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Lecture 8

- Sensitivity function
- Sampling time
- Sampled-data control

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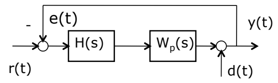
Sensitivity function



System equations:

$$E(s) = R(s) - Y(s)$$
$$Y(s) = H(s)W_p(s)E(s) + D(s)$$

Closed-loop system (SISO) with disturbance:

$$Y(s) = \frac{HW_p}{1+HW_p}R(s) + \frac{1}{1+HW_p}D(s)$$

$$W(s) = \frac{HW_p}{1+HW_p}$$
$$S(s) = \frac{1}{1+HW_p}$$

- *W(s)* is the closed-loop transfer function
- *S(s)* is the closed-loop sensitivity function
- Design objectives:
  - Disturbance rejection (stabilization controller)
  - Reference tracking (servo system)
- *H(s)* has to yield stability of the closed loop system and fulfill the design objective

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Sensitivity function, contd.

- **Disturbance rejection**: To minimize the impact of *d(t)* on *y(t)*, $H(j\omega)$ should be large in the frequency range of *d(t)*, (where $D(j\omega)$ is large)
- **Reference tracking**: To make y(t) follow r(t), $H(j\omega)$ should be large in the frequency range of r(t), (where $R(j\omega)$ is large)
- Stability problems usually arise for high gain design
- Assume *d=const* (measurement bias) and a pure integral in the transfer function of the controller

$$H(s) = \frac{1}{s}H_0(s)$$

$$S(s) = \frac{1}{1+\frac{1}{s}H_oW_p} = \frac{s}{s+H_0W_p} \qquad S(j\omega) = \frac{j\omega}{j\omega + H_0W_p}\Big|_{\omega=0} = 0$$

The controller is insensitive to constant disturbance.

- The sensitivity function should also kept low (by controller design) in the frequency range where the plant model W_p is uncertain.

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Sampling time

- Sampling time T – seconds; sampling frequency $\omega_c=2\pi/T$ – rad/s
- **Nyquist-Shannon principle**: A bandlimited analog signal that contains no frequencies higher than *B* rad/s can be perfectly reconstructed from an infinite sequence of samples if the sampling time is less or equal π/B seconds.
- **Alias phenomenon**: a sampled continuous time sinusoid with frequency above $\omega_c=\pi/T$ (Nyqvist frequency) cannot be distinguished from a signal with frequency below ω_c.
- **Anti-alias filter**: pre-sampling (continuous) low-pass filter minimizing higher frequency components
- **Thumb rule**: the sampling time is chosen as one tenth of the process time constant or less

$$W(s) = \frac{K}{Ts+1}$$

T – time constant (s)

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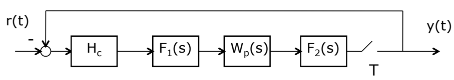
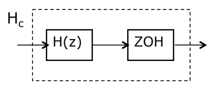
Sampled data control

Feedback sampled-data controller





- $W_p(s)$ – plant
- $F_1(s)$ – pre-filter
- $F_2(s)$ – anti-alias filter
- H(z) – discrete controller
- ZOH – zero-order hold

- The plant and the filters are continuous
- The controller H(z) is discrete
- Sampled-data design includes the controller and the filters

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Summary

- Sensitivity functions are the basis of modern controller design
- A good controller design is a trade-off between performance and sensitivity
- Sampling time in a control system has to follow Nyqvist-Shannon principle
- Sampling time is typically selected from an experiment or a process model
- Discrete controllers for continuous plants are most suitable to design via sampled-data theory