Systems and Control
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Introduction to computer control systems: Selected exercises for the problem solving sessions

Master program in embedded systems, period 2, 2010

Problem solving session V (Ex5)

1. (Exercise 3.4 from [1])

   Consider the discrete-time system

   $$x(t+1) = \begin{pmatrix} 0.2 & 0.1 \\ 0.2 & 0.3 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u(t)$$

   Determine, in case it is possible, the input $u(t)$ so that the state vector change from x(0) to $x'(t)$ in at most two sampling intervals, when

   (a)

   $$x(0) = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, x'(t) = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

   (b)

   $$x(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, x'(t) = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

   In both cases, it thus holds $t' = 1$ or $t' = 2$. Explain the achieved results.

2. Consider the system composed by three-tanks in series [2] shown in Fig. 1. The inputs of the system are the tank 1 input flow $f_i(t)$ and the tank 1 output flow $f_0(t)$. The output of the system is the output flow $f_3(t)$ from the tank 3. A_1, A_2 and A_3 are the cross-sectional area of the tanks, ρ is the density, and $h_1(t)$, $h_2(t)$ and $h_3(t)$ are the tank levels.

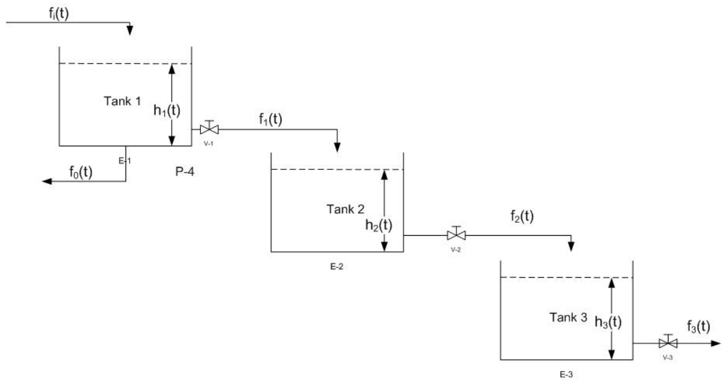

Figure 1: Three-tank process.

The system dynamic is described by

$$\begin{aligned}
\rho A_1 \frac{dh_1(t)}{dt} &= \rho f_i(t) - \rho f_1(t) - \rho f_0(t) \\
\rho A_2 \frac{dh_2(t)}{dt} &= \rho f_1(t) - \rho f_2(t) \\
\rho A_3 \frac{dh_3(t)}{dt} &= \rho f_2(t) - \rho f_3(t) \\
f_1(t) &= C_{v1}\sqrt{h_1(t)} \\
f_2(t) &= C_{v2}\sqrt{h_2(t)} \\
f_3(t) &= C_{v3}\sqrt{h_3(t)}
\end{aligned}$$

(a) Find the linear approximation in the state-space form at the equilibrium point $f_{i,0} = 5$ m^3/h and $f_{0,0} = 2$ m^3/h. The model parameters are: A_1=1.2 m^2, A_2=1.5 m^2, A_3=1 m^2, $C_{v1} = 3.15$, $C_{v2} = 2.8$ and $C_{v3} = 2.5$.

(b) Compare the dynamic behaviour for the linear and nonlinear model by simulations for a step change of +10% and +30% in the input flow $f_i(t)$. Explain the differences.

(c) Compute the static gain for the state space model.

3. Consider the pendulum shown in Fig. 2. The system consists of a ball of mass m located at the end of a massless rod with a length l. The moment of inertia of the pendulum about its pivot point is J, the viscous friction coefficient B and the applied torque is T. The rotated angle θ, which is the output variable and is taken as shown in Fig. 2.

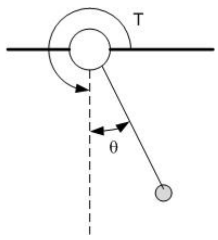

Figure 2: Pendulum.

The angle θ is determined by

$$T = J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} + mgl\sin(\theta(t))$$

This nonlinear differential equation of second order describes the dynamic behaviour of the pendulum. The model parameters are l=1 m, B=2 Nm/(rad/s), g=9.8 m/s^2, m=3 kg and $J = ml^2$ kg m^2.

(a) Obtain the state space form at the equilibrium point $\theta_0 = 0$.

(b) Obtain the transfer function.

(c) Obtain the poles and zeros of the system.

(d) Analyse the response of the linear system to a sinusoidal signal $T = A\sin(\omega t)$ for:

   i. A=0.5, $\omega = 0.1$ rad/s.

   ii. A=0.5, $\omega = 0.04$ rad/s.

   iii. $A = 29.4$, $\omega = 0.04$ rad/s.

(e) Compare by simulation the previous responses with the nonlinear system response. Explain the differences.

References

[1] Mikael Johansson and Torsten Söderström. *Exercises Control Theory.* Uppsala University and Royal Institute of Technology, 2010.

[2] Carlos A. Smith and Armando B. Corripio. *Principles and practice of automatic process control.* John Wiley & Sons, USA, 2 edition, 1997.