Intro. Computer Control Systems: F4

PID-controller and its poles, disturbances, root-locus

Dave Zachariah

Dept. Information Technology, Div. Systems and Control

dave.zachariah@it.uu.se

F3: Quiz!

1) Closed-loop systems $G_c(s)$ with complex-conjugated poles exhibit
   - a instability $\uparrow$
   - b oscillations $\uparrow$
   - c slow time-response $\downarrow$

dave.zachariah@it.uu.se

F3: Quiz!

1) Closed-loop systems $G_c(s)$ with complex-conjugated poles exhibit
   - a instability ↑
   - b oscillations ↑
   - c slow time-response ↓
2) Integral term $\frac{1}{s}$ in a control $F(s)$ can
   - a ensure quick closed-loop system ↑
   - b ensure zero stationary control error ↑
   - c ensure oscillations in the closed-loop system ↓

dave.zachariah@it.uu.se

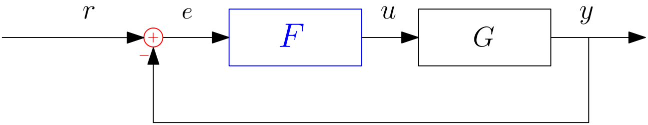
Poles of closed-loop system

dave.zachariah@it.uu.se

Ideal PID-controller

PID-controller with user parameters:

$$u(t) = \underbrace{K_p e(t)}_{P} + \underbrace{K_i \int_{\tau=0}^{t} e(\tau) d\tau}_{I} + \underbrace{K_d \dot{e}(t)}_{D}$$



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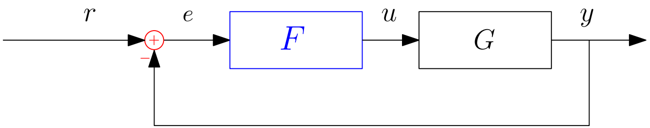

Laplace domain:

$$U(s) = \underbrace{\left(K_p + \frac{K_i}{s} + K_d s \right)}_{\text{controller} F(s)} E(s).$$

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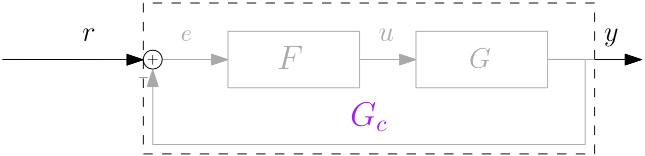
Simple feedback control

Closed-loop system $G_c(s)$:



Laplace domain:

$$Y(s) = \frac{G(s)F(s)}{1 + G(s)F(s)} R(s).$$

Determine user parameters K_p, K_i and K_d to obtain stable closed-loop system

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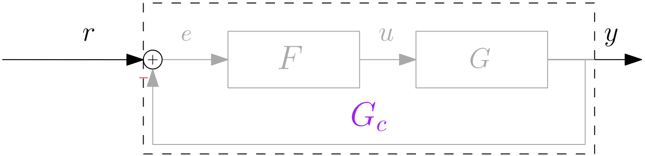
Simple feedback control

Closed-loop system $G_c(s)$:



Laplace domain:

$$Y(s) = \frac{G(s)F(s)}{1 + G(s)F(s)} R(s).$$

Determine user parameters K_p, K_i and K_d to obtain
stable closed-loop system
$\Leftrightarrow$ poles of $G_c(s)$ in left half-plane!

dave.zachariah@it.uu.se

Studying the poles of G_c

Poles of

$$Y(s) = \frac{G(s)F(s)}{1 + G(s)F(s)} R(s).$$

depend on user parameters in $F(s)$.

Classical design methods include:

- Solve roots of $G_c(s)$ as a function of parameters
- Routh's algorithm for stability check
- Root-locus plot for single parameter

dave.zachariah@it.uu.se

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Illustrated in example below

dave.zachariah@it.uu.se

Routh's algorithm

dave.zachariah@it.uu.se

Routh's algorithm

A method for analyzing system stability

Assume (closed-loop) system of order n
Poles obtained as roots to polynomial, $a_0 > 0$

$$a_0 s^n + b_0 s^{n-1} + a_1 s^{n-2} + b_1 s^{n-3} + \cdots = 0$$

dave.zachariah@it.uu.se

Routh's algorithm

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Routh's algorithm: i) Form table

$$\begin{array}{ccccc} a_0 & a_1 & a_2 & a_3 & \cdots \\ b_0 & b_1 & b_2 & b_3 & \cdots \end{array}$$

dave.zachariah@it.uu.se

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Routh's algorithm

A method for analyzing system stability

Assume (closed-loop) system of order n

ii) Fill the rows according to...

$$
\begin{array}{ccccc}
a_0 & a_1 & a_2 & a_3 & \cdots \\
b_0 & b_1 & b_2 & b_3 & \cdots \\
c_0 & c_1 & c_2 & \cdots &
\end{array}
$$

where

$$c_k = \frac{b_0 a_{k+1} - b_{k+1} a_0}{b_0}$$

dave.zachariah@it.uu.se

Routh's algorithm

A method for analyzing system stability

Assume (closed-loop) system of order n

iii) ...continue

$$\begin{array}{ccccc} a_0 & a_1 & a_2 & a_3 & \cdots \\ b_0 & b_1 & b_2 & b_3 & \cdots \\ c_0 & c_1 & c_2 & \cdots & \\ d_0 & d_1 & d_2 & \cdots & \end{array}$$

where

$$d_k = \frac{c_0 b_{k+1} - c_{k+1} b_0}{c_0},$$

until one ends with $(n+1)$ rows

dave.zachariah@it.uu.se

Routh's algorithm

A method for analyzing system stability

Assume (closed-loop) system of order n

iv) Check coefficients in the first column

$$\begin{array}{ccccc} \color{red}{a_0} & a_1 & a_2 & a_3 & \cdots \\ \color{red}{b_0} & b_1 & b_2 & b_3 & \cdots \\ \color{red}{c_0} & c_1 & c_2 & \cdots & \\ \color{red}{d_0} & d_1 & d_2 & \cdots & \\ \color{red}{\vdots} & & & & \end{array}$$

Theorem 2.3 (Routh):

If $a_0, b_0, c_0, d_0, \ldots > 0 \Leftrightarrow$ all poles lie in the left half-plane (stable system)

Useful for higher order systems (alt. use Matlab)!

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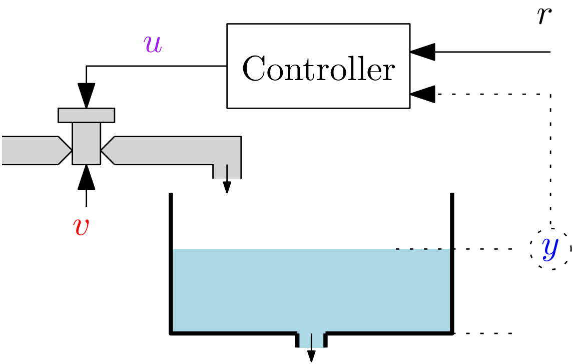
Controlling system with disturbance

dave.zachariah@it.uu.se

PID-control subject to external disturbance

Example: tank with value + disturbance



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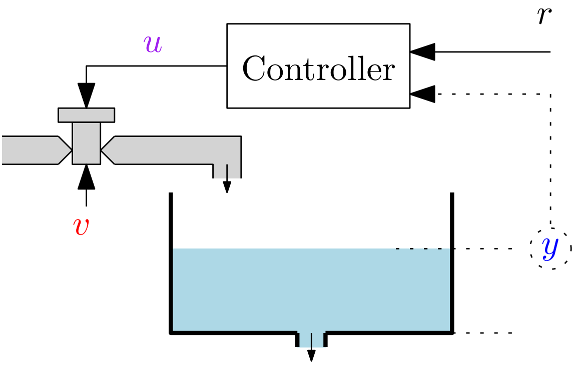

Linear *approximate* models (around an operating point):

- Tank (inflow to level): $G_1(s) = \frac{4}{s+1}$
- Valve (turn angle to ideal outflow): $G_2(s) = \frac{2}{s+2}$

dave.zachariah@it.uu.se

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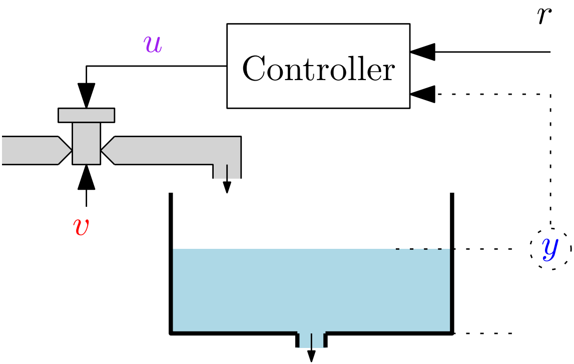

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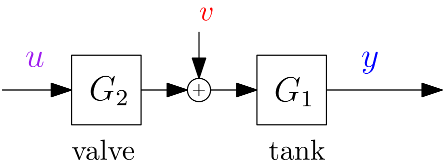
With pressure disturbance $v(t) \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad V(s)$

dave.zachariah@it.uu.se

Example: tank with valve + disturbance

Draw block diagram

Block diagram:



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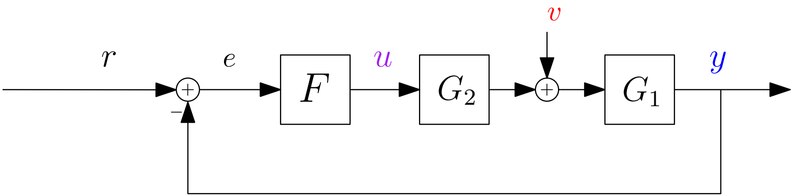
Example: tank with valve + disturbance

Draw block diagram

Closed-loop system:



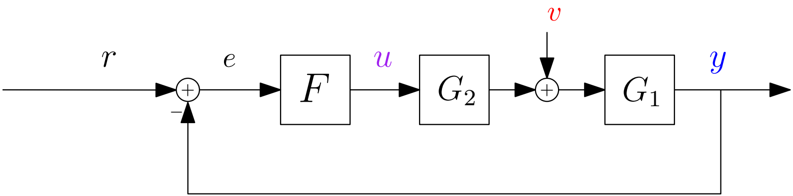
[Board: derive the closed-loop system]

dave.zachariah@it.uu.se

Example: tank with valve + disturbance

Draw block diagram

Closed-loop system:



$$Y(s) = \frac{G_o(s)}{1+G_o(s)} R(s) + \frac{G_1(s)}{1+G_o(s)} V(s),$$

where

- open-loop $G_o(s) \triangleq G_1(s) G_2(s) F(s)$
- controller $F(s)$

dave.zachariah@it.uu.se

Example: tank with valve + disturbance

Effect of disturbance on control of tank level

Let $r(t) \equiv 0 \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad R(s) = 0$ and study effect of disturbance

dave.zachariah@it.uu.se

Example: tank with valve + disturbance

Effect of disturbance on control of tank level

Let $r(t) \equiv 0 \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad R(s) = 0$ and study effect of disturbance

$$Y(s) = 0 + \frac{G_1(s)}{1 + G_1(s)G_2(s)F(s)} V(s),$$

where $F(s) = K_p + \frac{K_i}{s} + K_d s$ is PID-controller.

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Insertion yields

$$Y(s) = \frac{\frac{4}{s+1}}{1 + \frac{4}{s+1} \cdot \frac{2}{s+2} \cdot \left(K_p + \frac{K_i}{s} + K_d s \right)} V(s)$$
$$= \frac{4s(s+2)}{s^3 + (3 + 8K_d)s^2 + (2 + 8K_p)s + 8K_i} V(s)$$

dave.zachariah@it.uu.se

Example: tank with valve + disturbance

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Insertion yields

$$\begin{aligned} Y(s) &= \frac{\frac{4}{s+1}}{1 + \frac{4}{s+1} \cdot \frac{2}{s+2} \cdot \left(K_p + \frac{K_i}{s} + K_d s \right)} V(s) \\ &= \frac{4s(s+2)}{s^3 + (3 + 8K_d)s^2 + (2 + 8K_p)s + 8K_i} V(s) \end{aligned}$$

Where do the poles end up?
What happens when a sudden constant disturbance occurs?

dave.zachariah@it.uu.se

P-control

dave.zachariah@it.uu.se

Example: tank with valve + disturbance

P-control, $K_i = K_d = 0$

Insertion of $K_i = K_d = 0$ gives

$$Y(s) = \frac{4(s+2)}{s^2 + 3s + 2 + 8K_p} V(s)$$

Example: tank with valve + disturbance

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Closed-loop system poles directly given by:

$$s^2 + 3s + 2 + 8K_p = 0 \quad \Rightarrow s = -1.5 \pm \sqrt{0.25 - 8K_p}$$

dave.zachariah@it.uu.se

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Note choice of parameter K_p:

- $K_p \geq 0$ gives poles in left half-plane
- $8K_p > 0.25$ or $K_p > 0.03125$ gives complex-conjugated poles

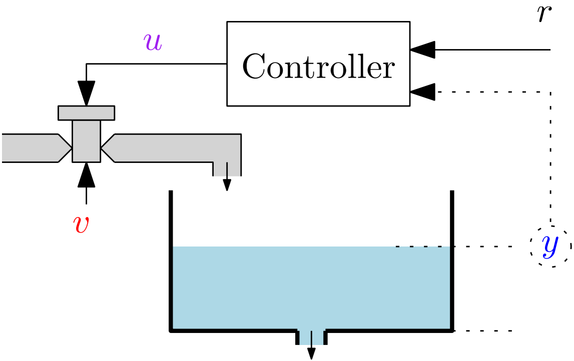
dave.zachariah@it.uu.se

Disturbance supression

P-control, $K_i = K_d = 0$, when $r(t) \equiv 0$

What happens to tank level $y(t)$ if a *sudden* pressure drop occurs in the pipe? Assume

$$v(t) = \begin{cases} -v_0 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad V(s) = -\frac{v_0}{s}$$



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When closed-loop system from $v(t)$ to $y(t)$ is stable: Use *final-value theorem*

$$y_f = \lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s)$$

[Board: derive final value of the closed-loop system]

dave.zachariah@it.uu.se

Disturbance supression

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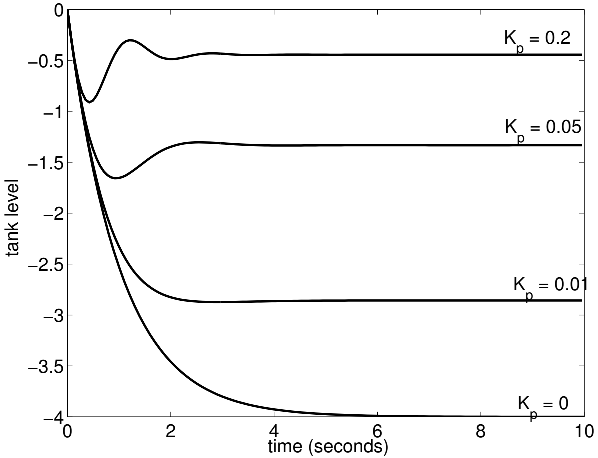

Note stability, quickness, final value and when oscillations occur

dave.zachariah@it.uu.se

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PI-control

dave.zachariah@it.uu.se

Example: tank with valve + disturance

PI-control, $K_d = 0$

Insertion of $K_d = 0$ gives

$$Y(s) = \frac{4s(s+2)}{s^3 + 3s^2 + (2 + 8K_p)s + 8K_i} V(s)$$

Closed-loop system poles given by:

$$s^3 + 3s^2 + (2 + 8K_p)s + 8K_i = 0 \qquad \text{left half-plane?}$$

dave.zachariah@it.uu.se

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Using Routh's algorithm: i) Tabulate

$$\begin{array}{ccc} 1 & 2+8K_p & 0 \\ 3 & 8K_i & 0 \end{array}$$

dave.zachariah@it.uu.se

Example: tank with valve + disturance

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Insertion of $K_d = 0$ gives

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Routh's algorithm: ii) Fill rows...

$$\begin{array}{ccc} 1 & 2 + 8K_p & 0 \\ 3 & 8K_i & 0 \\ (24K_p + 6 - 8K_i)/3 & 0 & 0 \end{array}$$

dave.zachariah@it.uu.se

Example: tank with valve + disturance

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dave.zachariah@it.uu.se

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Example: tank with valve + disturance

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Routh's algorithm: iv) Check coefficients in the first column

$$\begin{array}{ccc} 1 & 2+8K_p & 0 \\ 3 & 8K_i & 0 \\ (24K_p + 6 - 8K_i)/3 & 0 & 0 \\ 8K_i & 0 & \end{array}$$

To obtain poles in left half-plane:

$$\Leftrightarrow 24K_p + 6 - 8K_i > 0 \quad \text{och} \quad K_i > 0$$

dave.zachariah@it.uu.se

Example: tank with valve + disturance

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To obtain poles in left half-plane:

$$\Leftrightarrow 24K_p + 6 - 8K_i > 0 \quad \text{och} \quad K_i > 0$$

Choice of parameters K_p and K_i:

- $0 < K_i < \frac{3}{4} + 3K_p$ gives a stable closed-loop system.

Disturbance suppression, cont'd.

PI-control, $K_d = 0$, when $r(t) \equiv 0$

What happens to tank level $y(t)$ if a *sudden* pressure drop occurs in the pipe? Assume

$$v(t) = \begin{cases} -v_0 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \overset{\mathcal{L}}{\longleftrightarrow} \quad V(s) = -\frac{v_0}{s}$$

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Disturbance suppression, cont'd.

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dave.zachariah@it.uu.se

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$$\begin{aligned} y_f &= \lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) \\ &= \lim_{s \to 0} s \frac{4s(s+2)}{s^3 + 3s^2 + (2 + 8K_p)s + 8K_i} \frac{-v_0}{s} \\ &= \frac{-4 \cdot 0 \cdot v_0}{8K_i} = 0! \end{aligned}$$

dave.zachariah@it.uu.se

Disturbance suppression, cont'd.

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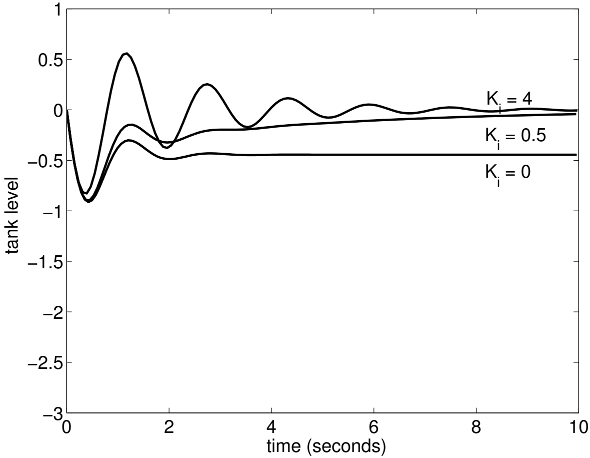
Effect of constant disturbance *vanishes* with integral!

dave.zachariah@it.uu.se

Disturbance suppression, cont'd.

PI-control, $K_d = 0$, when $r(t) \equiv 0$



Note stability and final value

dave.zachariah@it.uu.se

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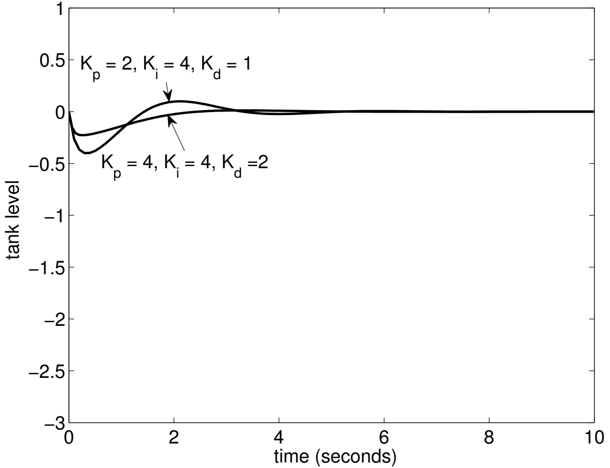
PID-control

dave.zachariah@it.uu.se

Disturbance suppression, cont'd.

PID-control, when $r(t) \equiv 0$



PID-control gives in *this case* free choice of poles using K_p,K_i and K_d

dave.zachariah@it.uu.se

Root locus

dave.zachariah@it.uu.se

Example: PI-control of a simple system

Poles on $\mathbb{C}$-plane as a function of single user-parameter

Assume system $Y(s) = \underbrace{\frac{1}{s+1}}_{G(s)} U(s)$ with PI-controller:

$$U(s) = \underbrace{\left(K + \frac{10K}{s}\right)}_{F(s)} (R(s) - Y(s)) \quad (K_p = K,\ K_i = 10K)$$

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gives closed-loop system

$$Y(s) = \frac{G(s)F(s)}{1 + G(s)F(s)} R(s) = \frac{K \frac{s+10}{s(s+1)}}{1 + K \frac{s+10}{s(s+1)}} R(s)$$

$$= \frac{K(s+10)}{s(s+1) + K(s+10)} R(s)$$

with single parameter K

dave.zachariah@it.uu.se

Root locus

A graphical method to study how poles depend on K

Closed-loop system can be written on the form:

$$G_c(s) = \frac{\cdots}{P(s) + KQ(s)}$$

A root locus shows the roots to the polynomial equation

$$P(s) + KQ(s) = 0, \quad 0 \leq K < \infty \qquad (*)$$

as a function of parameter K.

dave.zachariah@it.uu.se

Root locus

A graphical method to study how poles depend on K

Closed-loop system can be written on the form:

$$G_c(s) = \frac{\cdots}{P(s) + KQ(s)}$$

A root locus shows the roots to the polynomial equation

$$P(s) + KQ(s) = 0, \quad 0 \leq K < \infty \qquad (*)$$

as a function of parameter K.

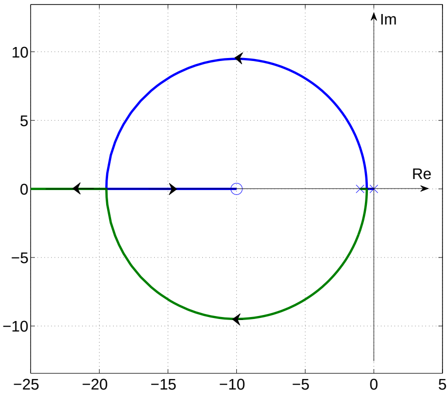
Definition:

Root locus = *the points* s that solve $P(s) + KQ(s) = 0$ (roots/poles) as we vary K.

dave.zachariah@it.uu.se

Root locus
Example: PI-control of simple system

Poles of $G_c(s)$ change when K goes from 0 to ∞:



dave.zachariah@it.uu.se

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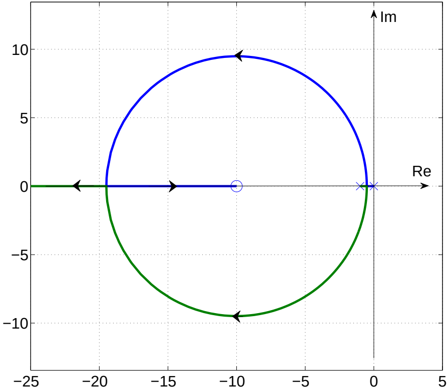


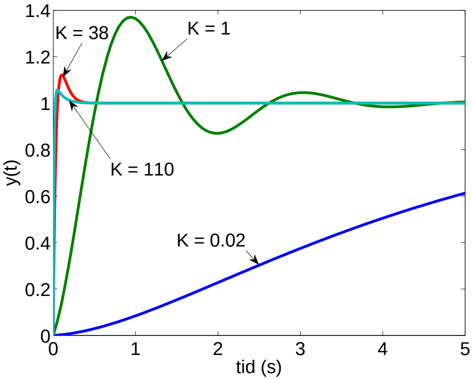
How do we expect closed-loop system to behave at different K?

dave.zachariah@it.uu.se

Corresponding step responses

Exemple: PI-control of simple system

Closed-loop system response to step $R(s) = \frac{1}{s}$ for different K:



dave.zachariah@it.uu.se

Characteristics of root locus

- Polynomial $P(s)$ and $Q(s)$ have order n and m, resp.
- Equation $(*)$ has always n roots, which constitute n *branches* in the root locus.
- $P(s)$ and $Q(s)$ have real-valued coefficients $\Rightarrow$ complex-valued roots to $(*)$ are all complex-conjugated pairs $\Rightarrow$ root locus is *symmetric around the real-axis*.

dave.zachariah@it.uu.se

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- The root locus is characterized by:
  1. starting points
  2. end points
  3. asymptotes
  4. parts on the real-axis

dave.zachariah@it.uu.se

Characteristics of root locus, cont'd.

Results 3.1 and 3.2

dave.zachariah@it.uu.se

Characteristics of root locus, cont'd.

UPPSALA UNIVERSITET

Results 3.1 and 3.2

- **Starting point:** The n roots to $(*)$ for $K = 0$, given by $P(s) = 0$. Use '$\times$'.

dave.zachariah@it.uu.se

Characteristics of root locus, cont'd.

Results 3.1 and 3.2

- **Starting point:** The n roots to $(*)$ for $K = 0$, given by $P(s) = 0$. Use '$\times$'.
- **End points:** The m finite roots to $(*)$ when $K \to \infty$, given by $Q(s) = 0$. Use '$\circ$'.

dave.zachariah@it.uu.se

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- **Asymptotes:** $n - m$ branches that extend to infinity. The asymptotes extend (symmetrically) from a point on the real axis.

dave.zachariah@it.uu.se

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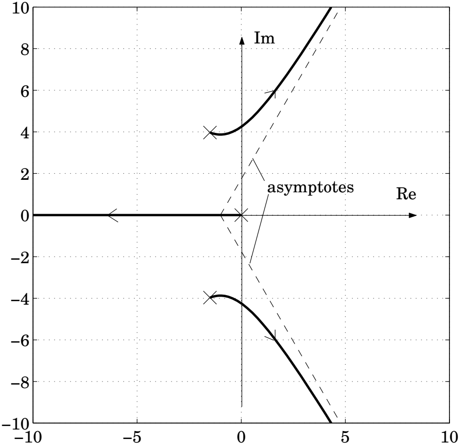
- **Asymptotes:** $n - m$ branches that extend to infinity. The asymptotes extend (symmetrically) from a point on the real axis.
- **Real axis:** The parts of the real-axis with an odd number of real-valued starting- and end points to their right, belong to the root locus.
- **Branches** of root locus cannot cross. Thus if two brances meet on the real axis they must 'split' in the complex plane.

dave.zachariah@it.uu.se

PI-control of the tank

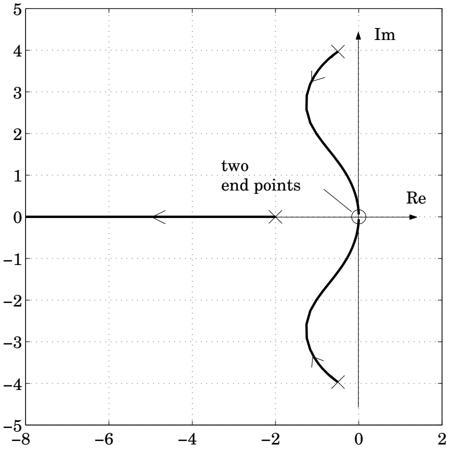
Root locus for $G_c(s)$ with respect to K_i (fix $K_p = 2$ and $K_d = 0$).



dave.zachariah@it.uu.se

PID-control of the tank

Root locus for $G_c(s)$ with respect to K_d (fix $K_p = 2$ and $K_i = 4$).



dave.zachariah@it.uu.se

Summary and recap

- PID-controller and closed-loop system stability
- Example of PID-controller and disturbance suppression
- Closed-loop system poles as a function of parameters K

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