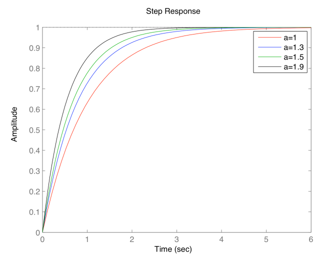
Lecture 7

- First order system
- Second order system
- PID controller

First order system

Single real pole:

$$W(s) = \frac{a}{s+a}$$



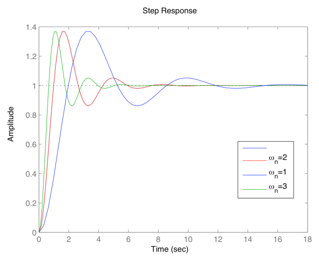
- Static gain is one
- The greater *a* is, the faster the system
- A static system is infinitely fast

Second order system

Transfer function:

$$W(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

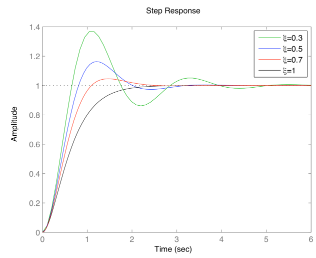
ω_n – resonance frequency
ξ – damping factor



- Static gain is 1.
- The higher is resonance frequency, the faster are the oscillations
- $\xi = 0.3$

Second order system, contd.

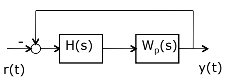
$$W(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$



- The higher the damping factor is, the lesser the oscillations
- $\xi = 1$ corresponds to a fully damped system
- $\xi = 0$ corresponds to undamped oscillations

PID controller

Feedback controller



$$H(s) = k_p + \frac{k_i}{s} + k_d s$$
$$U(s) = H(s)E(s)$$
$$E(s) = R(s) - Y(s)$$

In time domain $H(p) = k_p + \frac{k_i}{p} + k_d p$

$$u(t) = k_p e(t) + \frac{k_i}{p} e(t) + k_d p e(t)$$
$$= k_p e(t) + k_i \int e(\theta) d\theta + k_d p \dot{e}(t)$$

Control parameters: k_p, k_i, k_d
Control error: *e(t)*

- Proportional term – related to the present value of *e(t)*
- Integral term – related to the history of *e(t)*, eliminates the static error $\lim_{t\to\infty} e(t)$
- Derivative term – related to the velocity (signed) of *e(t)*

PID controller, contd.

- The PID controller is the most commonly used controller.
- The derivative term is usually implemented with a filter. Only those controllers have state space representation.

$$H(s) = k_p + \frac{k_i}{s} + \frac{k_d s}{1 + \epsilon k_d s} \qquad \text{for a small } \varepsilon$$

- PID controllers are implemented in discrete time but tuned using a continuous formulation
- PID controllers have typically bad performance when there is a significant time delay in the loop. They work though nicely for relatively small time delays.
- Most of the tuning methods for PID controllers are based on experiments with the process plant (step response, parameters of oscillations, etc.)
- There are also formal methods for the design of PID controllers that demand a mathematical model of the plant.

Informationsteknologi

UPPSALA UNIVERSITET

Summary

- Normalized first and second order systems are instructive in selection of desired pole location
- PID controller is the most widespread feedback controller
- Design of PID controllers can be performed both from experiments and by means of a mathematical plant model