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Intro. Computer Control Systems: F5

Control structures, frequency descriptions

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F4: Quiz!

1) PI-control
   - a May suppress constant disturbances $\uparrow$
   - b Guarantees zero stationary control errors $\uparrow$
   - c Guarantees zero stable closed-loop system $\downarrow$

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F4: Quiz!

1) PI-control
   a May suppress constant disturbances ↑
   b Guarantees zero stationary control errors ↑
   c Guarantees zero stable closed-loop system ↓
2) Routh's algorithm is a method that
   a Checks for oscillations in systems ↑
   b Aids design of stable closed-loop systems ↑
   c Guarantees stable systems ↓

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Simple vs. general linear feedback

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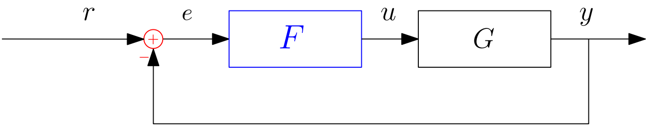
Simple linear feedback control

Exemple: PID-controller

Simple feedback using *control error*:

$$U(s) = F(s)\underbrace{(R(s) - Y(s))}_{=E(s)} = F(s)R(s) - F(s)Y(s).$$



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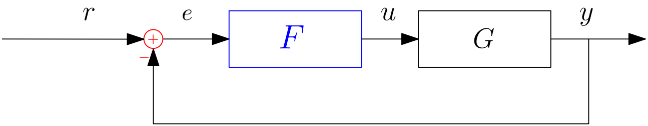

Closed-loop system:

$$Y(s) = \frac{G(s)F(s)}{1 + G(s)F(s)} R(s)$$

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General linear feedback control

General feedback with *all measured* signals:

$$U(s) = F_r(s)R(s) - F_y(s)Y(s).$$

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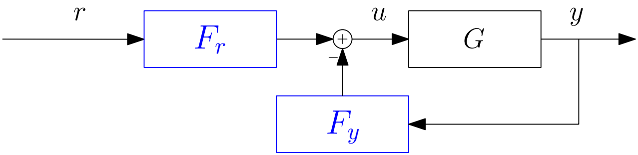

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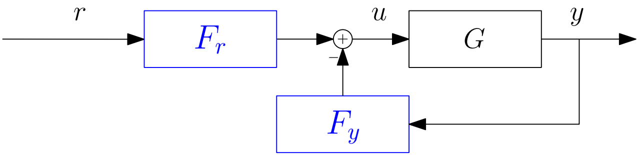

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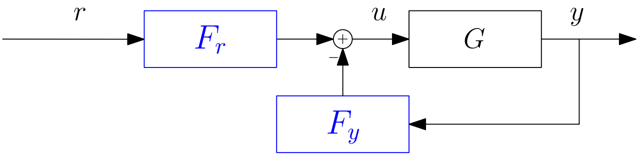

Closed-loop system:

$$Y(s) = \frac{G(s)F_r(s)}{1 + G(s)F_y(s)}R(s)$$

- $F_y(s) = F_r(s) = F(s) \Rightarrow$ simple linear feedback
- $F_y(s) \neq F_r(s) \Rightarrow$ more *degrees of freedom*.

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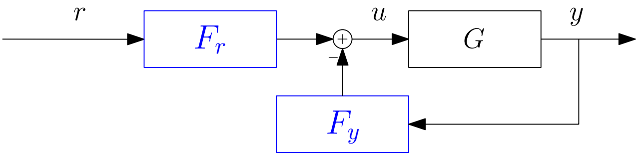

Closed-loop system:

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See state-feedback controller F8-F10.

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Feedback with measureable disturbances

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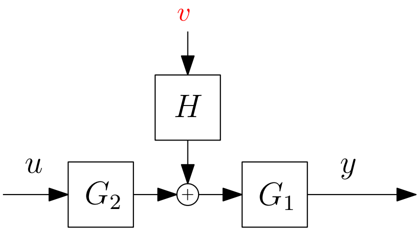
Feedforward control

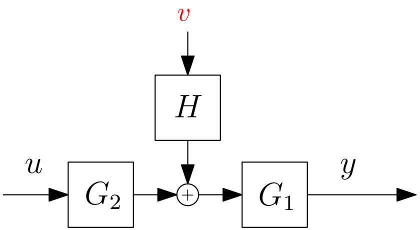
Scenarios with measurable disturbances

In some systems we are also able to measure disturbances.

Example:



Feedforward control
Scenarios with measurable disturbances



General feedback with *measurable* signals *including* disturbance:

$$U(s) = F_r(s)R(s) - F_y(s)Y(s) + \underbrace{F_f(s)V(s)}_{\text{control using also measured disturbance}}$$

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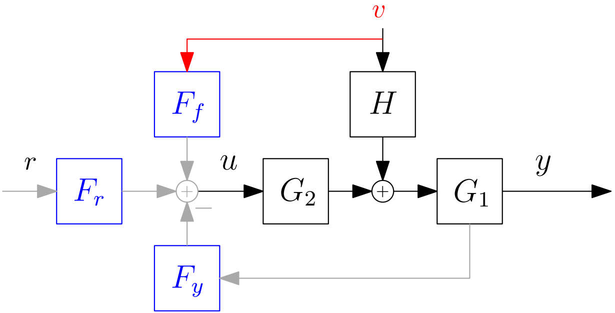
Feedforward control

Scenarios with measurable disturbances

General feedback with *measurable* signals *including* disturbance:

$$U(s) = F_r(s)R(s) - F_y(s)Y(s) + F_f(s)V(s).$$



[Board: derive the closed-loop system $r, v \to y$]

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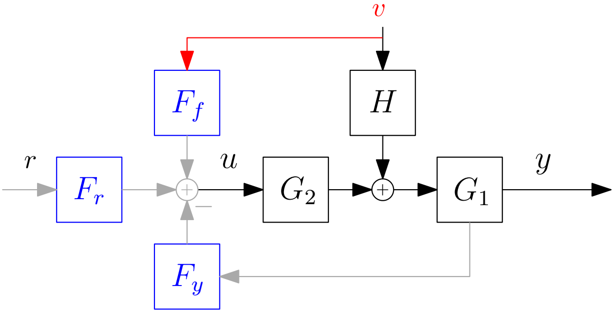
Feedforward control

Scenarios with measurable disturbances



Ideal controller:

$$Y(s) = \underbrace{\frac{G_1(s)G_2(s)F_r(s)}{1+G_1(s)G_2(s)F_y(s)}}_{\approx 1} R(s) + \underbrace{\frac{G_1(s)\left(H(s)+G_2(s)F_f(s)\right)}{1+G_1(s)G_2(s)F_y(s)}}_{\approx 0} V(s)$$

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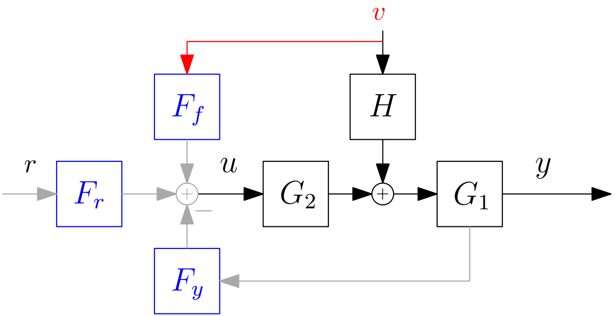
Feedforward control

Scenarios with measurable disturbances



Ideal disturbance suppression:

$$H(s) + G_2(s)F_f(s) = 0 \quad \Leftrightarrow \quad F_f(s) = -\frac{H(s)}{G_2(s)}$$

Explicit disturbance compensation, but often *hard* to implement, due to *high order in the numerator* of $F_f(s)$.

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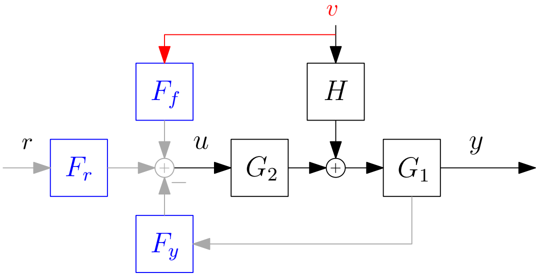
Example: tank with valve + disturbance



Example:

- Static feedforward:

$$F_f(s) = -\frac{H(0)}{G_2(0)}$$

- Dynamic feedforward:

$$F_f(s) = -\frac{H(s)}{G_2(s)}\frac{20}{s+20}$$

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Example: tank with valve + disturbance

Control using feedforward, when $r(t) \equiv 0$ and $v(t)$ is a step

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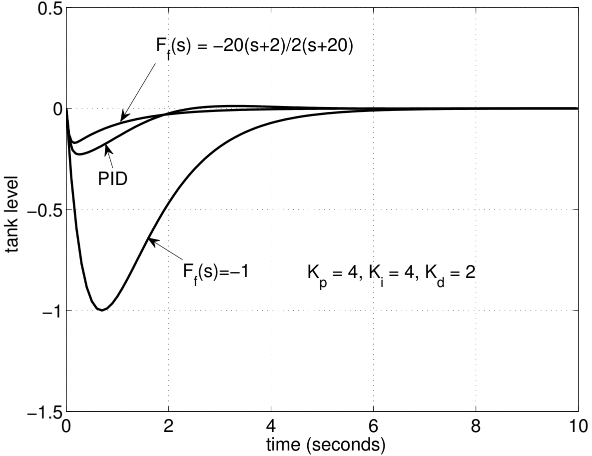
Example: tank with valve + disturbance

Control using feedforward, when $r(t) \equiv 0$ and $v(t)$ is a step



Static/dynamic feedforward vs. PID control

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Feedback with intermediate signals

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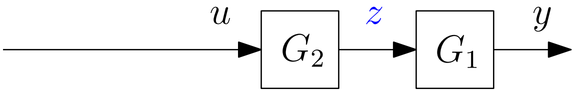
Cascade control
Measurable intermediate signals

In certain systems we can measure internal or intermediate signals:



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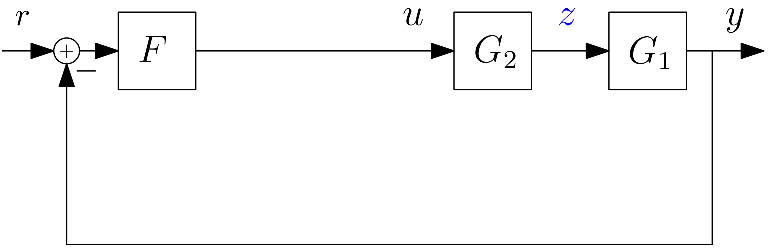
Cascade control
Measurable intermediate signals

Simple linear feedback:

$$U(s) = F(s)R(s) - F(s)Y(s) \\ = F(s)E(s)$$



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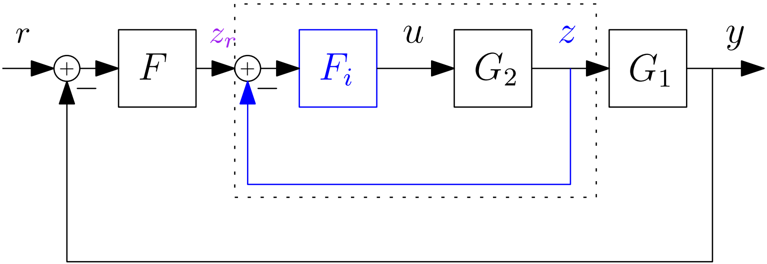
Cascade control
Measurable intermediate signals

Simple linear feedback control with *measured* signals:

$$U(s) = F_i(s)F(s)R(s) - F_i(s)F(s)Y(s) - F_i(s)Z(s)$$
$$= F_i(s)\big(\underbrace{F(s)E(s)}_{Z_r(s)} - Z(s)\big)$$



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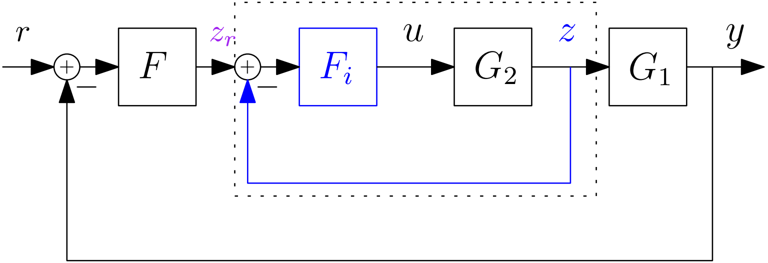
Cascade control

Measurable intermediate signals



Control strategy: easier to control subsystems

- Design $F_i(s)$ so that $Z(s) \approx Z_r(s)$
- Design $F(s)$ with respect to $G_1(s)$, neglect internal loop.

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Combining all above

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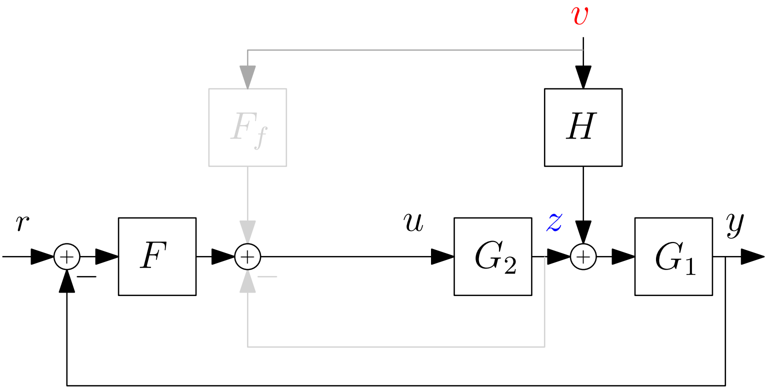
Measurable signals and disturbances

Total controller

Control with respect to $R(s)$ and $Y(s)$



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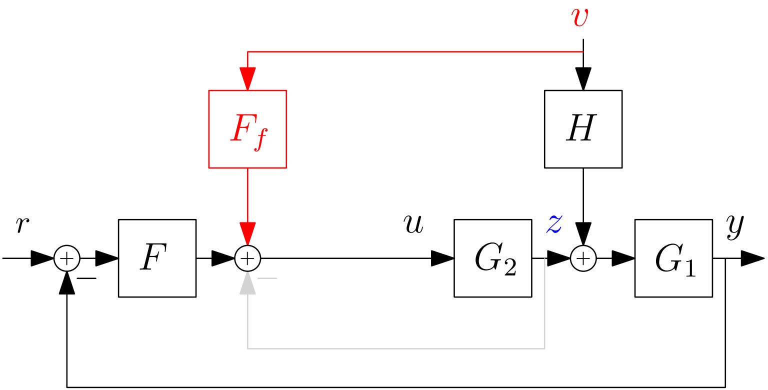
Measurable signals and disturbances
Total controller

Control with respect to $R(s)$ and $Y(s)$ as well as measurable disturbance $V(s)$



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Measurable signals and disturbances

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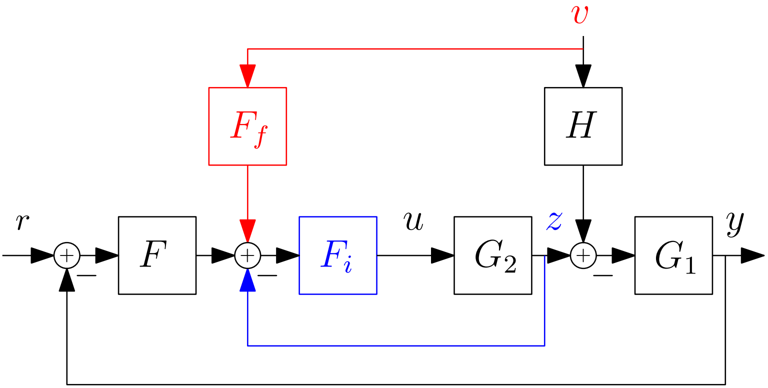

Exploits all available information!

Time-varying signals and frequency descriptions

Frequency description of systems

(Closed-loop) system response to oscillating signals

- Recall representation of cosine and sine signals:

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

$$\omega = 2\pi f \quad \text{(frequency)}$$

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- Any signal $x(t)$ can be decomposed into sum of cosine and sine signals:

$$x(t) = \int \underbrace{X(i\omega)}_{\text{weights}} \overbrace{e^{i\omega t}}^{\text{periodic}} \, d\omega,$$

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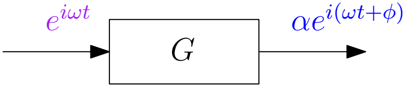
- $e^{i\omega t}$ is an eigen-function to linear time-invariant systems



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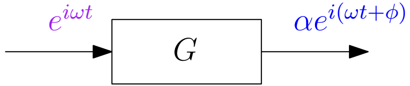

Fundamental property of LTI systems

Output is also a sum of the *input* cosine- and sine signals!

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Frequency properties

Sine in/sine out

[Board: derive $y(t)$ when $u(t) = A\sin(\omega t)$]

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Frequency properties
Sine in/sine out

[Board: derive $y(t)$ when $u(t) = A\sin(\omega t)$]

Sine in-sine out

Assume stable system $Y(s) = G(s)U(s)$, where $u(t) = A\sin(\omega t)$. *After* all transients vanish, we obtain output:

$$y(t) = \underbrace{|G(i\omega)|}_{\text{amplification}} \cdot A\sin(\omega t + \underbrace{\arg G(i\omega)}_{\text{phase shift}})$$

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Frequency properties
Sine in/sine out

[Board: derive $y(t)$ when $u(t) = A\sin(\omega t)$]

Sine in-sine out

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- Same applies to closed-loop system

$$Y(s) = G_c(s)R(s),$$

  with $r(t) = A\sin(\omega t)$
- Yields *interpretation* of complex-valued

$$\boxed{G_c(s) = |G_c(s)|e^{i\arg G_c(s)} \text{ when } s = i\omega}$$

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Frequency properties of closed-loop system

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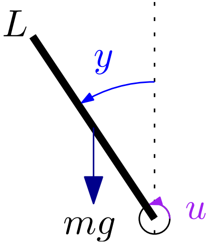
Example: inverted pendulum



Linearized model (around $y \approx 0$):

$$\ddot{y} - \left(\frac{3g}{2L}\right) y = \left(\frac{3}{mL^2}\right) u \Leftrightarrow Y(s) = \frac{\frac{3}{mL^2}}{s^2 - \frac{3g}{L}} U(s) = \frac{1}{s^2 - 1} U(s) s$$

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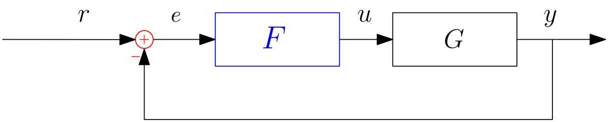
Example: inverted pendulum
Response of PD-control

System $Y(s) = \frac{1}{s^2-1}U(s)$.



- PD-control $U(s) = K(s+3)(R(s) - Y(s))$ give closed-loop system

$$Y(s) = G_c(s)R(s) \quad \text{where} \quad G_c(s) = \frac{K(s+3)}{s^2 + Ks + 3K - 1}.$$

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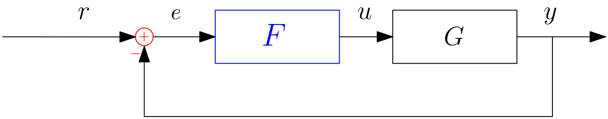
Example: inverted pendulum

Response of PD-control

System $Y(s) = \frac{1}{s^2-1}U(s)$.



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- With reference $r(t) = \sin(\omega t)$ yields output

$$y(t) = |G_c(i\omega)| \sin\left(\omega t + \arg(G_c(i\omega))\right)$$

Affected by user parameter K!

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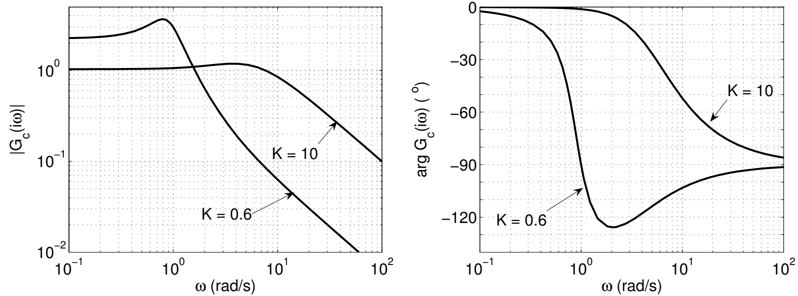
Amplitude- and phase plot

Example: PD-control of inverted pendulum

$|G_c(i\omega)|$ and $\arg(G_c(i\omega))$ as a function of frequency ω



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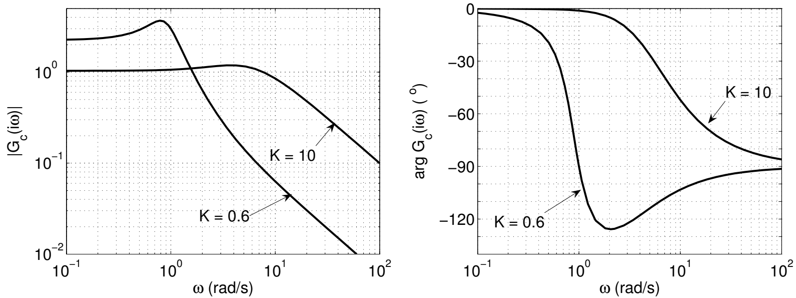

Note amplification and phase shift at $\omega = 1$

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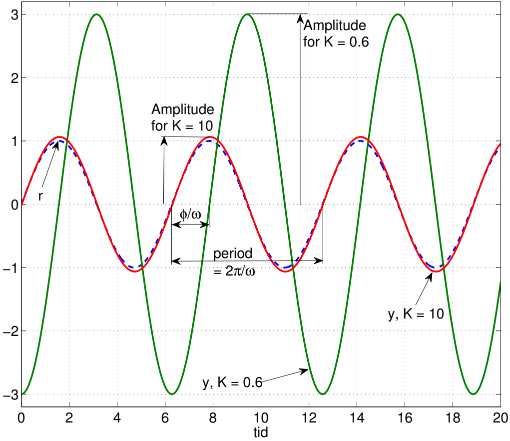
Example: PD-control of inverted pendulum

Amplification/attenuation and phase shift of $r(t)$

PD-control inverted pendulum when $r(t) = \sin(1t)$.



Cf. amplitude- and phase plots for different K

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Summary and recap

- General linear feedback control
  - Measured disturbances $\rightarrow$ feedforward control
  - Measured intermediate signals $\rightarrow$ cascade control
- Frequency description of system:
  - Sine in/sine out
  - System amplitude- and phase plots (Bode diagram)

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