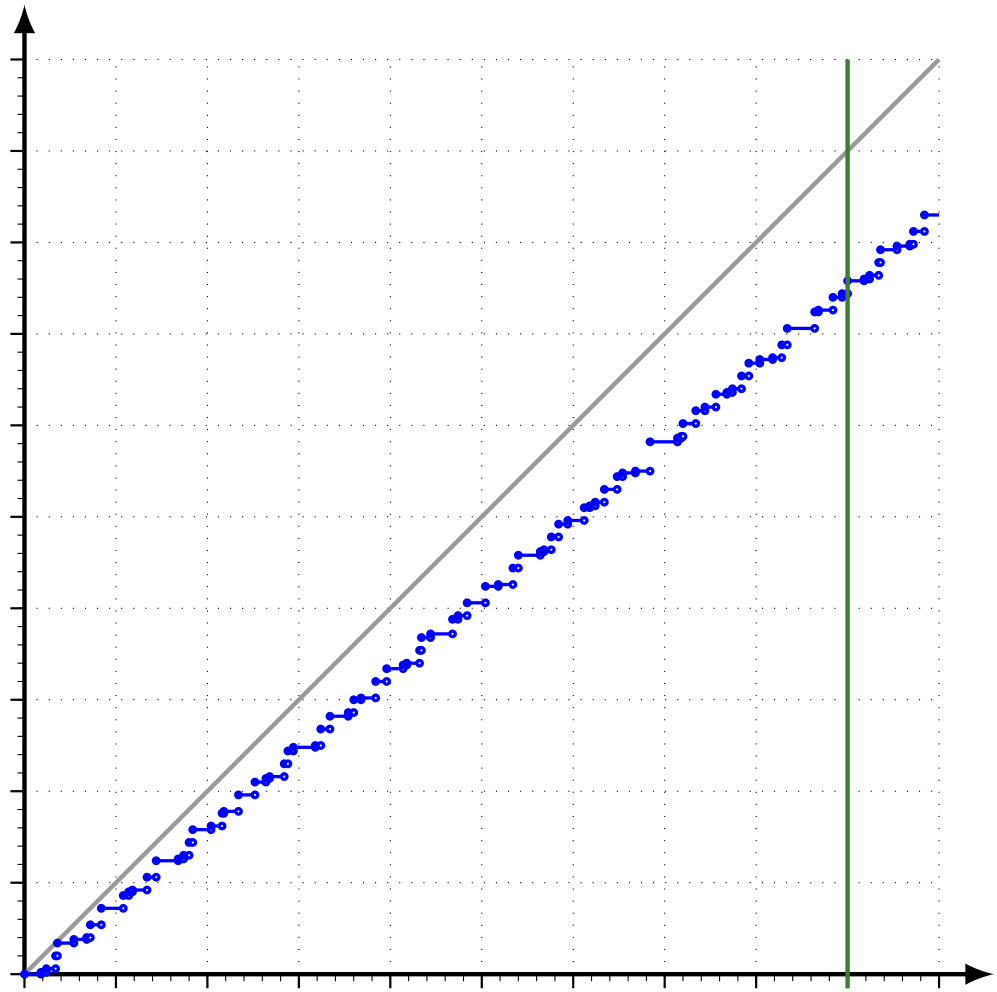

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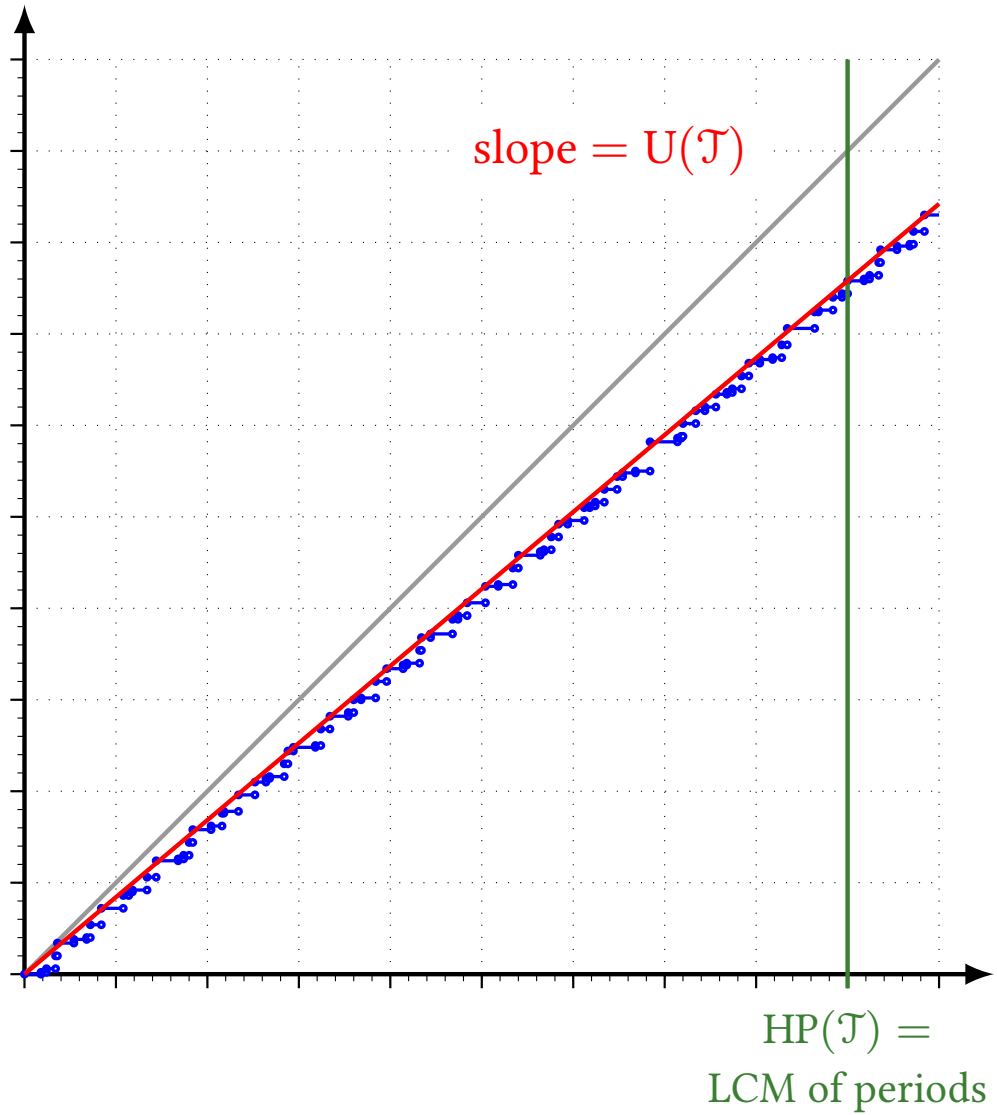

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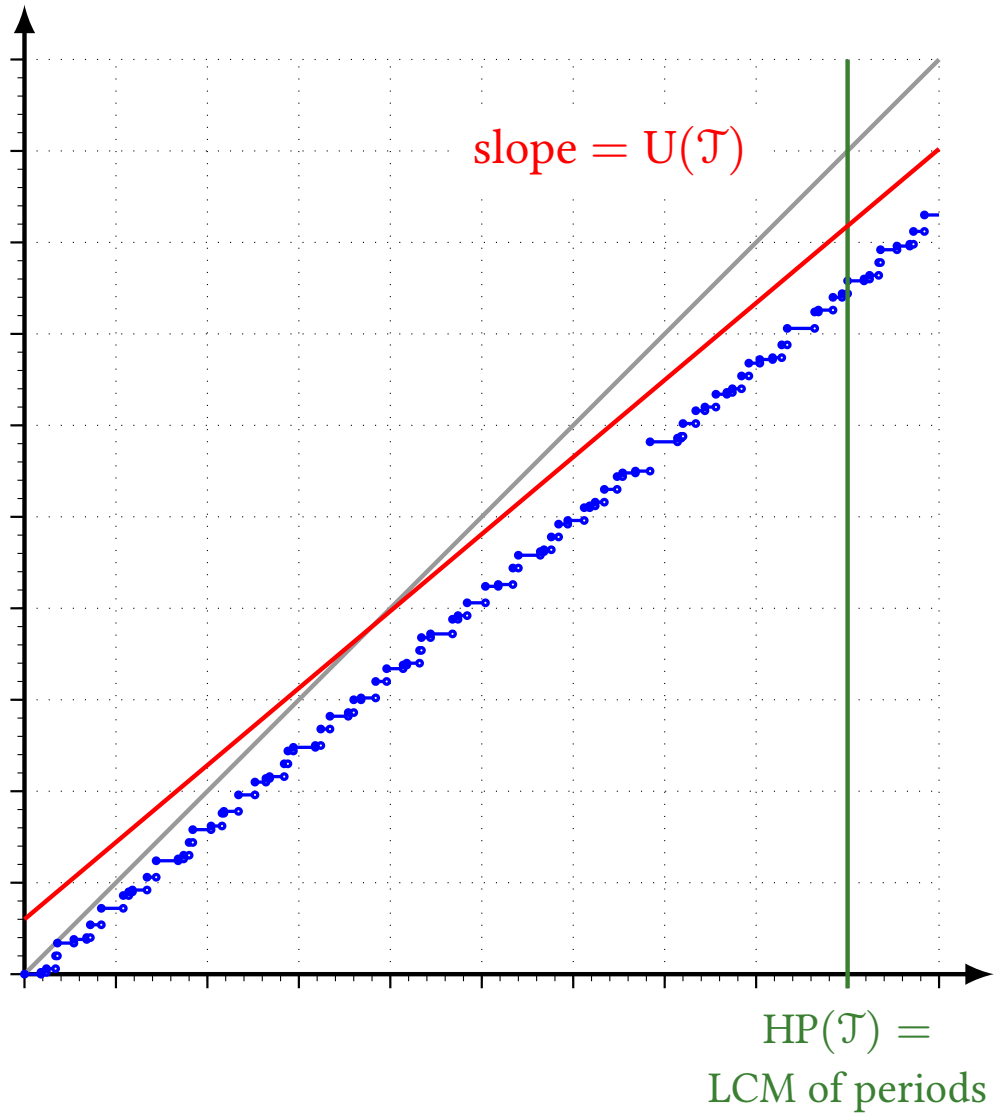

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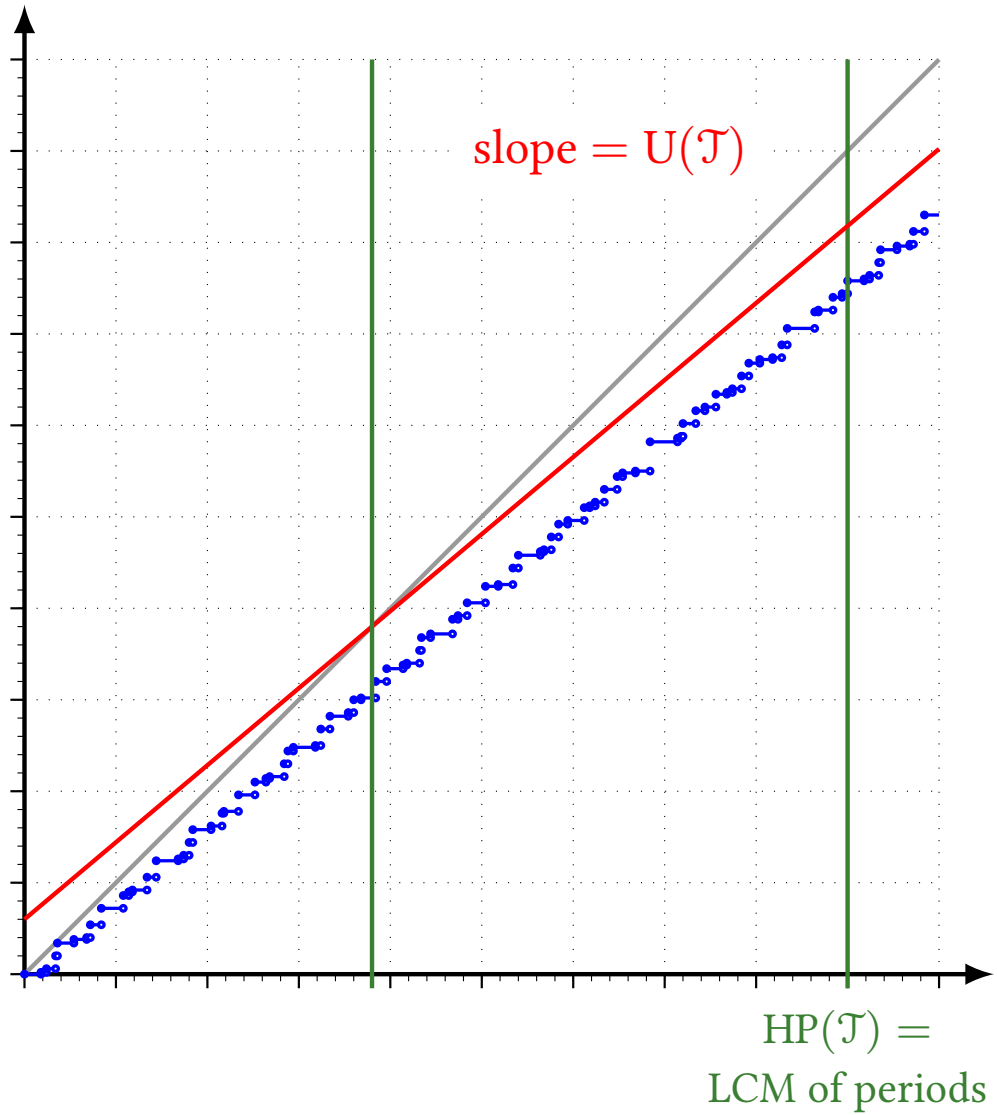

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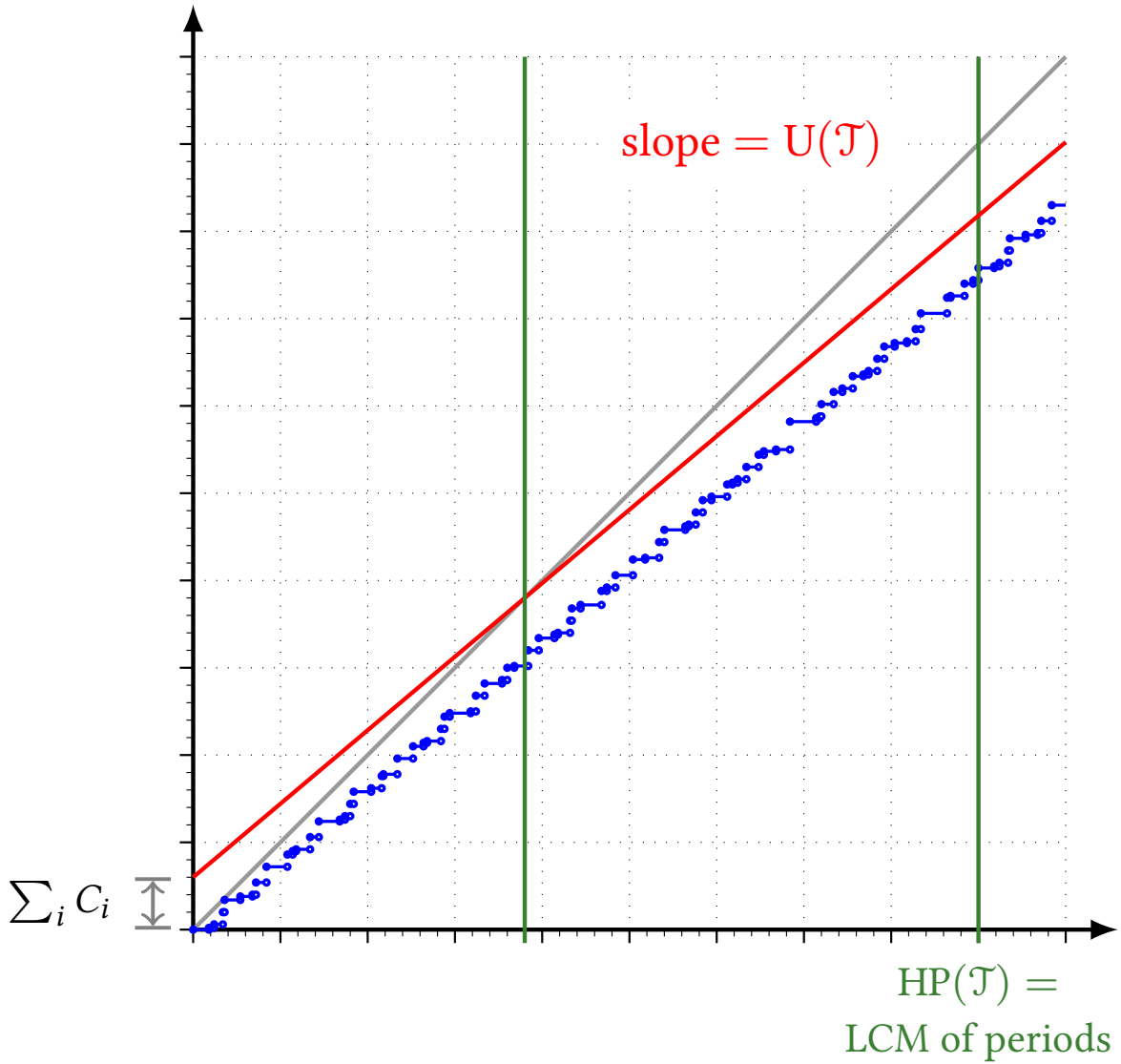

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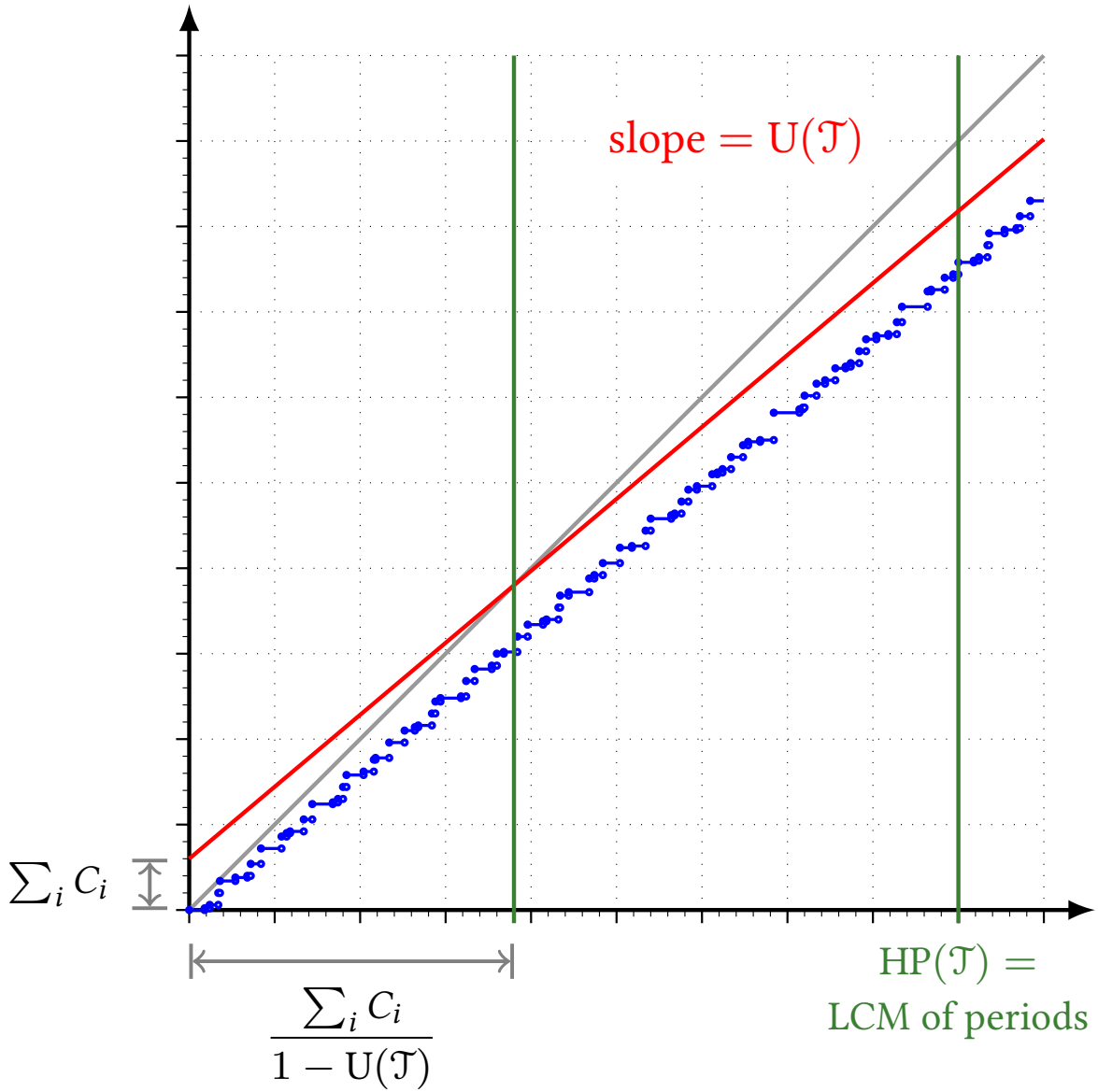

Generalizing sporadic tasks

- Example code structure for a periodic/sporadic task:

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loop

    // Execute some function for, e.g.,
    // up to 11ms
    // (obtained via WCET analysis)

    delay until Previous_Period + 50ms;

end loop;
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- What if the structure is more complicated?

A more complicated control structure

- Code is not always periodic:

```
loop

    // Execute some function
    delay until Period_Start + 50ms;
    // Execute another function
    delay until Prev_Function + 30ms;
    // Execute yet another function
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A more complicated control structure

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```

- Here a task is split in *frames*, each with own
  - Execution time
  - Inter-release separation until next frame
  - Relative deadline

The Generalized Multiframe (GMF) Task Model

- Each task *cycles* through job types
  - Vector for WCET $(e^{(1)}, \ldots, e^{(n)})$
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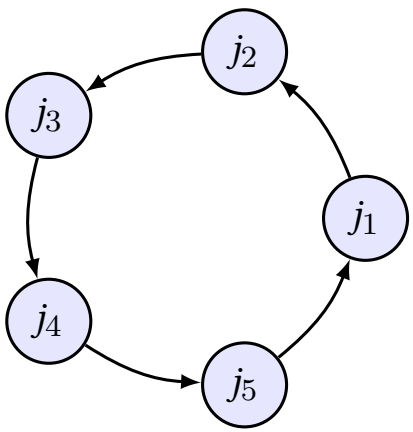

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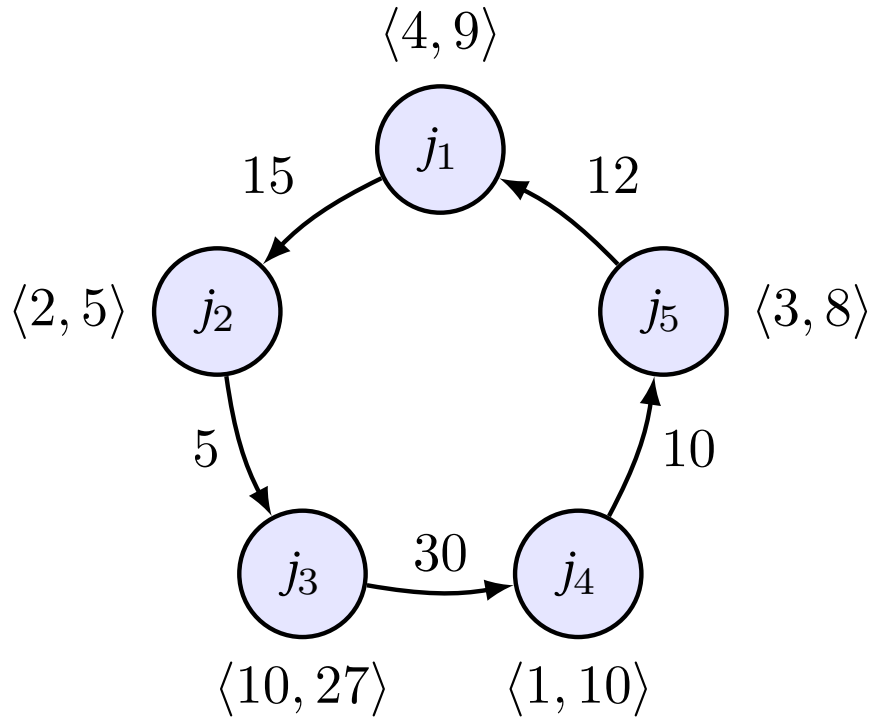

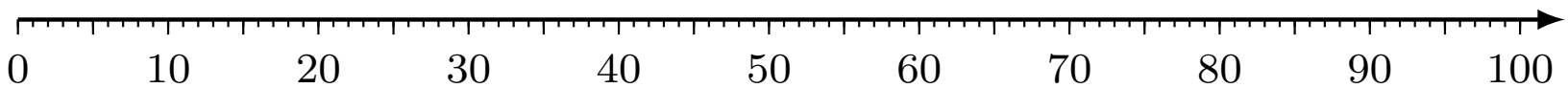

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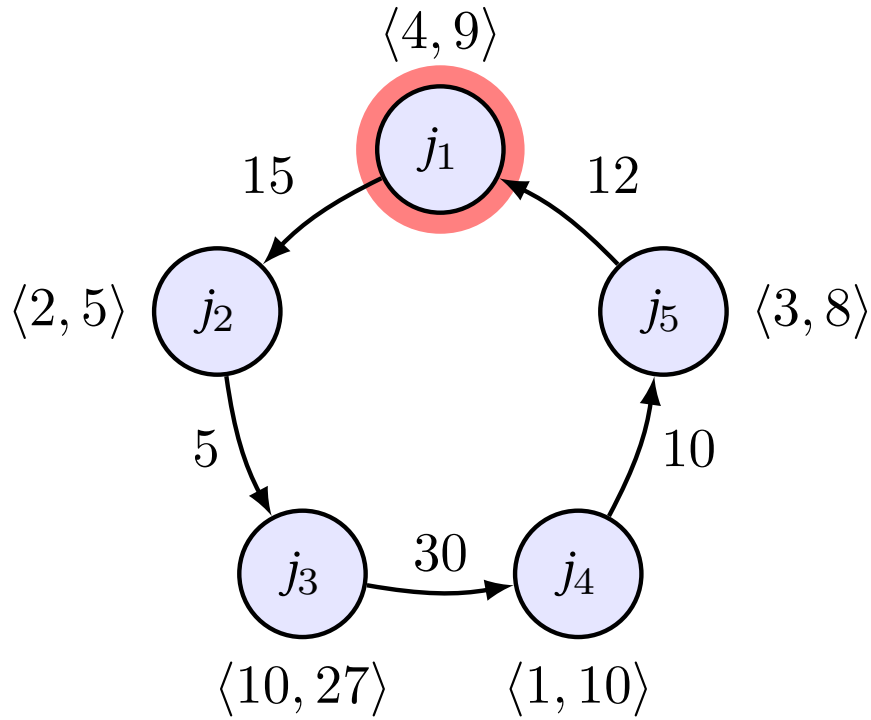

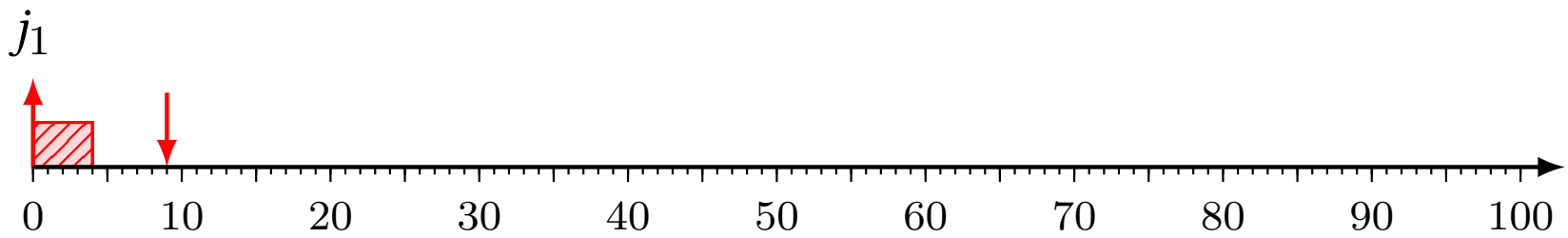

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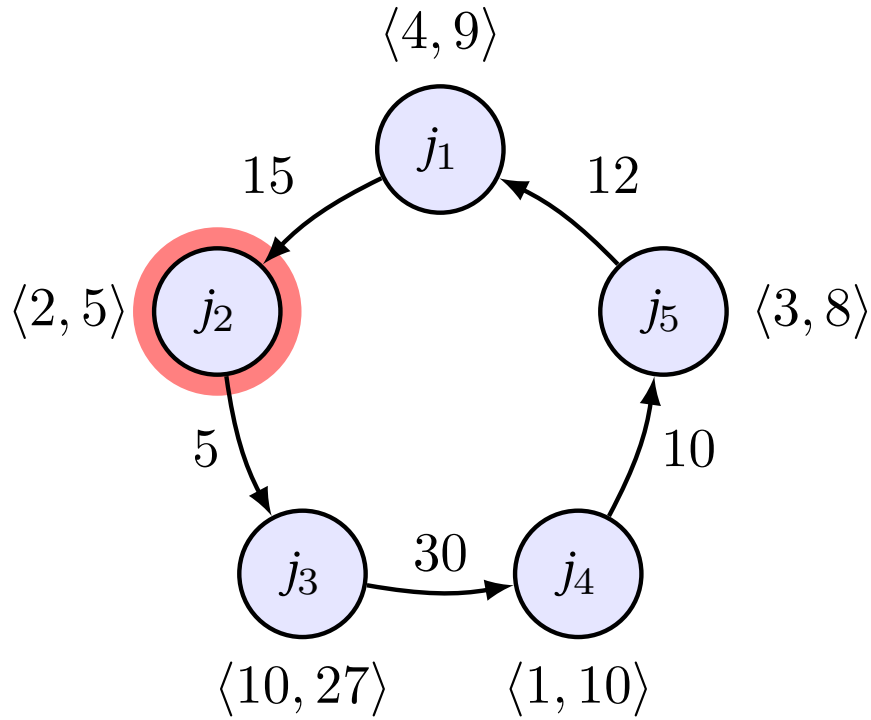

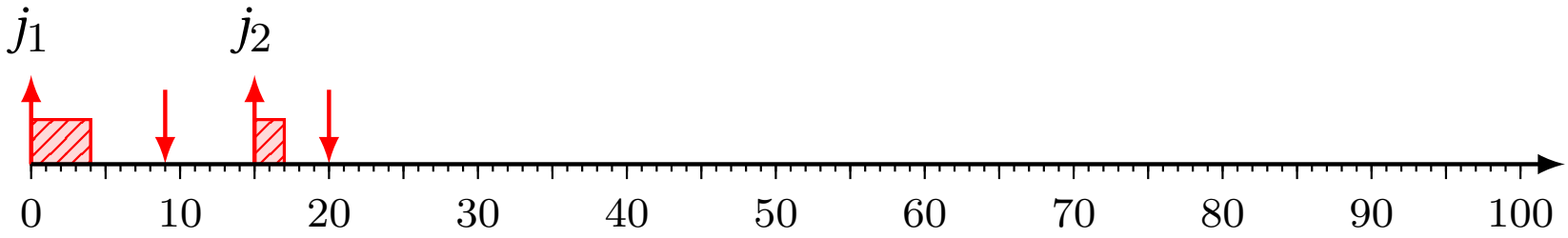

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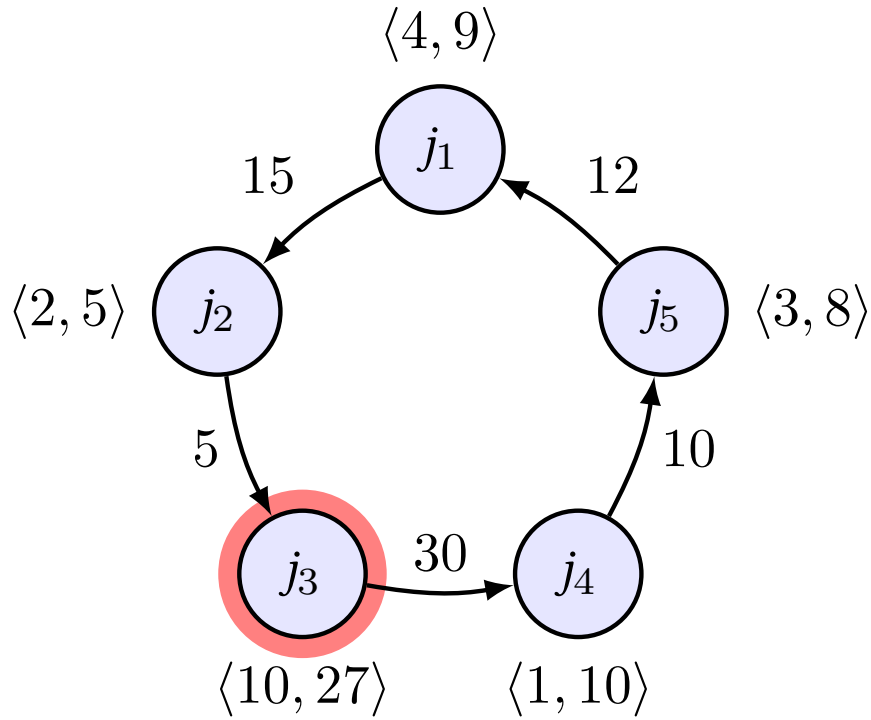

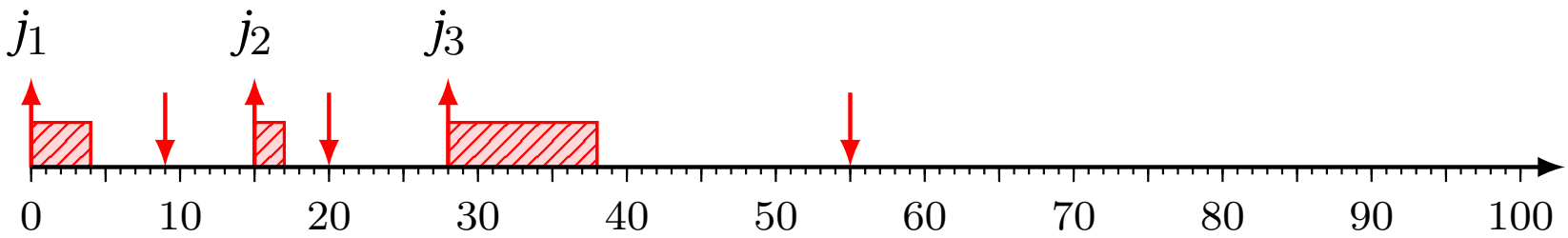

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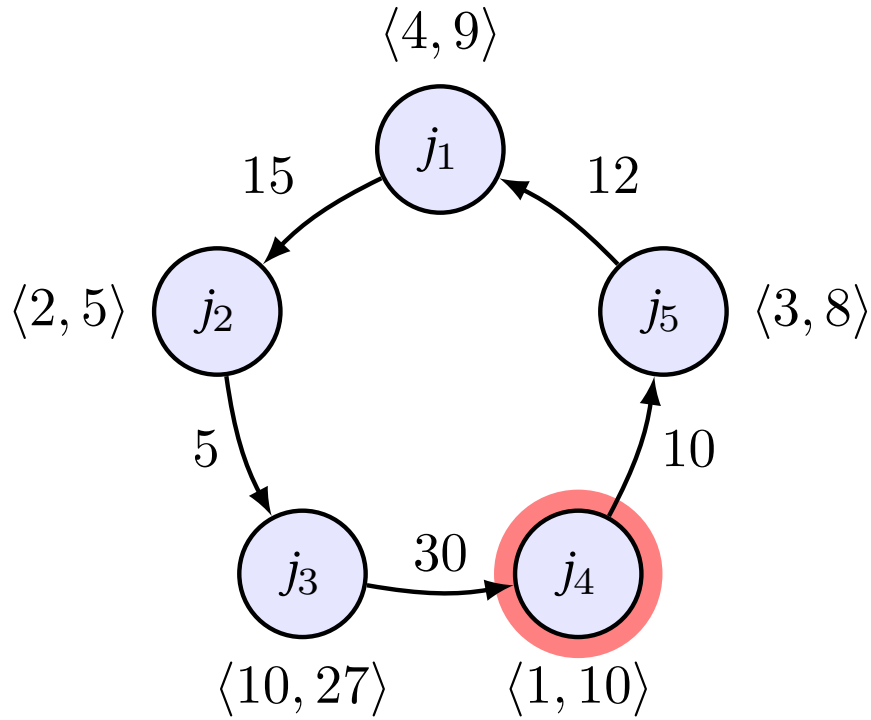

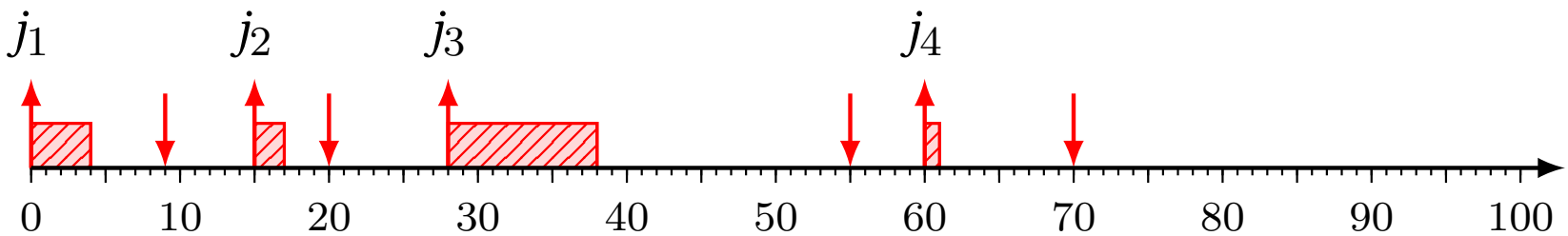

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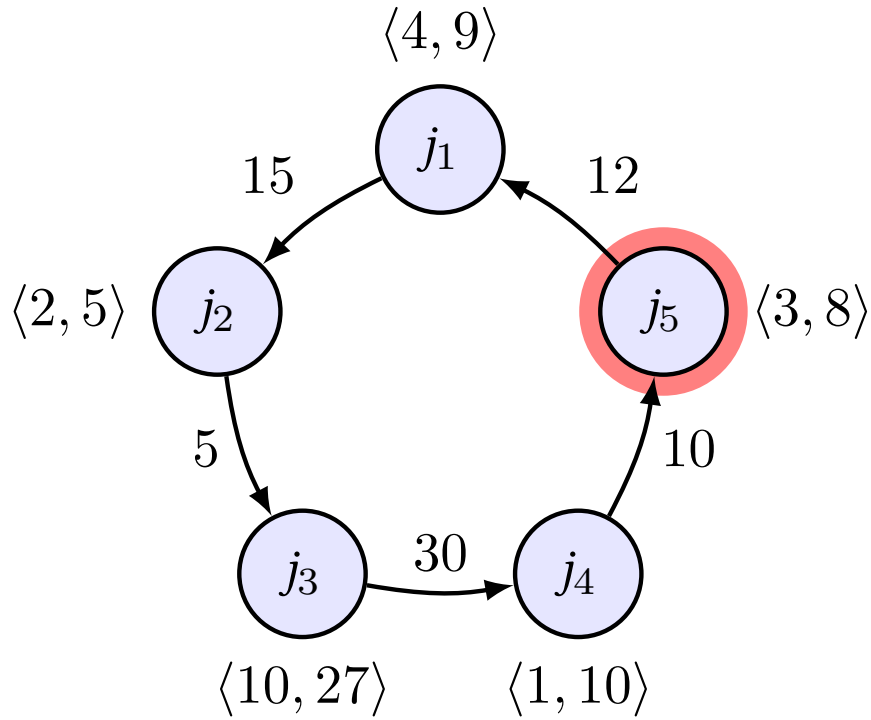

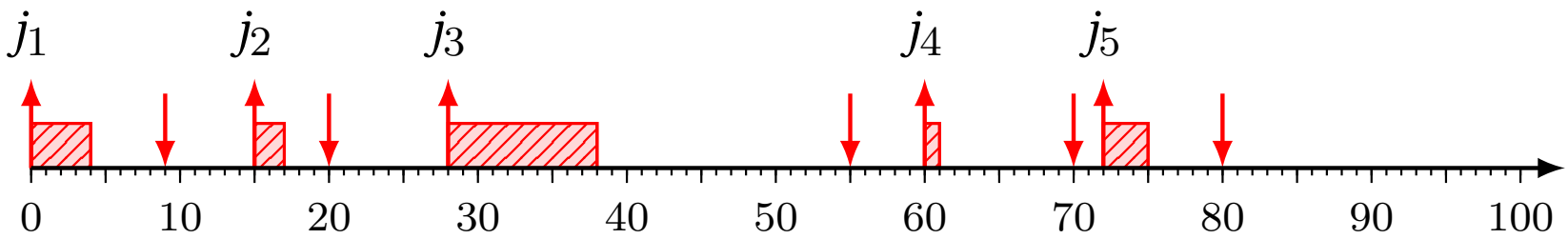

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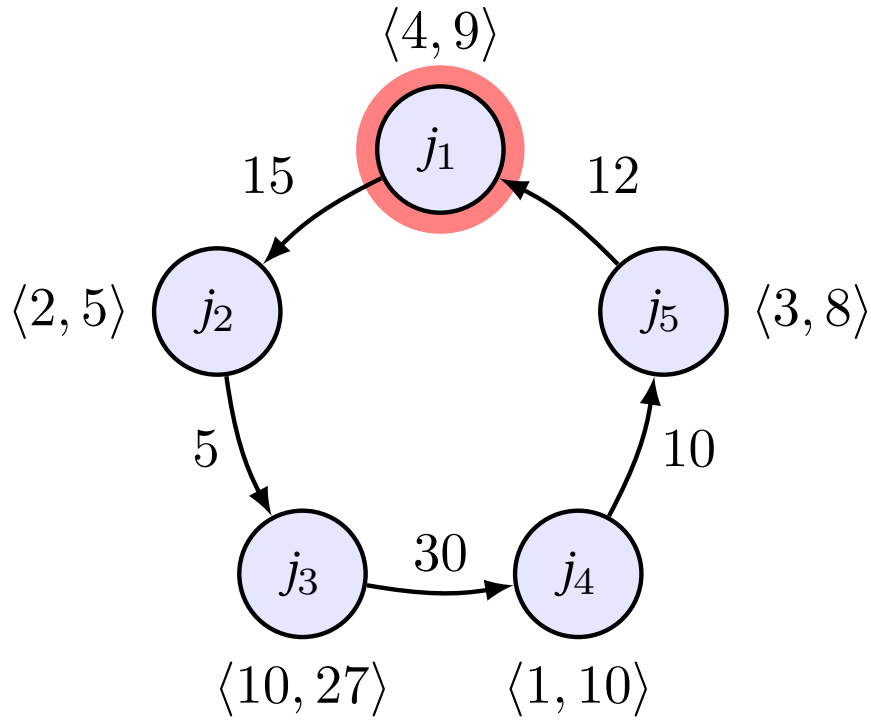

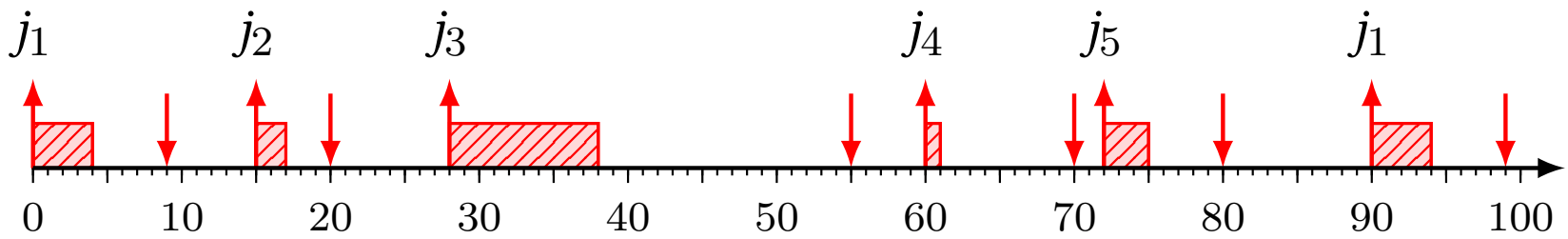

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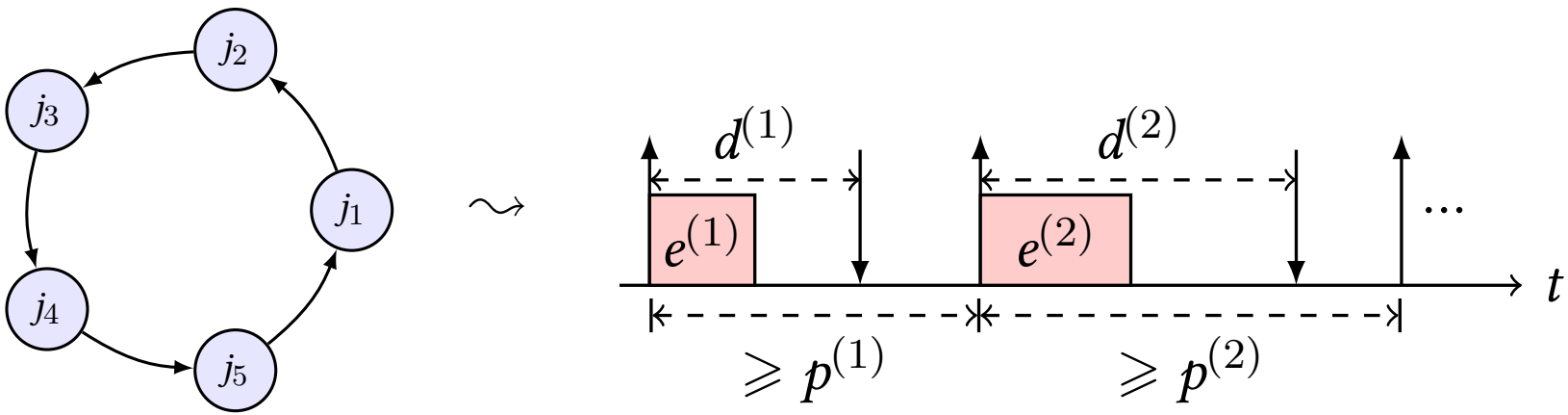

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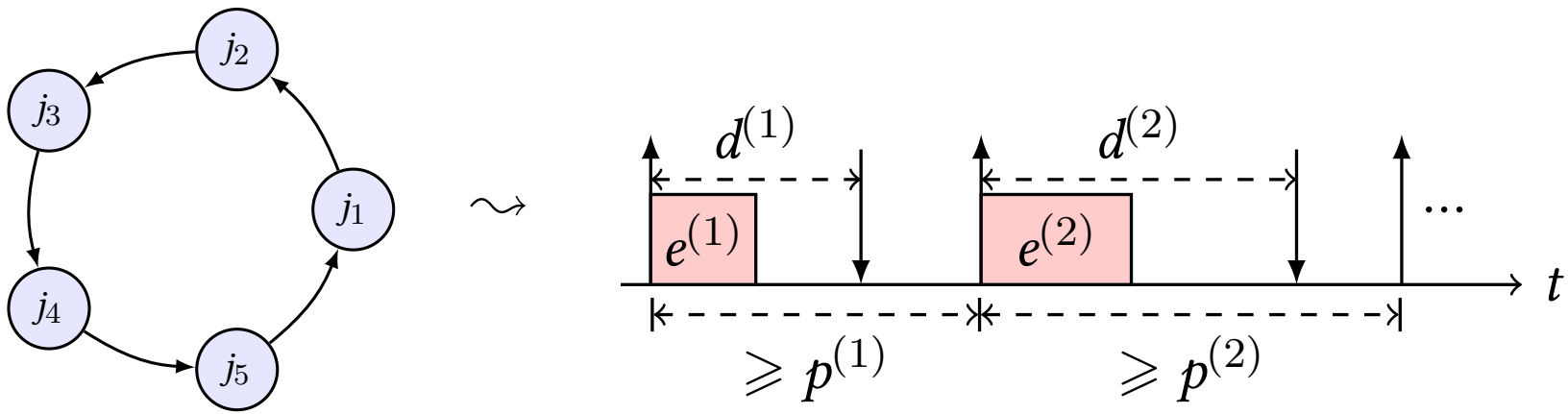

- Schedulability analysis for EDF?
  - Use the demand bound function, like for sporadic tasks
  - How to calculate dbf? What about the bound?
  - Exercise for the interested!
  - Read more in *Generalized Multiframe Tasks* (Baruah et al., 1999)

Generalize differently?

- What about *branches*?

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    if (condition) then {
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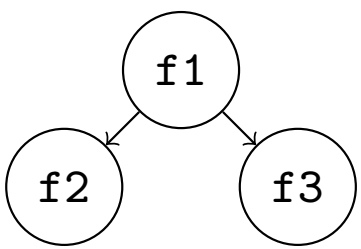
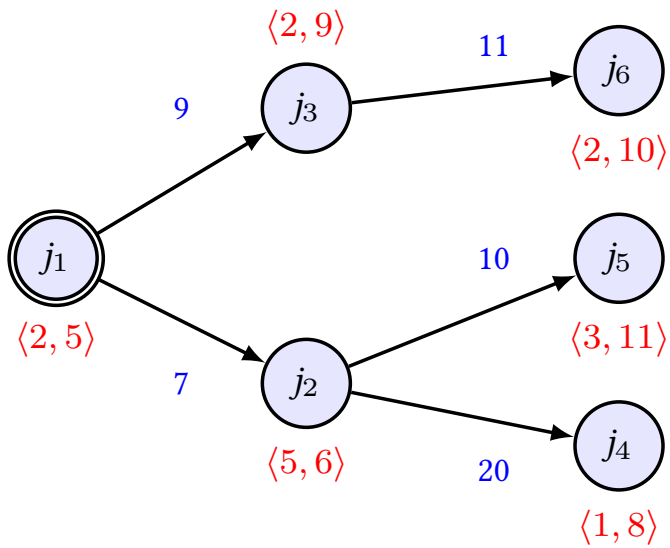
- Each task is a *tree*
  - Vertices represent jobs
  - Edges represent control flow and delays
  - Restarted once a leaf is reached



The Recurring Branching (RB) Task Model

- Introduces *branching* structures
- A *tree* for each task
  - Vertices j: job types with WCET and deadline $\langle e(j), d(j) \rangle$
  - Edges (j_i, j_k): minimum inter-release delays $p(j_i, j_k)$



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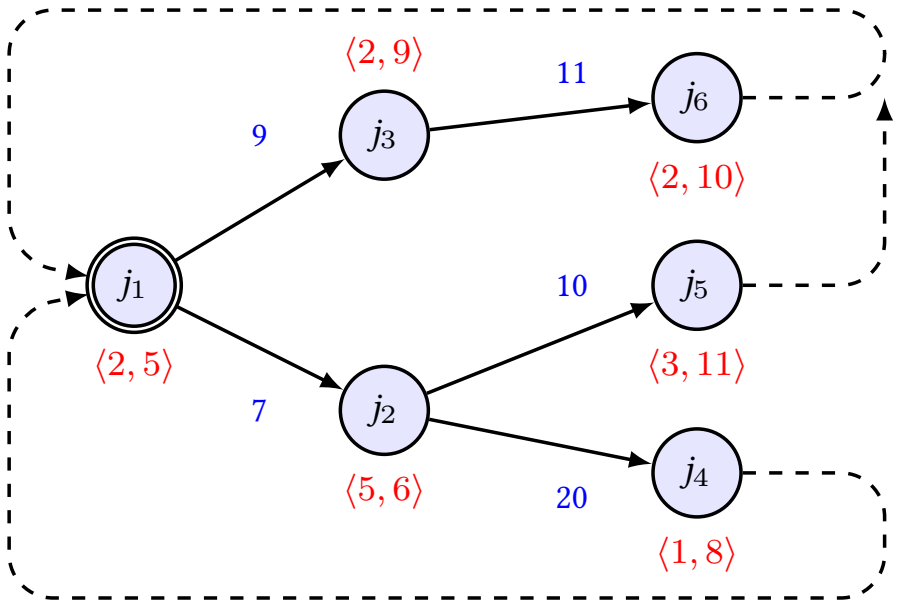

Period $P = 57$

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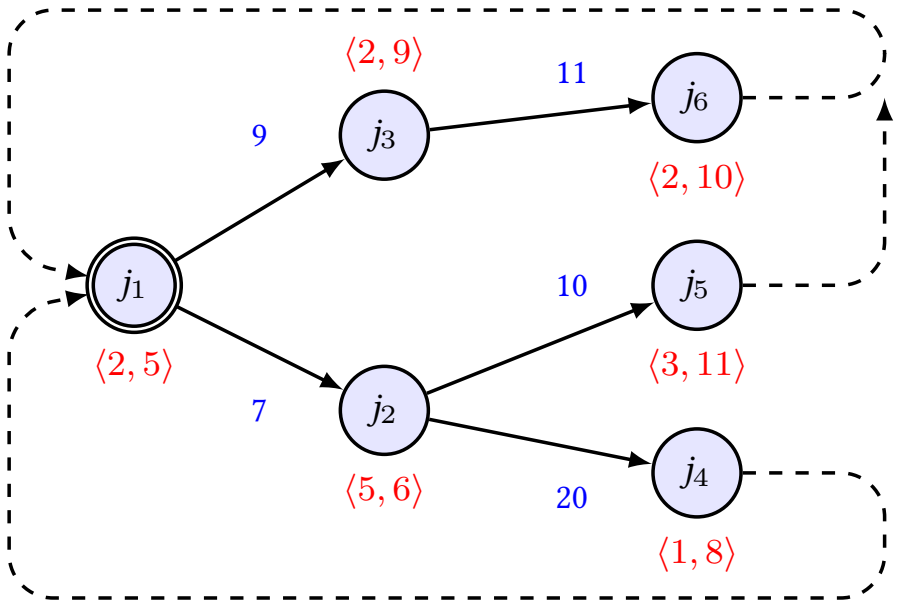


- Feasibility analysis is also dbf-based, rather involved
- Read more in *Feasibility analysis of recurring branching tasks* (Baruah, 1998)

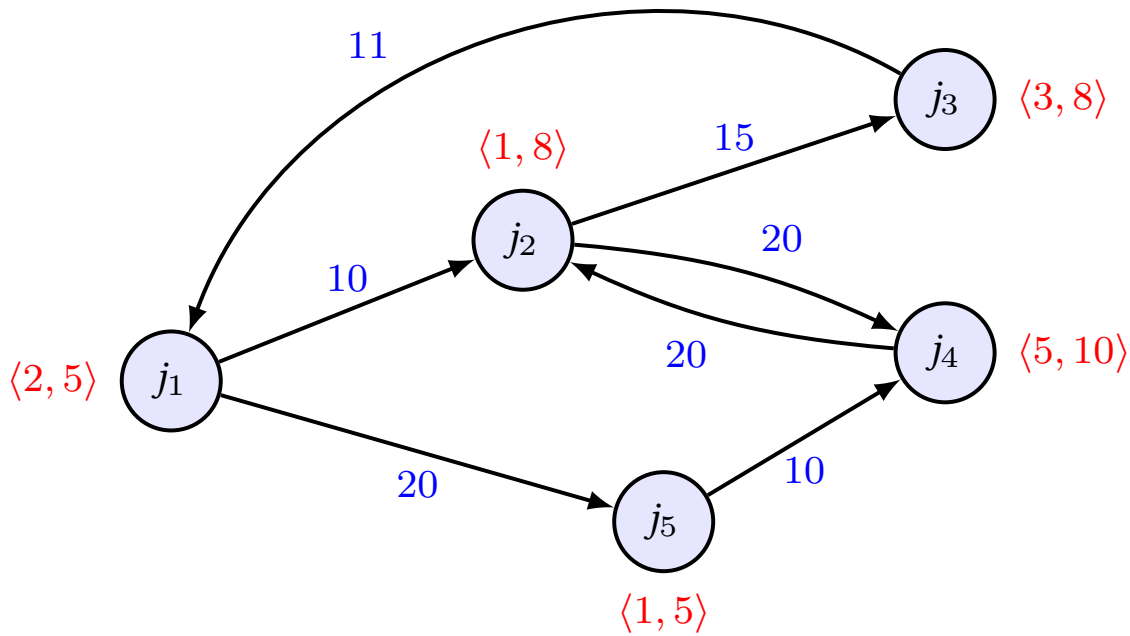
Generalize further?

- Now we can model:
  - Periodic behavior
  - Multiple frames
  - Branching behavior
- Still not possible:
  - Local loops
  - Local modes
  - ...

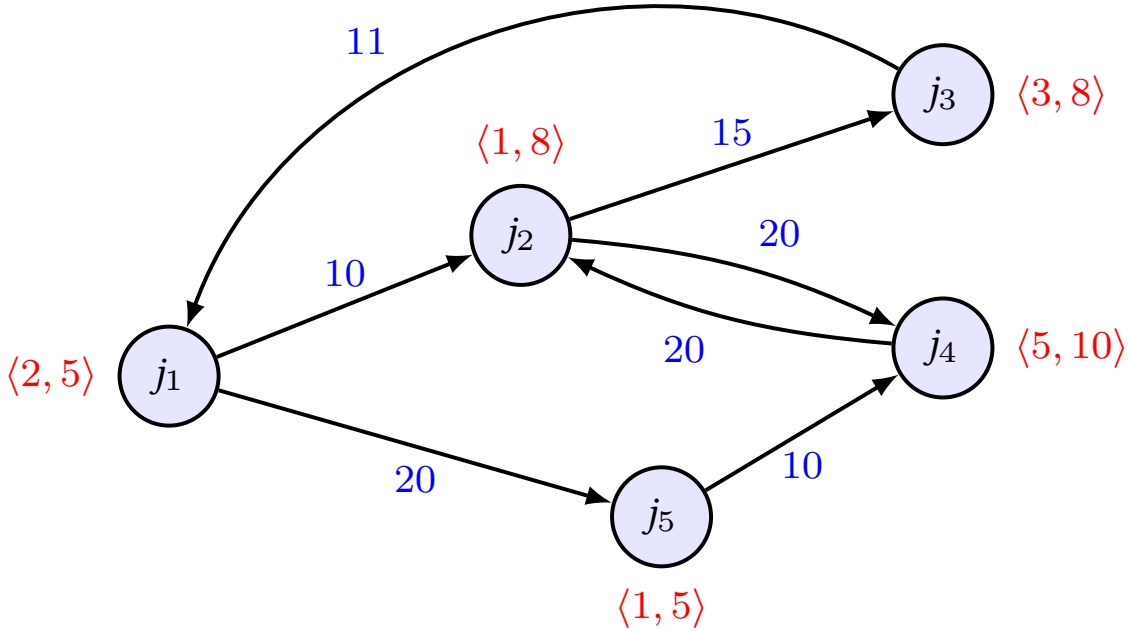
```
loop
    f1();
    delay ...
    if (cond) then {
        while (cond2) {
            f2();
            delay ...
        }
    } else {
        f3();
        delay ...
    }
end loop;
```

The Digraph Real-Time (DRT) Task Model

- Generalizes sporadic, GMF, RRT (almost), ...
- *Directed graph* for each task
  - Vertices j: job types with WCET and deadline $\langle e(j), d(j) \rangle$
  - Edges (j_i, j_k): minimum inter-release delays $p(j_i, j_k)$



DRT: Semantics



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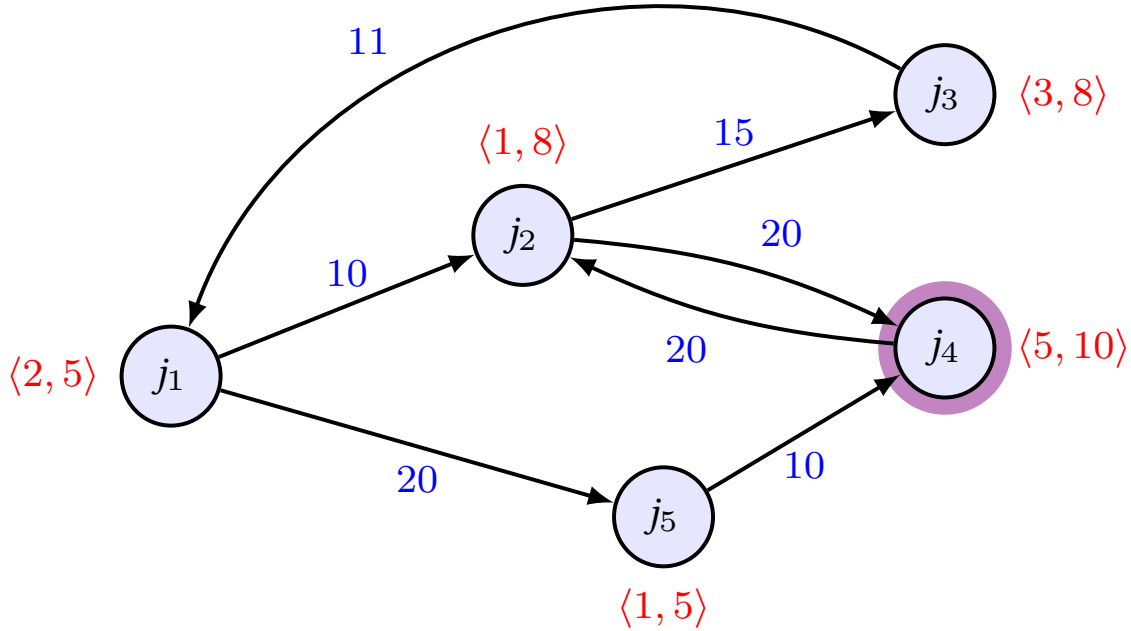
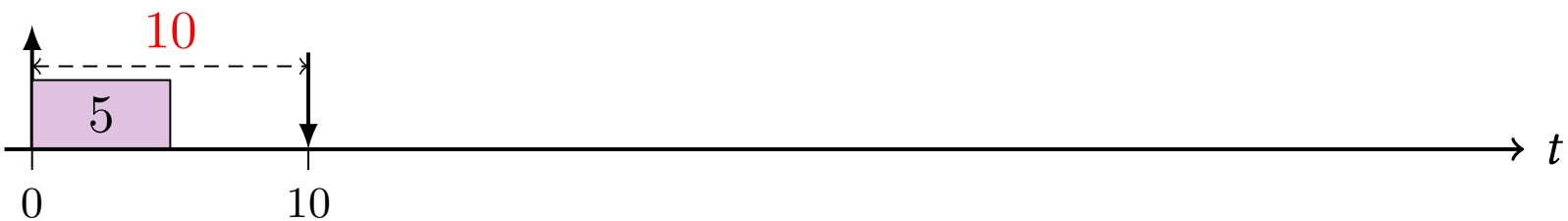


Path $\pi = (j_4)$



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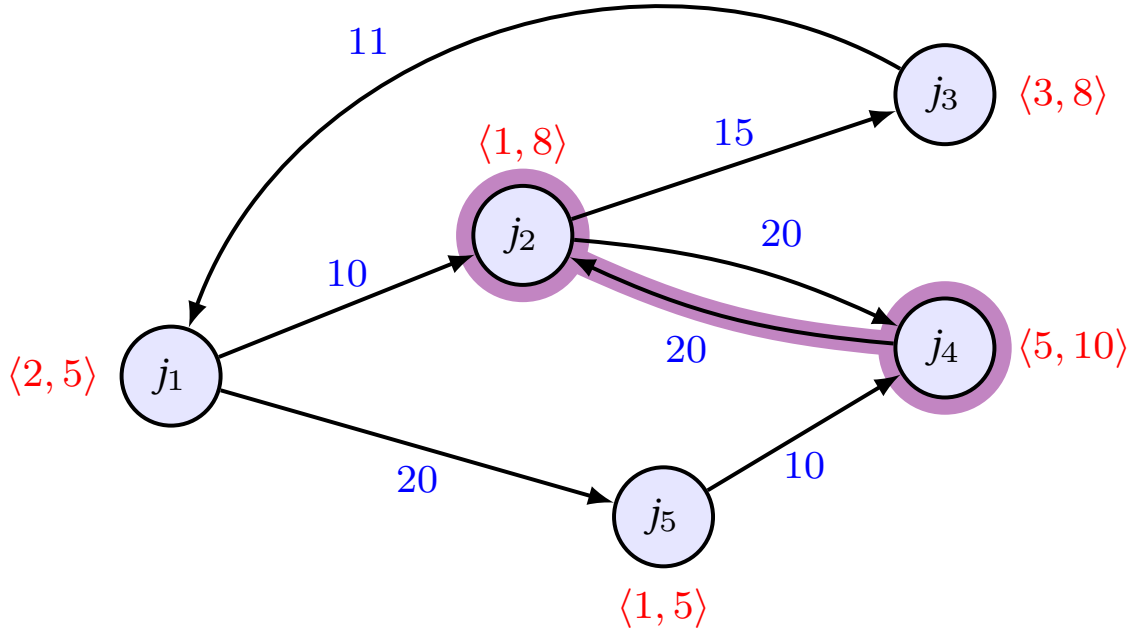
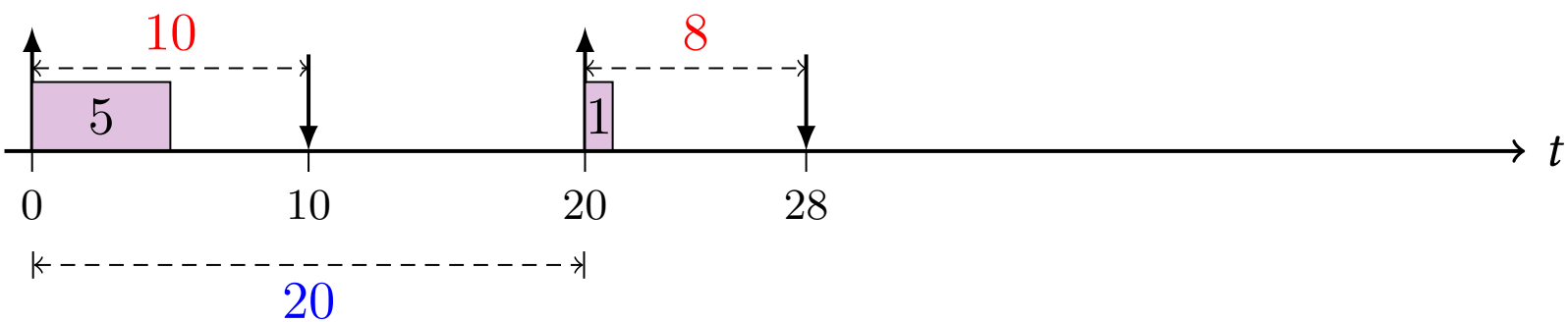


Path $\pi = (j_4, j_2)$



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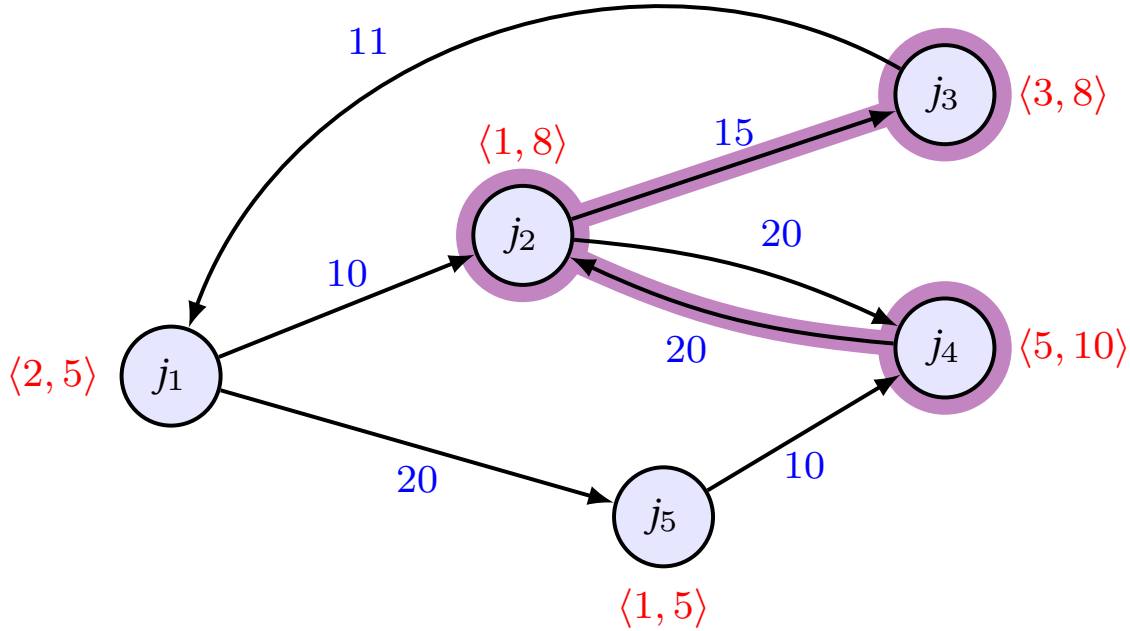
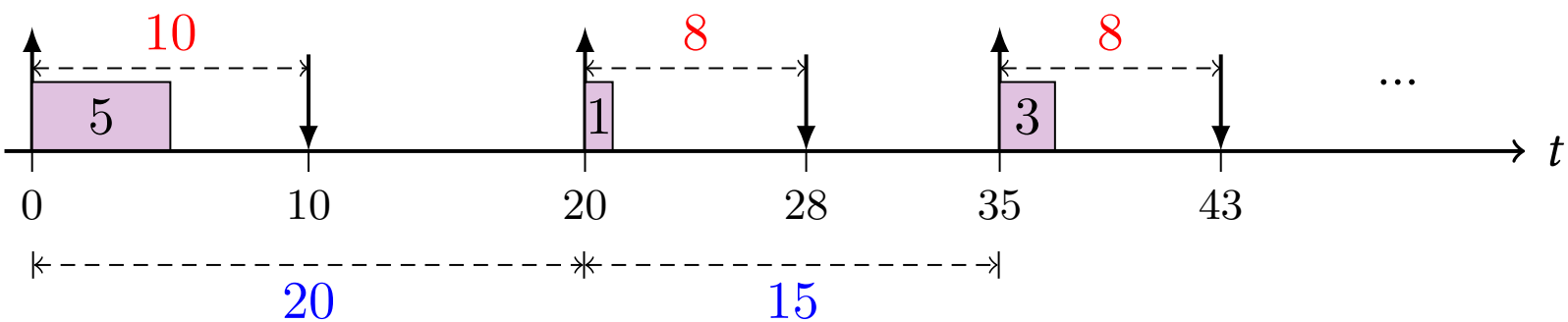


Path $\pi = (j_4, j_2, j_3)$



DRT: Feasibility analysis

- The main result about demand bound functions still holds:

Theorem

A task set $\mathcal{T} = \{\tau_1, \ldots, \tau_n\}$ of DRT tasks is feasible on a single preemptive processor iff

$$\forall t \geqslant 0 : \qquad \sum_{\tau_i \in \mathcal{T}} \mathrm{dbf}(\tau_i, t) \leqslant t.$$

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- Thus, do as before:
  1. Compute demand bound functions $\mathrm{dbf}(\tau_i, t)$.
     - How to do that for a given t?
  2. Test the inequality for all $t \leqslant B$ for some bound B.
     - How to derive the bound B?

Demand pairs

- Recall demand bound function:
  - Interval length t, sum all demand …
  - … of jobs *released* and with *deadline* inside

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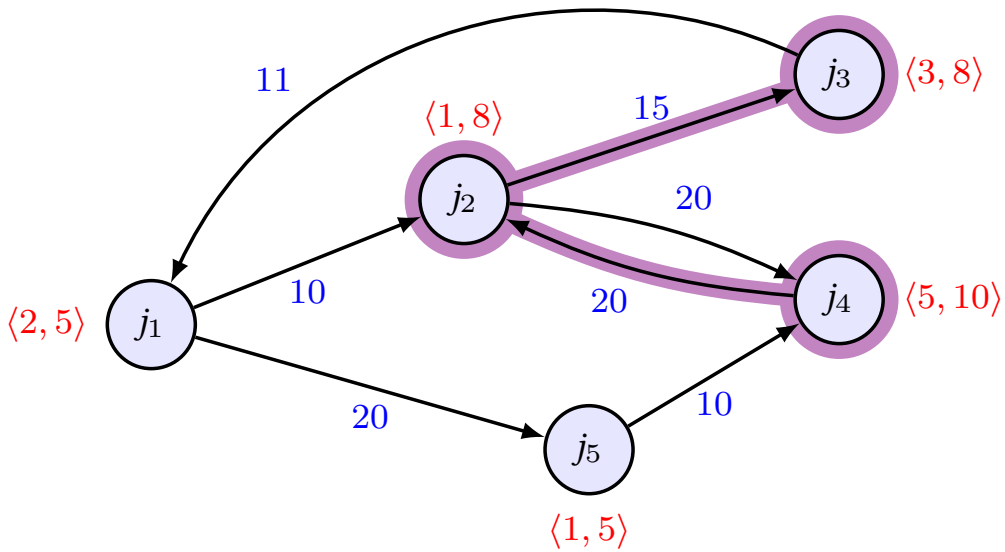

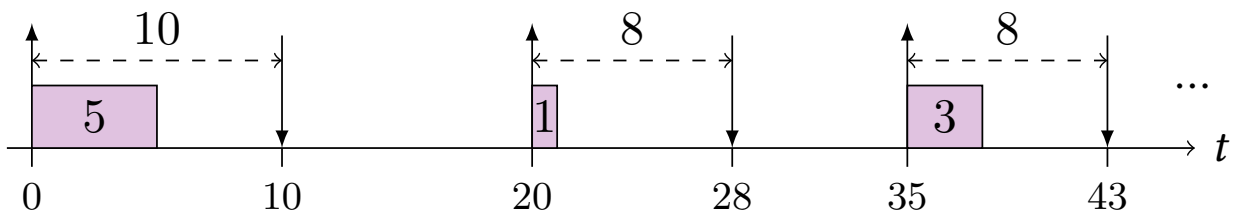

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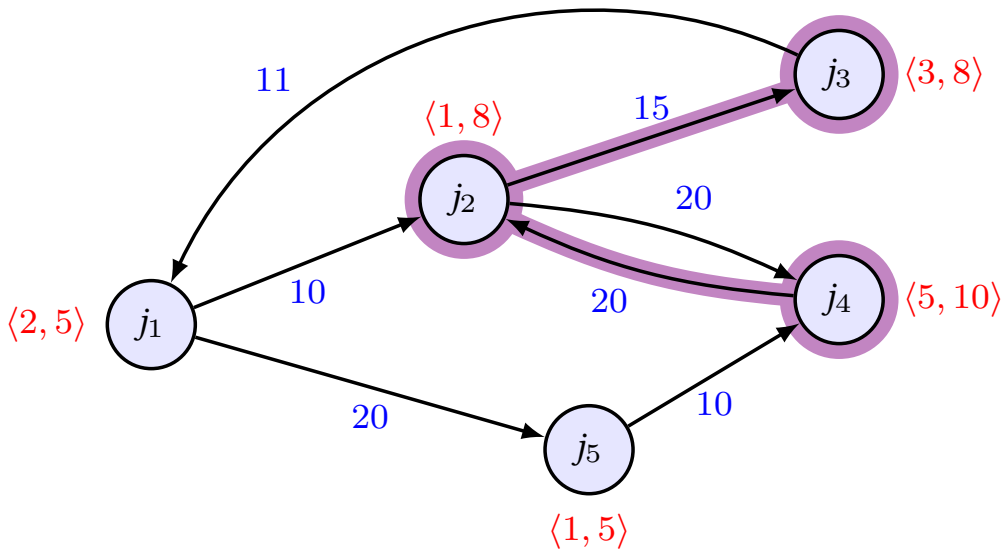


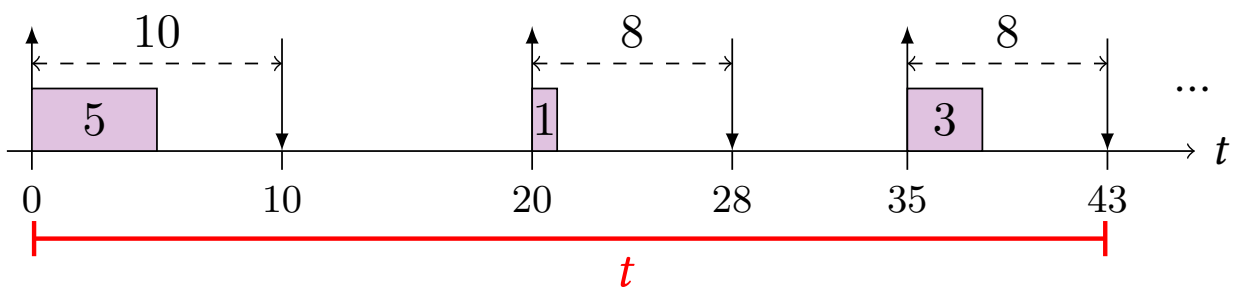
Execution demand:

$5 + 1 + 3 = 9$

Interval size:

$20 + 15 + 8 = 43$

Demand pair $\langle 9, 43\rangle$



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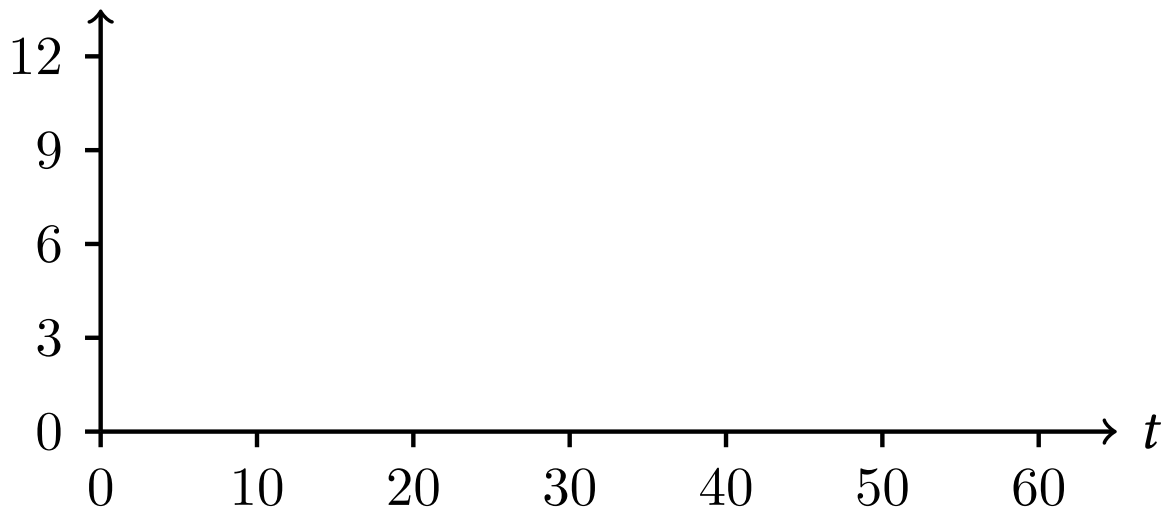

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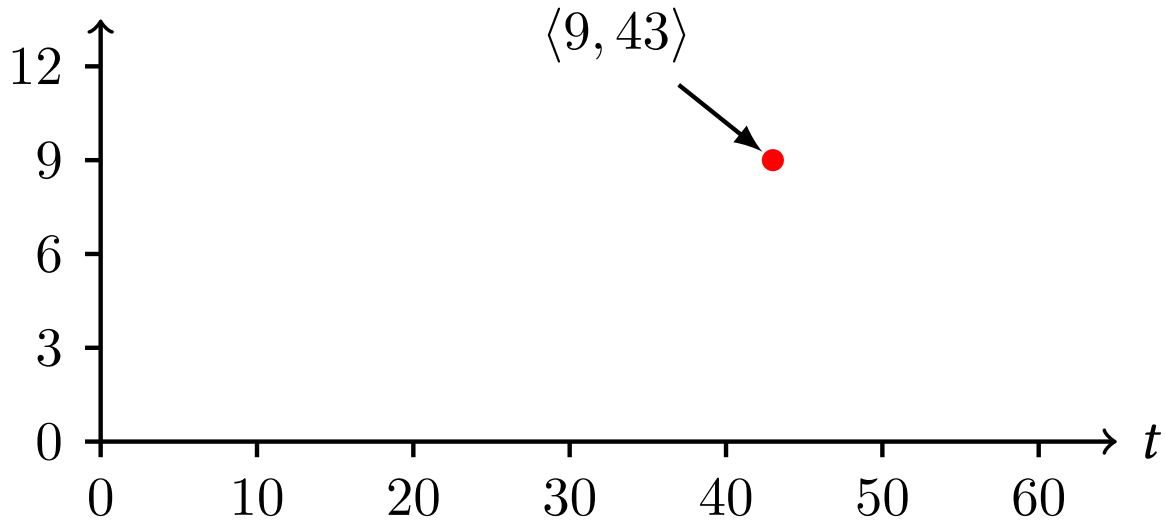

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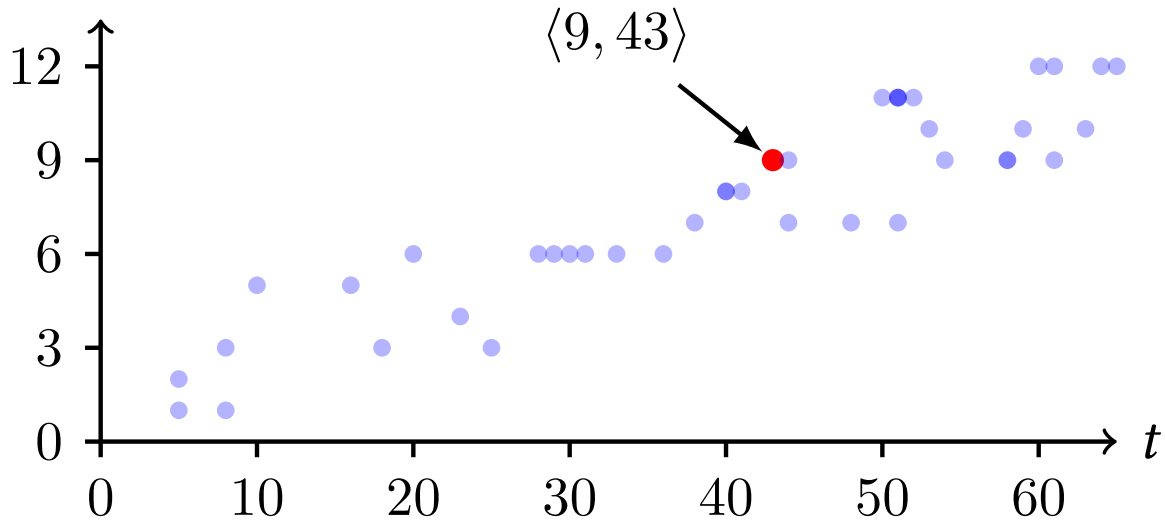

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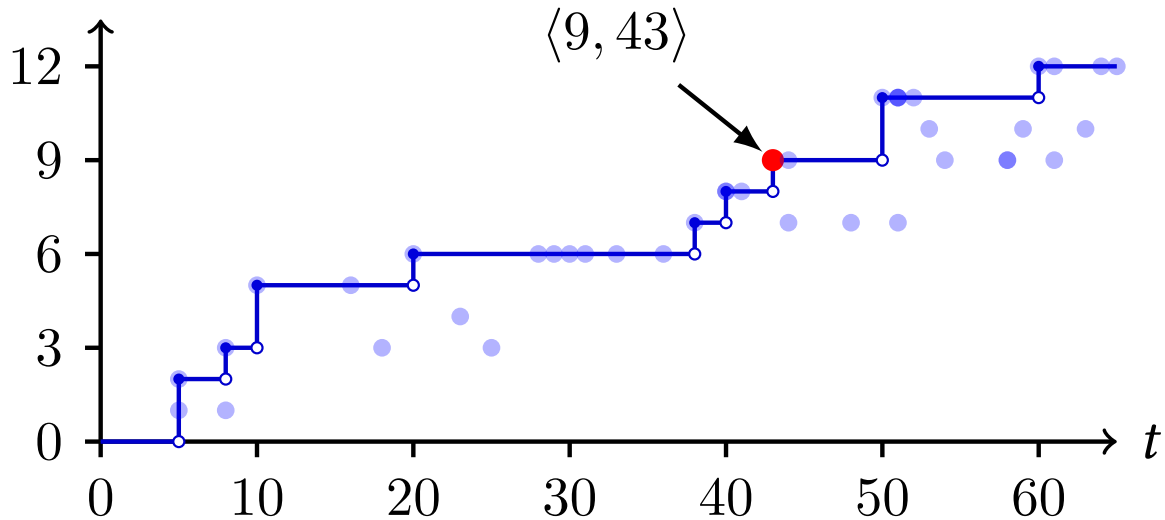

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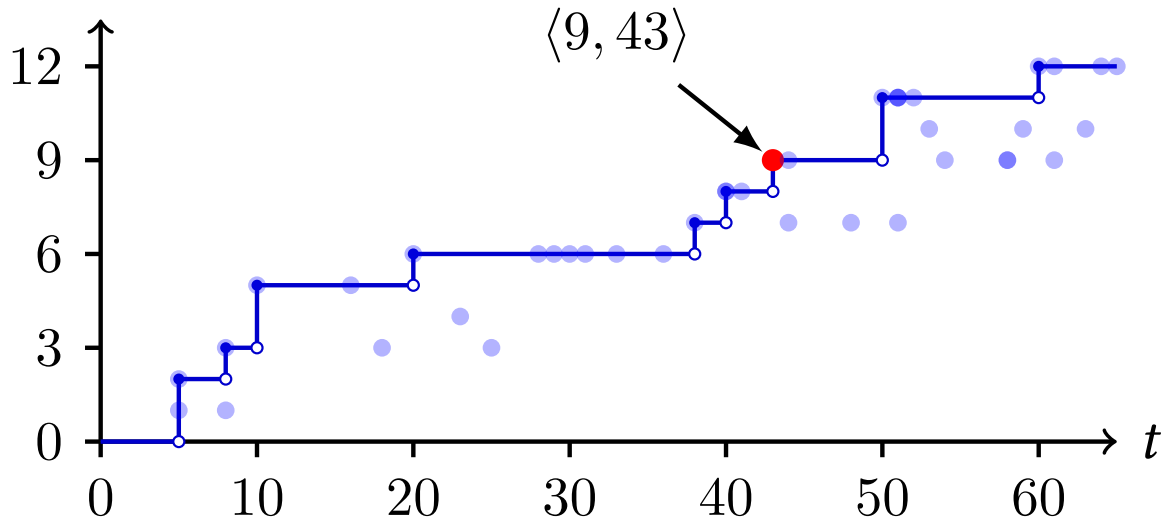

- Thus: Compute *all demand pairs*, then take "maximum"

$$\mathrm{dbf}(\tau_i, t) = \max \{ e \mid \langle e, d \rangle \text{ demand pair with } d \leqslant t \}$$

Demand pairs (cont. 2)

More formally

- Given path $\pi = (\pi_1, \ldots, \pi_k)$
- *Execution demand:* $e(\pi) := \sum_{i=1}^{k} e(\pi_i)$
- *Deadline:* $d(\pi) := \sum_{i=1}^{k-1} p(\pi_i, \pi_{i+1}) + d(\pi_k)$
- $\langle e(\pi), d(\pi) \rangle$ is a *demand pair* for π

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How to compute all demand pairs?

- Enumerate all paths? Too expensive! (Exponential)
- Better: Iteration using *abstraction*
- (Remark: Demand pairs are abstractions of paths)

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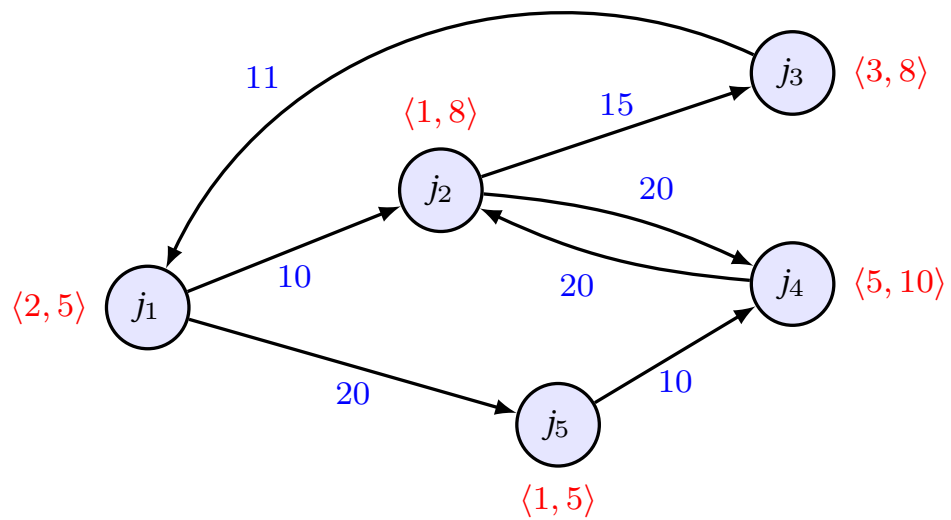
- Demand triple $\langle e(\pi), d(\pi), \pi_k \rangle$ is another path abstraction!



Path (j_4)
$\rightsquigarrow \langle 5, 10, j_4 \rangle$

Path (j_4, j_2)
$\rightsquigarrow \langle 6, 28, j_2 \rangle$

Path (j_4, j_2, j_3)
$\rightsquigarrow \langle 9, 43, j_3 \rangle$

Iterative procedure

- Create all demand triples up to bound B:
  1. Store all *0-paths*, i.e., $\langle e(j), d(j), j \rangle$ for all vertices j
  2. Pick some stored unmarked demand triple $\langle e, d, j_i \rangle$, then *mark* it
  3. *Create new demand triples*:
     - For each successor vertex j_k of j_i
     - $e' = e + e(j_k)$
     - $d' = d - d(j_i) + p(j_i, j_k) + d(j_k)$
     - $\langle e', d', j_k \rangle$ is a new demand triple!
  4. Store each new $\langle e', d', j_k \rangle$ if
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- More *efficient* procedure than enumerating all paths!
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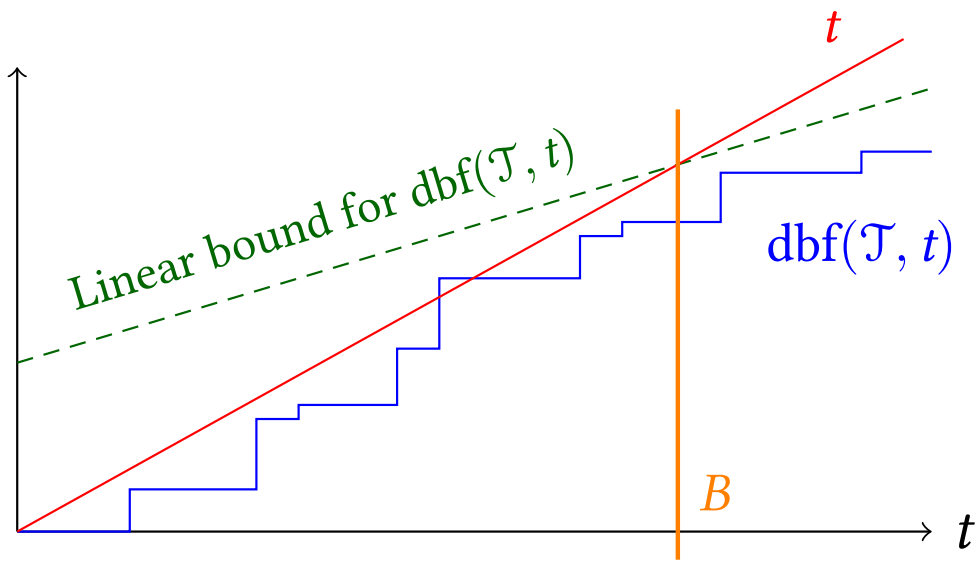
  - Optimizations: Discard non-critical triples along the way
- Exercise: What's $\mathrm{dbf}(\tau_i, 26)$ if τ_i is the task on previous slide?

Which t to check?

- Recall: Want to check $\mathrm{dbf}(\mathcal{T}, t) \leqslant t$ for all t
- Find a bound for t just like for sporadic tasks!

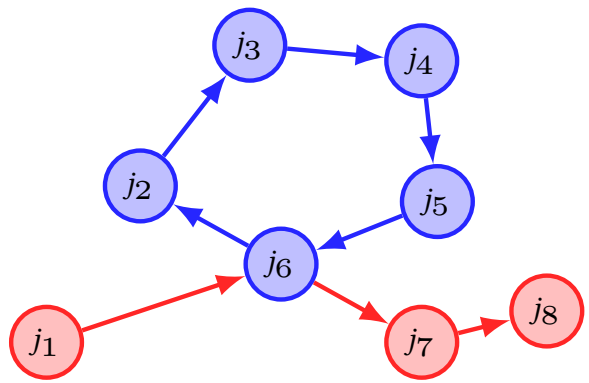


$$\forall t : \mathrm{dbf}(\mathcal{T}, t) \leqslant t$$

- Derive linear bound for $\mathrm{dbf}(\mathcal{T}, t)$
  - Intersection with t gives bound B
  - How to find the linear bound?

Linear bound for dbf

- Bound is based on *utilization*
  - Long-term demand "rate" (asymptotic)
  - For DRT: "most dense" cycle
  - Highest ratio execution demand (vertices) vs. duration (edges)
    - How to find this value? (Exercise!)



- Any path can be split into vertices in cycles and not in cycles
- Leads to

$$\mathrm{dbf}(\tau_i, t) \leqslant \mathrm{U}(\tau_i) \cdot t + \sum_k e(j_k)$$

- So, check $\mathrm{dbf}(\tau_i, t)$ for which t? (Exercise!)

DRT feasibility: Summary

- Feasibility test (or schedulability test for EDF), based on dbf
- First, compute the *utilization* for all tasks
  - Based on most dense cycles in graphs
- Derive *bound B*
- Compute $\mathrm{dbf}(\mathcal{T}, t)$ for all $t \leqslant B$
  - Uses iterative procedure with *demand triples*
  - Path abstraction to reduce complexity
- If $t \leqslant B$ with $\mathrm{dbf}(\mathcal{T}, t) > t$ is found, then $\mathcal{T}$ is infeasible
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- Read more in *The Digraph Real-Time Task Model* (Stigge et al., 2011)
- ...or in Martin Stigge's PhD thesis
- Play with a Python implementation: libdrt

Generalize further? Timed automata!

Idea

Annotate locations on timed automata with job parameters (WCET, relative deadline) let a job be created every time such a location is visited.

Analysis is generally costly, but sometimes fast enough.

Academic tool based on Uppaal: `http://www.timestool.com/`