Systems and Control
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Introduction to computer control systems: Selected exercises for the problem solving sessions

Master program in embedded systems, period 2, 2010

Problem solving session IV (Ex4)

1. Demonstrate that for a system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

with a diagonal $A-$matrix given by

$$A = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{pmatrix}$$

The transition matrix is given by

$$e^{At} = \begin{pmatrix} e^{\lambda_1 t} & 0 & \dots & 0 \\ 0 & e^{\lambda_2 t} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & e^{\lambda_n t} \end{pmatrix}$$

2. (Based on Exercise 3.17 from [1])

Determine a coordinate transformation $z = Tx$ that transfers the system

$$\begin{aligned} x(k+1) &= \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} x(k) + \begin{pmatrix} 3 \\ 4 \end{pmatrix} u(k) \\ y(k) &= \begin{pmatrix} 5 & 6 \end{pmatrix} x(k) \end{aligned}$$

(a) To diagonal canonical form

(b) To controllable canonical form

(c) To observable canonical form

3. (Exercise 3.2 from [2])

Consider the system

$$x(t+1) = \begin{pmatrix} -1 & 1.5 \\ -3 & 3.5 \end{pmatrix} x(t) + \begin{pmatrix} -1 & 0.5 \\ -2 & 0.5 \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$$

(a) Is the system stable?

(b) Is there a state feedback stabilizing the system?

(c) Can the system be stabilized only using input u_1?

4. (Exercise 3.4 from [2])

Consider the discrete-time system

$$x(t+1) = \begin{pmatrix} 0.2 & 0.1 \\ 0.2 & 0.3 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u(t)$$

Determine, in case it is possible, the input $u(t)$ so that the state vector change from x(0) to $x'(t)$ in at most two sampling intervals, when

(a)

$$x(0) = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, x'(t) = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

(b)

$$x(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, x'(t) = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

In both cases, it thus holds $t' = 1$ or $t' = 2$. Explain the achieved results.

References

[1] Karl J. Åström and Björn Wittenmark. *Computer-Controlled Systems.* Prentice Hall, 1997.

[2] Mikael Johansson and Torsten Söderström. *Exercises Control Theory.* Uppsala University and Royal Institute of Technology, 2010.