Systems and Control
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Introduction to computer control systems: Selected exercises for the problem solving sessions

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Problem solving session III (Ex3)

1. (Based on Exercise 3.20 from [3])

   Consider the electrical circuit with the input voltage u shown in Figure 1.

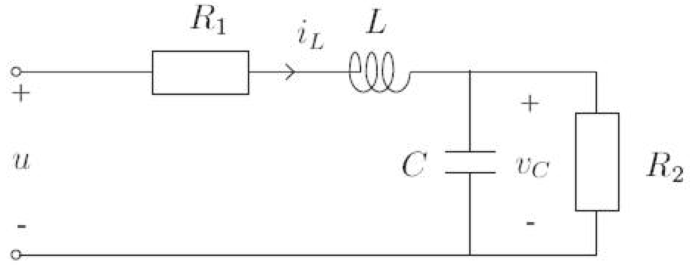

Figure 1: Electric circuit

   (a) Select the current through the inductor and the voltage across the capacitance as state variables, and the capacitance's voltage, and determine the state space equations

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

   (b) Calculate the transition matrix $\Phi(t) = e^{At}$ assuming the following values: $R_1 = R_2 = 1\ \Omega$, $L = 1\ H$, and $C = 1\ F$.

   (c) Obtain the time response for the state variables if an step change is produced at the input, that is the input changes from 2 V to 0 V at t=0 s (Consider that the system is in stationary conditions for t<0 s). Draw the circuit state variables in a diagram for $0 \leq t \leq 5$ s.

(d) Repeat the previous item but considering that the input voltage changes from 2 V to 1 V. Draw the circuit state variables in a diagram for $0 \le t \le 5$ s.

2. (Based on [2])

   Analise the controllability and unobservability for the following continuous-time systems:

   (a)

$$
\begin{aligned}
\dot{x} &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} u \\
y &= \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} x
\end{aligned}
$$

   (b)

$$
\begin{aligned}
\dot{x} &= \begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix} x + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} u \\
y &= \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} x
\end{aligned}
$$

3. (Based on [2])

   Consider the system

$$
\begin{aligned}
\dot{x} &= \begin{pmatrix} -2 & 3 \\ 1 & -4 \end{pmatrix} x + \begin{pmatrix} 1 & 8 \\ 2 & 6 \end{pmatrix} u \\
y &= \begin{pmatrix} 3 & 7 \end{pmatrix} x
\end{aligned}
$$

   (a) Is the system controllable?

   (b) Suppose that we have only access to the input u_1. Is the system controllable in this case?

   (c) Suppose that the two control signals are coupled so that $u_1 + 2u_2 = 0$ always holds. Is the system controllable in this case?

4. (Exercise 3.6 from [1])

   Is the following system (a) observable, (b) controllable?

$$
\begin{aligned}
x(k+1) &= \begin{pmatrix} 0.5 & -0.5 \\ 0 & 0.25 \end{pmatrix} x(k) + \begin{pmatrix} 6 \\ 4 \end{pmatrix} u(k) \\
y(k) &= \begin{pmatrix} 2 & -4 \end{pmatrix} x(k)
\end{aligned}
$$

5. (Exercise 3.7 from [1])

Is the following system controllable?

$$x(k+1) = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix} x(k) + \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} u(k)$$

Assume that a scalar input $u'(k)$ such that

$$u(k) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} u'(k)$$

is introduced. Is the system controllable from $u'(k)$?

References

[1] Karl J. Åström and Björn Wittenmark. *Computer-Controlled Systems.* Prentice Hall, 1997.

[2] Automatic Control Group (Linköpings University) and Systems and Control Group (Uppsala University). *Exercise Manual for Automatic Control.* Uppsala University, 2001.

[3] Mikael Johansson and Torsten Söderström. *Exercises Control Theory.* Uppsala University and Royal Institute of Technology, 2010.