Systems and Control
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Introduction to computer control systems: Selected exercises for the problem solving sessions

Master program in embedded systems, period 2, 2010

Problem solving session IX (Ex9)

1. Consider the following continuous-time second order linear system

$$G(s) = \frac{4}{4s^2 + 12s + 8}$$

and the PID controller given by

$$H(s) = k_P + \frac{k_I}{s} + k_D s$$

   (a) Obtain the resonance frequency ω_n and the damping factor ξ for system $G(s)$.

   (b) Obtain the closed-loop transfer function.

   (c) Obtain the PID parameters such that the closed-loop poles are in $p_{1,2} = -5 \pm 2i$ and p_3=-10.

2. Consider the continuous-time linear system given by

$$G(s) = \frac{100}{5s + 10}$$

   (a) Design a PI controller

$$H(s) = k_P + \frac{k_I}{s}$$

   so that the closed-loop poles have a resonance frequency $\omega_n = 4$ and a damping factor $\xi = 0.707$.

   (b) Compute the static gain for the closed loop system.

3. Consider the following continuous-time system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1.6537 & 0 & 0 \\ 1.6537 & -1.3067 & 0 \\ 0 & 1.3067 & -1.0417 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u \\ y &= (0 \quad 0 \quad 1.0417)x \end{aligned}$$

which is controlled by $u = -K\hat{x}$ ($\hat{x}$ are the estimated states), where $K = [5.50 \quad 6.86 \quad 3.91]$. The observer is given by

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$

where $L = [2743.3 \quad 756.0 \quad 53.8]^T$.

(a) Compute the closed loop system poles. Compare with the poles of the open loop system.