Systems and Control
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Introduction to computer control systems: Selected exercises for the problem solving sessions

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Problem solving session II (Ex2)

1. (Exercise 2.10 from [1])

   Consider a system of two tanks, where the input signal is the flow to the first tank and the output is the level in the second tank (the output flow of the first tank is the input flow of the second tank). Use of the levels as state variables gives the system

$$\begin{aligned} \frac{dx}{dt} &= \begin{pmatrix} -0.0197 & 0 \\ 0.0178 & -0.0129 \end{pmatrix} x + \begin{pmatrix} 0.0263 \\ 0 \end{pmatrix} u \\ y &= \begin{pmatrix} 0 & 1 \end{pmatrix} x \end{aligned}$$

   (a) Sample the system with the sampling period h=12.

   (b) Verify that the pulse-transfer operator for the system is

$$H_0(q) = \frac{0.030q + 0.026}{q^2 - 1.65q + 0.68}$$

2. (Based on Exercise 3.19 from [2])

   For a second order system given by the state space equations

$$\begin{aligned} \frac{dx}{dt} &= Ax + Bu \\ y &= Cx \end{aligned}$$

   where the matrices B and C, and the system's transition matrix, are given by

$$\begin{aligned} B &= \begin{bmatrix} -0.5 \\ 1 \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 \end{bmatrix} \\ e^{At} &= \begin{bmatrix} e^{-t}(\cos t - \sin t) & e^{-t}\sin t \\ -2e^{-t}\sin t & e^{-t}(\cos t \sin t) \end{bmatrix} \end{aligned}$$

   Determine the system impulse response.

3. (Based on Exercise 2.10 from [1])(Notice that A and B matrices are the same as in Exercise 1)

   Consider a system of two tanks, where the input signal is the flow to the first tank and the output is the level in the second tank (the output flow of the first tank is the input flow of the second tank). The levels are the state variables of the system.

   Consider the time-continuous system given by

$$\begin{aligned} \frac{dx}{dt} &= \begin{pmatrix} -0.0197 & 0 \\ 0.0178 & -0.0129 \end{pmatrix} x + \begin{pmatrix} 0.0263 \\ 0 \end{pmatrix} u \\ y &= \begin{pmatrix} 0 & 0.5429 \end{pmatrix} x \end{aligned}$$

   and its discrete-time representation (using sampling time T=12) is given by

$$\begin{aligned} x(t+1) &= \begin{pmatrix} 0.7895 & 0 \\ 0.1757 & 0.8566 \end{pmatrix} x(t) + \begin{pmatrix} 0.2810 \\ 0.0296 \end{pmatrix} u \\ y(t) &= \begin{pmatrix} 0 & 0.5429 \end{pmatrix} x(t) \end{aligned}$$

   (a) Determine the impulse response for the continuous-time and the discrete-time systems.

   (b) Consider that the system is in stationary conditions, and the input flow are 3.8 L/min. Calculate the steady state values for the tank levels and for the output flow.

   (c) Consider the steady state values as initial values (x_0) for the tank levels. Determine the time response for the state variables if the input flow changes from 3.8 L/min to 5 L/min at time $t_0 = 0$, for the continuous-time system and for the sampled data system. Compare both responses.

References

[1] Karl J. Åström and Björn Wittenmark. *Computer-Controlled Systems*. Prentice Hall, 1997.

[2] Mikael Johansson and Torsten Söderström. *Exercises Control Theory*. Uppsala University and Royal Institute of Technology, 2010.