Systems and Control
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Introduction to computer control systems: Selected exercises for the problem solving sessions

Master program in embedded systems, period 2, 2010

Assignment: Solve the exercises listed below individually for next Wednesday (2010/12/15).

Problem solving session X (Ex10)

1. Given the Volterra predator-prey mathematical model

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_1 x_2 \\ \dot{x}_2 &= x_2 - x_1 x_2 \end{aligned}$$

   (a) Obtain the linear space-state representation at the equilibrium points: $(x_{10}, x_{20}) = (0, 0)$ and $(x_{10}, x_{20}) = (1, 1)$.

   (b) Are the equilibrium points stable?

2. Consider the continuous-time linear system given by

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -2 & -1 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u \\ y &= (0 \quad 1)x \end{aligned}$$

   with a transition matrix given by

$$\Phi = \begin{pmatrix} e^{-t}(1-t) & -te^{-t} \\ te^{-t} & e^{-t}(1+t) \end{pmatrix}$$

   (a) Obtain the output response of the system, $y(t)$ (for $t > 0$), if a unit step input is applied at $t_0 = 0$ and the initial conditions are $x(0) = [1, 0]^T$.

   (b) Obtain the transfer function of the system.

   (c) Design a PID controller so that the closed-loop poles are at $(p_1, p_2, p_3) = (-4 \pm 5j, -6)$. Obtain the values k_P, k_I and k_D.

$$H(p) = k_P + \frac{k_I}{p} + k_D p$$

3. Consider a continuous-time linear system given by

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -4 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u \\ y &= (2 \quad 5)x \end{aligned}$$

(a) Sample the system with a sampling time T=0.1.

(b) Is the sampled system observable and controllable?

(c) Design an observer for the discrete-time system so that the poles of the observer are in $\lambda_{1,2}$=0.35.

(d) Obtain a control law $u(t) = -K\hat{x}(t) + Hr(t)$ ($\hat{x}(t)$ are the estimated states and $r(t)$ is the reference signal), so that the poles of the controller are in $\lambda_{1,2}$=0.5 and the static gain of the closed-loop system is 1.