

Introduction to computer control systems:
Selected exercises for the problem solving sessions
Master program in embedded systems, period 2, 2010

Problem solving session VII (Ex7)

1. (Based on Exercise 5.9 from [1])

The system (sampled double integrator)

$$\begin{aligned}x(t+T) &= \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} x(t) + \begin{pmatrix} T^2/2 \\ T \end{pmatrix} u(t) \\ y(t) &= (1 \quad 0)x(t)\end{aligned}$$

is given.

Determine

$$K = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$$

in the observer,

$$\hat{x}(t+T) = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} \hat{x}(t) + \begin{pmatrix} T^2/2 \\ T \end{pmatrix} u(t) + K(y(t) - (1 \quad 0)\hat{x}(t))$$

so that estimation error becomes zero at two sampling intervals. (Dead-beat estimation)

2. (Based on Exercise 5.12 from [1])

Given the system

$$\begin{aligned}x(t+1) &= \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix} x(t) + \begin{pmatrix} 2 \\ 3 \end{pmatrix} u(t) \\ y(t) &= (1 \quad 0)x(t)\end{aligned}$$

You build an observer for the state x_2 as

$$\begin{aligned}\hat{x}_2(t+1) &= 0.9\hat{x}_2(t) + 0.1y(t) + 3u(t) + K(y(t+1) - 0.8y(t) \\ &\quad - 0.2\hat{x}_2(t) - 2u(t))\end{aligned}$$

Select the number K in this reduced observer so that the estimation error becomes zero after a finite number of steps.

3. (Based on Exercise 6.7 from [1])

Consider the process described by the following state model:

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y &= (-1 \quad 1) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{aligned}$$

With state feedback here refers to the matrix $L = (l_1 \quad l_2)$ such a control law of the form $u = Kr - Lx$ obtained.

- (a) Determine the matrix L so that the closed-loop system poles may $p = -1.6 \pm j1.2$.
 - (b) Determine the transfer function from the $R(s)$ to $Y(s)$ for the closed loop system.
 - (c) Determine the gain parameter K so that the closed-loop static gain is one.
4. Consider the three-tank system analysed in Exercise 2 of problem solving session V (Ex5). Suppose $f_i(t)$ as the input and $f_3(t)$ as the output. The process has two flowmeters, which measure $f_i(t)$ and $f_3(t)$. So, the tank levels are not measured. The state space representation obtained at the equilibrium point $f_{i,0} = 5 \text{ m}^3/\text{h}$ and $f_{0,0} = 2 \text{ m}^3/\text{h}$ is given by

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1.6537 & 0 & 0 \\ 1.6537 & -1.3067 & 0 \\ 0 & 1.3067 & -1.0417 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u \\ y &= (0 \quad 0 \quad 1.0417)x \end{aligned}$$

- (a) Design a control law $u = -K\hat{x}$ (\hat{x} are the estimated states), so that the closed loop system poles are in $p_1 = -2.5$, $p_2 = -3$ and $p_3 = -4$.
- (b) Draw a block diagram of the closed-loop system.
- (c) Validate the designed observer and state feedback controller by simulations. Analyse the results.

References

- [1] Mikael Johansson and Torsten Söderström. *Exercises Control Theory*. Uppsala University and Royal Institute of Technology, 2010.