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Intro. Computer Control Systems: F10

Sensitivity and robustness

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F9: Quiz!

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1) When a system is observerable
   a the states can be estimated arbitrarily well $\uparrow$
   b the states can be controlled arbitrarily well $\uparrow$
   c the system is also stable $\downarrow$

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2) State estimation using an observer
   - a does not handle initial errors of the state $\uparrow$
   - b can be described as a differential equation $\uparrow$
   - c is an unstable process $\downarrow$

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1) When a system is observerable
   - a the states can be estimated arbitrarily well ↑
   - b the states can be controlled arbitrarily well ↑
   - c the system is also stable ↓

2) State estimation using an observer
   - a does not handle initial errors of the state ↑
   - b can be described as a differential equation ↑
   - c is an unstable process ↓

3) The transfer function for a control system with estimated states
   - a is different from that of control system with known states ↑
   - b is the same as that of control system with known states↑
   - c is real-valued ↓

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Sensitivity to disturbance and noise

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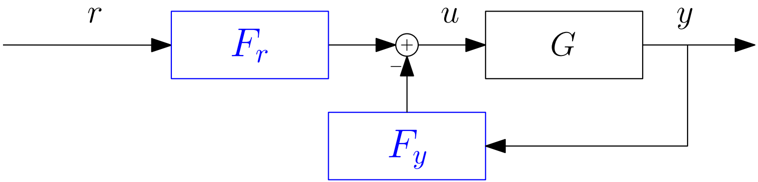
Control system with disturbances and noise

Using general linear feedback



Closed-loop system using general linear feedback:

$$G_c(s) = \frac{G(s)F_r(s)}{1 + G(s)F_y(s)}$$

General open-loop system: $G_o(s) \triangleq F_y(s)G(s)$

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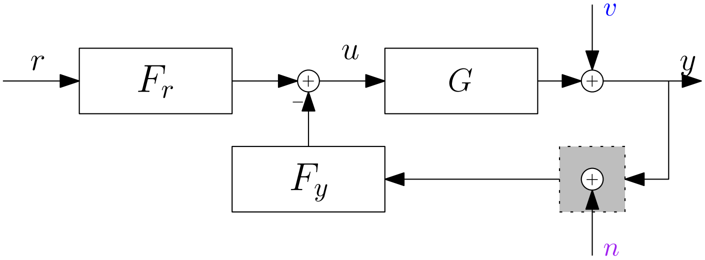

How will the control system cope with *unknown* disturbances and noise?

[Board: the closed-loop system with $V(s)$ and $N(s)$]

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Defining sensitivity functions

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- **Complementary sensitivity function:**

$$T(s) \triangleq 1 - S(s) = \frac{G_o(s)}{1 + G_o(s)}$$

- Consequence:

$$\boxed{S(s) + T(s) \equiv 1, \qquad \forall s}$$

- $S(s)$ and $T(s)$ affected by controller $F_y(s)$.

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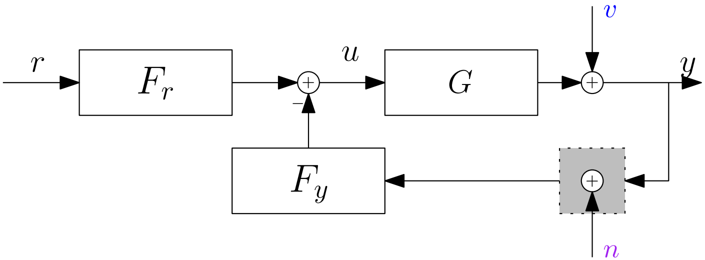
Closed-loop system and the sensitivity functions



- Closed-loop system:

$$Y(s) = G_c(s)R(s) + S(s)V(s) - T(s)N(s)$$

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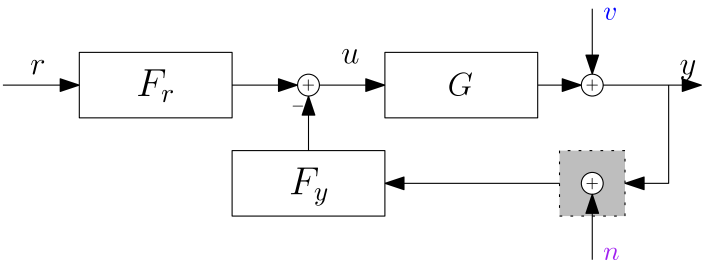
Closed-loop system and the sensitivity functions



- Closed-loop system:

$$Y(s) = G_c(s)R(s) + S(s)V(s) - T(s)N(s)$$

- Want both $|S(i\omega)|$ and $|T(i\omega)| \ll 1$ *simultanously*...

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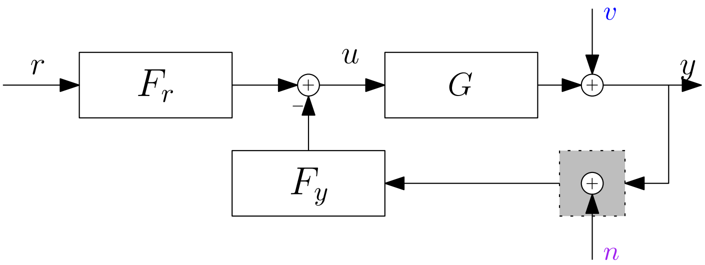
Closed-loop system and the sensitivity functions



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- Want both $|S(i\omega)|$ and $|T(i\omega)| \ll 1$ *simultanously*...
- ...but *impossible* since

$$\boxed{|S(i\omega)| + |T(i\omega)| \geq |S(i\omega) + T(i\omega)| \equiv 1}$$

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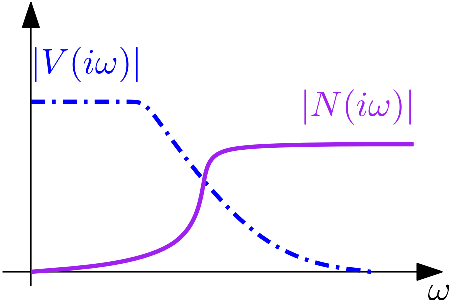
Sensitivity functions in frequency domain
Design trade-off

Example:

- Disturbance $v(t)$ with energy at low frequencies
- Noise $n(t)$ with energy at high frequencies



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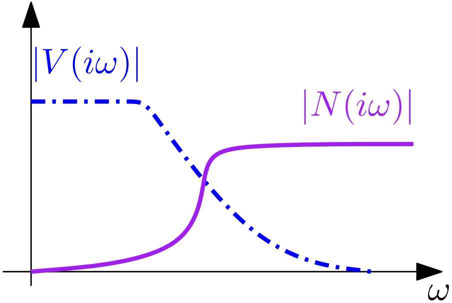

Typical design trade-off is then:

- *low* ω: $|S(i\omega)| \ll 1$ to suppress $V(i\omega)$.
- *high* ω: $|T(i\omega)| \ll 1$ to suppress $N(i\omega)$.

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Sensitivity functions in frequency domain

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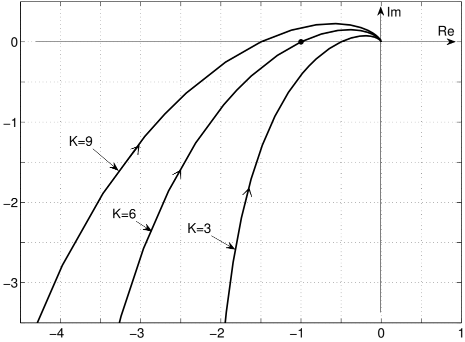
In addition we want Nyquist contour

$$G_o(i\omega) = F_y(i\omega)G(i\omega) = \frac{T(i\omega)}{S(i\omega)}$$

far from -1. (Cf. F6 and F7.)



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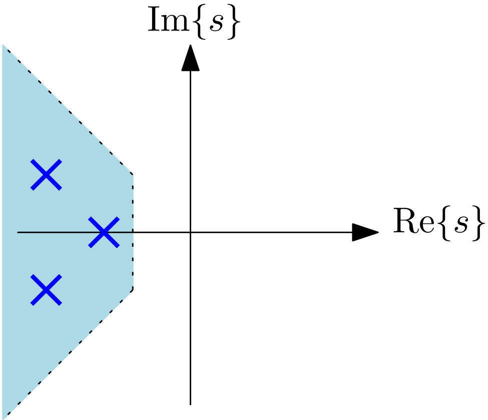
Sensitivity functions in frequency domain
Design trade-off

Design of poles and zeros via F_r and F_y affects also S and T



E.g. quick system means also increased sensitivity

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Robustness to model errors

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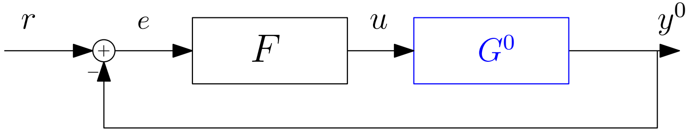
Control systems with model errors

Model G is an *approximation*



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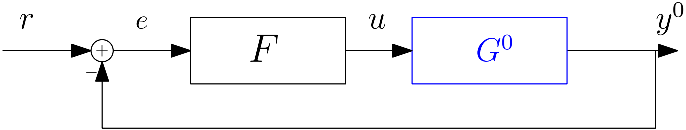

- *Assume* that the real system can be written as

$$G^0(s) = G(s)(1 + \Delta_G(s))$$

- The relative model error of $G(s)$:

$$\boxed{\Delta_G(s) = \frac{G^0(s) - G(s)}{G(s)}}$$

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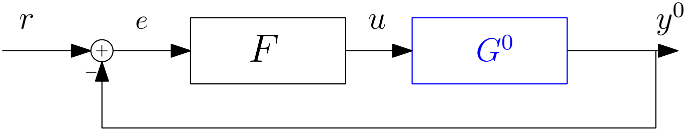

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- How is stability of $G_c^0(s)$ affected by unknown error $\Delta_G(s)$?

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Model errors and stability

Using the complementary sensitivity function

Assume:

1. Controller $F(s)$ stabilizes the *assumed* system $G(s)$
2. $G(s)$ and $G^0(s)$ have same number of poles in right half-plane.
3. Open-loop: $F(s)G(s) \to 0$ and $F(s)G^0(s) \to 0$ where $|s| \to \infty$

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(Result 6.2) Robustness criterium

If assumptions are valid and $T(i\omega)$ fulfills

$$|T(i\omega)| < \frac{1}{|\Delta_G(i\omega)|}, \quad -\infty \leq \omega \leq \infty$$

$\Rightarrow$ the *real* closed-loop system $G_c^0(s)$ is also stable!

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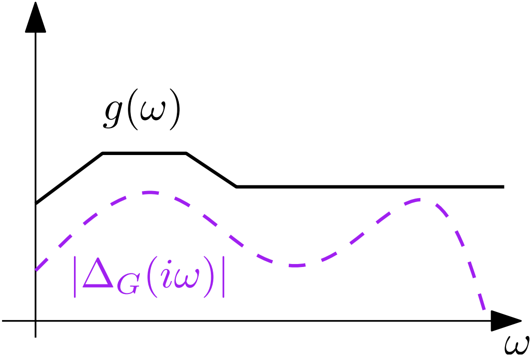
Model errors and stability

Bounding the model errors

$\Delta_G(i\omega)$ is unknown but suppose we can cap it by $g(\omega) > |\Delta_G(i\omega)|$



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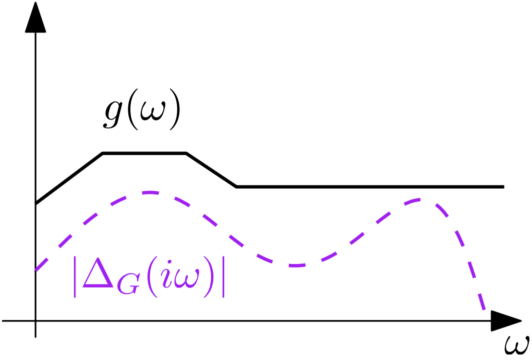

$$|T(i\omega)| < \frac{1}{g(\omega)} < \frac{1}{|\Delta_G(i\omega)|}$$

$\Rightarrow$ real closed-loop system $G_c^0(s)$ is also stable

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Summary and recap

- Sensitivity with respect to disturbances and noise
- Sensitivity functions and their impact on control
- Robustness with respect to model errors
- Robustness criterion in the frequency domain

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