Final exam
Automatic Control II
Reglerteknik II 5hp

Date: August 23, 2014

Venue: Bergsbrunnagatan 15, sal 2

Responsible teacher: Hans Norlander.

Aiding material: Calculator, mathematical handbooks, textbooks by Glad & Ljung (Reglerteori/Control theory & Reglerteknik). Additional notes in the textbooks are allowed.

Preliminary grades: 23p for grade 3, 33p for grade 4, 43p for grade 5.

Use separate sheets for each problem, i.e. no more than one problem per sheet. Write your exam code on every sheet.

Important: Your solutions should be well motivated unless else is stated in the problem formulation! Vague or lacking motivations may lead to a reduced number of points.

Problem 6 is an alternative to the homework assignments. (In case you choose to hand in a solution to Problem 6 you will be accounted for the best performance of the homework assignments and Problem 6.)

Good luck!

Problem 1

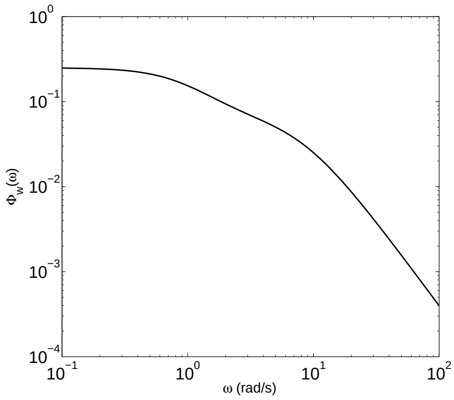
(a) A discrete-time stochastic process is described by

$$\begin{aligned} x(k+1) &= -0.6x(k) + 2v(k), \\ w(k) &= x(k), \end{aligned}$$

where $v(k)$ is zero mean white noise, and $Ev(k)^2 = 1$. Determine the variance of w, $Ew(k)^2$. **(2p)**

(b) What is the spectrum, $\Phi_w(\omega)$, of the stochastic process w in (a)? **(2p)**

(c) The spectrum of a stationary continuous-time stochastic process, $w(t)$, is obtained on an empirical basis — see the plot below.



It is found that

$$\Phi_w(\omega) = \frac{4\omega^2 + 16}{\omega^4 + 65\omega^2 + 64}$$

is a very good approximation of the spectrum. Determine a minimum phase transfer function $G(s)$ such that $G(0) > 0$, and that the model $w(t) = G(p)v(t)$ is stationary and has the spectrum $\Phi_w(\omega)$ above. Here $v(t)$ is zero mean white noise with intensity $\Phi_v(\omega) = 1$. **(4p)**

Problem 2 Specify for each of the following statements whether it is true or false. No motivations required — only answers "true" / "false" are considered!

(a) It is always preferable to have the Nyquist frequency greater than the sampling frequency.

(b) A drawback with MPC is that it is only applicable for systems with sufficiently small time constants.

(c) MPC yields a linear time-invariant controller.

(d) LQG yields a linear time-invariant controller.

(e) LQG handles control constraints better than MPC does.

(f) The weighting matrices $\{Q_1, Q_2\}$ and $\{kQ_1, kQ_2\}$ (with $k > 0$) give identical LQG controllers.

(g) White noise is always periodic.

Each correct answer scores +1, each incorrect answer scores −1, and omitted answers score 0 points. (Minimal total score is 0 points.) **(7p)**

Problem 3 A continuous-time system has the state space representation

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix}-3 & 0\\ 0 & -8\end{bmatrix} x(t) + \begin{bmatrix}0\\1\end{bmatrix} u(t) + \begin{bmatrix}4\\0\end{bmatrix} v_1(t),\\ z(t) &= \begin{bmatrix}1 & 1\end{bmatrix} x(t),\\ y(t) &= \begin{bmatrix}1 & 0\end{bmatrix} x(t) + v_2(t),\end{aligned}$$

where u is the input, z is the performance variable and y is the measured output. Here v_1 and v_2 are Gaussian, zero mean white noise processes, with intensities $R_1 = 1$ and $R_2 = 1$ respectively, and they are uncorrelated so $R_{12} = 0$. Determine the controller that minimizes the criterion

$$V = E[u^2 + \gamma^2 z^2]$$

Specifically, give the controllers for $\gamma = 6$ and $\gamma = 15$ respectively. **(8p)**

Problem 4 Consider the ARMA process

$$y(k) - y(k-1) = e(k) - 2e(k-1),$$

where $e(k)$ is zero mean white noise with intensity $Ee(k)^2 = 1$. The output $y(k)$ is measured. A Kalman filter is used for computing $\hat{y}(k+1|k)$, the prediction of $y(k+1)$ based on $y(k), y(k-1), y(k-2), \ldots$. The following state space representation is used:

$$\begin{aligned}x(k+1) &= \begin{bmatrix}1 & -2\\ 0 & 0\end{bmatrix} x(k) + \begin{bmatrix}1\\1\end{bmatrix} v(k),\\ y(k) &= \begin{bmatrix}1 & 0\end{bmatrix} x(k).\end{aligned}$$

Here $v(k) = e(k+1)$.

(a) Show that the state space representation is equivalent to the ARMA process above. **(2p)**

(b) Determine the Kalman filter for the state space representation given above. **(4p)**

(c) Given the Kalman filter in (b) the one-step predictor can be written as $\hat{y}(k+1|k) = F(q)y(k)$. Give the transfer function/operator $F(q)$. **(2p)**

(d) An alternative state space representation of the ARMA process above is

$$\begin{aligned}x(k+1) &= x(k) + e(k),\\ y(k) &= -x(k) + e(k).\end{aligned}$$

Determine the Kalman filter for this state space representation. **(3p)**

(e) Again we can write $\hat{y}(k+1|k) = F(q)y(k)$. Give the transfer function for this case, and compare it to the one in (c). Are the two one-step predictors equivalent? **(2p)**

Problem 5 Consider the system

$$y(t) = \frac{4}{p+4}\left(\frac{1}{p}u_1(t) + u_2(t) + w(t)\right).$$

Here $w(t)$ is a stochastic process that can be modeled as

$$w(t) = \frac{p+2}{p^2+3p+5}v(t),$$

where $v(t)$ is zero mean white noise with intensity $\Phi_v(\omega) = R_v$.

(a) Give a state space representation for the system with y as output, $u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T$ as input and v as system noise. Let $x_1 = y$.
Hint: First determine one state space representation with y as output and u and w as inputs, and another with w as output and v as input. Then combine these two into one total state space representation. **(5p)**

(b) Is your state space representation in (a) controllable from u? **(2p)**

Problem 6 *The HW bonus points are exchangeable for this problem.*
Consider the continuous-time system

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} 1 & -2 & 0.5 \end{bmatrix} x(t).$$

(a) Determine the poles and zeros of the system. Is the system minimum phase? **(2p)**

(b) The system is controlled by a sampling controller, using zero-order hold sampling (i.e. the input $u(t)$ is kept constant between the sampling instants). The sampling period is $h = 0.2$ seconds. Determine the corresponding sampled, discrete-time state space model for the system. **(3p)**

(c) Determine the poles and zeros of the discrete-time model in (b). Is the system minimum phase? **(2p)**

Solutions to the exam in Automatic Control II, 2014-08-23:

1. **(a)** Use that $Ew^2 = Ex^2 = \Pi_x$, and that Π_x is the solution of the discrete-time Lyapunov equation, $\Pi_x = F\Pi_x F^T + GRG^T$. Here $F = -0.6$, $G = 2$ and $R = 1 \Rightarrow$

$$\Pi_x = (-0.6)^2\Pi_x + 2^2 \qquad \Leftrightarrow \qquad \Pi_x = \frac{4}{1-0.36} = \frac{4}{0.64} = 6.25.$$

Thus, $Ew^2 = 6.25$.
(b) The spectrum of $w(k) = G(q)v(k)$ is $\Phi_w(\omega) = G(e^{i\omega})\Phi_v(\omega)G^*(e^{i\omega})$. Here $G(q) = H(qI-F)^{-1}G = \frac{2}{q+0.6}$ and $\Phi_v(\omega) = 1$, so

$$\begin{aligned}\Phi_w(\omega) = G(e^{i\omega})G(e^{-i\omega}) &= \frac{2}{e^{i\omega}+0.6}\frac{2}{e^{-i\omega}+0.6} \\ &= \frac{4}{1+0.6^2+2\cdot 0.6(e^{i\omega}+e^{-i\omega})/2} = \frac{4}{1.36+1.20\cos\omega}.\end{aligned}$$

(c) For a continuous-time system, $w(t) = G(p)v(t)$, we have that

$$\Phi_w(\omega) = G(i\omega)\Phi_v(\omega)G^*(i\omega) = G(i\omega)G(-i\omega),$$

where the latter equality follows since $G(s)$ is scalar and $\Phi_v(\omega) = 1$. Since $\Phi_w(\omega)$ is rational in ω^2 it is possible to find a rational, stable and minimum phase $G(s)$ such that the relation above holds (Theorem 5.1). Since the numerator of $\Phi_w(\omega)$ has degree 2, and the denominator has degree 4, a clever guess is that

$$G(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}.$$

Then

$$\begin{aligned}G(i\omega)G(-i\omega) &= \frac{ib_1\omega + b_2}{-\omega^2 + ia_1\omega + a_2}\frac{-ib_1\omega + b_2}{-\omega^2 - ia_1\omega + a_2} \\ &= \frac{b_1^2\omega^2 + b_2^2}{(a_2-\omega^2)^2 + a_1^2\omega^2} = \frac{b_1^2\omega^2 + b_2^2}{\omega^4 + (a_1^2 - 2a_2)\omega^2 + a_2^2}.\end{aligned}$$

By comparing numerator and denominator, per power of ω, the following equation systems are obtained:

$$\begin{cases} 4 &= b_1^2, \\ 16 &= b_2^2, \end{cases} \qquad \text{and} \qquad \begin{cases} 65 &= a_1^2 - 2a_2, \\ 64 &= a_2^2. \end{cases}$$

Stationarity of $w \Rightarrow G(s)$ must be stable $\Rightarrow a_1, a_2 > 0$. This, together with $G(0) > 0$ implies that $b_2 > 0$, which together with the minimum phase requirement (no zeros in the RHP) implies that $b_1 \geq 0$. Given these conditions, the solution is

$$\begin{cases} b_1 &= 2, \\ b_2 &= 4, \end{cases} \qquad \begin{cases} a_1 &= 9, \\ a_2 &= 8 \end{cases} \qquad \Leftrightarrow \qquad G(s) = \frac{2s+4}{s^2+9s+8} = \frac{2(s+2)}{(s+1)(s+8)}.$$

2. **(a)** False (nonsense — per definition $\omega_n = 0.5\omega_s$.); **(b)** False (rather the other way around); **(c)** False (MPC = nonlinear and time-varying); **(d)** True; **(e)** False (the other way around); **(f)** True (only the realtive sizes of Q_1 and Q_2 matter); **(g)** False (for white noise $Ew(t)w(s) = 0$ for all $s \neq t$.).

3. **(a)** The optimal controller is the LQG control law $u = -L\hat{x}$, where $\hat{x}$ is obtained from the corresponding Kalman filter (Theorem 9.1). The Kalman filter is (Theorem 5.4)

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x}),$$

where the Kalman gain is $K = PC^T R_2^{-1}$, and $P = P^T \geq 0$ solves the CARE $0 = AP + PA^T + NR_1N^T - PC^TR_2^{-1}CP$. Here the CARE is

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -3p_1 & -3p_{12} \\ -8p_{12} & -8p_2 \end{bmatrix} + \begin{bmatrix} -3p_1 & -8p_{12} \\ -3p_{12} & -8p_2 \end{bmatrix} + \begin{bmatrix} 16 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} p_1^2 & p_1p_{12} \\ p_1p_{12} & p_{12}^2 \end{bmatrix},$$

resulting in the equation system

$$\begin{cases} 0 & = -6p_1 + 16 - p_1^2, \\ 0 & = -11p_{12} - p_1p_{12}, \\ 0 & = -16p_2 - p_{12}^2, \end{cases} \quad \Leftrightarrow \quad \begin{cases} 0 & = p_1^2 + 6p_1 - 16, \\ 0 & = (11 + p_1)p_{12}, \\ 16p_2 & = p_{12}^2. \end{cases}$$

From this follows that $p_{12} = p_2 = 0$, and $p_1 = -3 \pm 5$. Since $P \geq 0$ requires $p_1 \geq 0$, the negative root is omitted. Thus

$$P = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{and} \quad K = PC^T = \begin{bmatrix} p_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

The optimal state feedback gain is $L = Q_2^{-1}B^TS$, where $S = S^T \geq 0$ is the solution to the CARE $0 = A^TS + SA + M^TQ_1M - SBQ_2^{-1}B^TS$. Here $Q_1 = \gamma^2$ and $Q_2 = 1$, so the CARE becomes

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -3s_1 & -3s_{12} \\ -8s_{12} & -8s_2 \end{bmatrix} + \begin{bmatrix} -3s_1 & -8s_{12} \\ -3s_{12} & -8s_2 \end{bmatrix} + \begin{bmatrix} \gamma^2 & \gamma^2 \\ \gamma^2 & \gamma^2 \end{bmatrix} - \begin{bmatrix} s_{12}^2 & s_{12}s_2 \\ s_{12}s_2 & s_2^2 \end{bmatrix},$$

resulting in the equation system

$$\begin{cases} 0 & = -6s_1 + \gamma^2 - s_{12}^2, \\ 0 & = -11s_{12} + \gamma^2 - s_{12}s_2, \\ 0 & = -16s_2 + \gamma^2 - s_2^2, \end{cases} \quad \Leftrightarrow \quad \begin{cases} s_1 & = (\gamma^2 - s_{12}^2)/6, \\ s_{12} & = \gamma^2/(11 + s_2), \\ 0 & = s_2^2 + 16s_2 - \gamma^2. \end{cases}$$

To have $S \geq 0$, we must have $s_2 \geq 0$. Thus $s_2 = -8 + \sqrt{64 + \gamma^2}$ (the negative root is omitted), and $s_{12} = \gamma^2/(3 + \sqrt{64 + \gamma^2})$. Since

$$L = B^TS = \begin{bmatrix} 0 & 1 \end{bmatrix} S = \begin{bmatrix} s_{12} & s_2 \end{bmatrix},$$

s_1 is not really needed explicitely. With $\gamma = 6$ we have $L = \begin{bmatrix} 36/13 & 2 \end{bmatrix}$, and with $\gamma = 15$ we get $L = \begin{bmatrix} 225/20 & 9 \end{bmatrix}$.

4. (a) The state space representation is

$$\begin{aligned} x(k+1) &= Fx(k) + Nv(k), \\ y(k) &= Hx(k). \end{aligned}$$

Then we have $y(k) = G(q)v(k)$, with

$$G(q) = H(qI - F)^{-1}N = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q-1 & 2 \\ 0 & q \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{q-2}{q(q-1)}.$$

Hence, by noting that $v(k) = qe(k)$, we have

$$\begin{aligned} q(q-1)y(k) = (q-2)v(k) &= (q-2)qe(k) && \Leftrightarrow \\ y(k+2) - y(k+1) &= e(k+2) - 2e(k+1) && \Leftrightarrow \\ y(k) - y(k-1) &= e(k) - 2e(k-1), \end{aligned}$$

which is the given ARMA process.
(b) The Kalman filter is $\hat{x}(k+1|k) = F\hat{x}(k|k-1) + K(y(k) - H\hat{x}(k|k-1))$, with $K = (FPH^T + NR_{12})(HPH^T + R_2)^{-1}$ where $P = P^T \geq 0$ is the solution of the DARE

$$P = FPF^T + NR_1N^T - (FPH^T + NR_{12})(HPH^T + R_2)^{-1}(FPH^T + NR_{12})^T.$$

Since there is no measurement noise in the state space representation we have $R_{12} = R_2 = 0$. Spelled out the DARE then is

$$\begin{bmatrix} p_1 & p_{12} \\ p_{12} & p_2 \end{bmatrix} = \begin{bmatrix} p_1 - 4p_{12} + 4p_2 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \frac{(p_1 - 2p_{12})^2}{p_1} & 0 \\ 0 & 0 \end{bmatrix}.$$

From the 1-2 and 2-2 elements we directly see that $p_{12} = p_2 = 1$, and when put in the 1-1 element this gives

$$p_1 = p_1 - 4 + 4 + 1 - \frac{(p_1 - 2)^2}{p_1} \quad \Leftrightarrow \quad p_1^2 - 5p_1 + 4 = 0,$$

with solutions $p_1 = 2.5 \pm 1.5$. We must have $\det P \geq 0$ and $F - KH$ stable. With $p_1 = 1$ we get $\det P = 0$ and $F - KH$ unstable (a pole in 2), so the sought solution is $p_1 = 4$ ($\Rightarrow \det P = 3$). Then

$$K = FPH^T(HPH^T)^{-1} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}.$$

(c) We have $\hat{y}(k+1|k) = H\hat{x}(k+1|k) = qH\hat{x}(k|k-1)$, so

$$\begin{aligned} \hat{y}(k+1|k) &= qH(qI - F + KH)^{-1}Ky(k) \\ &= q \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q-0.5 & 2 \\ 0 & q \end{bmatrix}^{-1} \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} y(k) = q\frac{0.5q}{q(q-0.5)}y(k) = \frac{0.5q}{q-0.5}y(k). \end{aligned}$$

Thus, $F(q) = \frac{0.5q}{q-0.5}$.
(d) Now $F = 1$, $N = 1$, $H = -1$, $R_1 = R_{12} = R_2 = 1$, so the DARE becomes

$$P = P + 1 - \frac{(-P+1)^2}{P+1} \quad \Leftrightarrow \quad P^2 + 3P = 0,$$

with solutions $P = 3$ and $P = 0$. Since $K = \frac{-P+1}{P+1} = -1 + \frac{2}{P+1}$ we have that $P = 0 \Rightarrow F - KH = 2$ and $P = 3 \Rightarrow F - KH = 0.5$. The sought solution is $P = 3 \Rightarrow K = -0.5$.
(e) Again, $\hat{y}(k+1|k) = H\hat{x}(k+1|k) = qH\hat{x}(k|k-1)$, so

$$\begin{aligned}\hat{y}(k+1|k) &= qH(qI - F + KH)^{-1}Ky(k) \\ &= q(-1)(q-0.5)^{-1}(-0.5)y(k) = q\frac{0.5}{q-0.5}y(k) = \frac{0.5q}{q-0.5}y(k).\end{aligned}$$

Thus, $F(q) = \frac{0.5q}{q-0.5}$, which is identical to $F(q)$ in (c).

5. (a) With $x_1 = y$ we have $(p+4)x_1 = 4(\frac{1}{p}u_1 + u_2 + w) \Leftrightarrow \dot{x}_1 = -4x_1 + 4\frac{1}{p}u_1 + 4u_2 + 4w$. Now set (for example) $x_2 = \frac{1}{p}u_1 \Rightarrow \dot{x}_2 = u_1$. Using e.g. the controller canonical form for describing w we get

$$\begin{aligned}\dot{x}_3 &= -3x_3 - 5x_4 + v, \\ \dot{x}_4 &= x_3, \\ w &= x_3 + 2x_4,\end{aligned}$$

and in total we get

$$\begin{aligned}\dot{x}_1 &= -4x_1 + 4x_2 + 4x_3 + 8x_4 + 4u_2, \\ \dot{x}_2 &= u_1, \\ \dot{x}_3 &= -3x_3 - 5x_4 + v, \\ \dot{x}_4 &= x_3, \\ y &= x_1.\end{aligned}$$

In vector form this is

$$\begin{aligned}\dot{x} &= \begin{bmatrix} -4 & 4 & 4 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & -5 \\ 0 & 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 4 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} v, \\ y &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x.\end{aligned}$$

(b) The examine the controllability, compute the controllability matrix:

$$\mathcal{S} = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix} = \begin{bmatrix} 0 & 4 & 4 & -16 & -16 & 64 & 64 & -256 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\mathcal{S}$ has rank two, so the system is not controllable.

6. **(a)** The transfer function is $G(s) = C(sI - A)^{-1}B$, so

$$G(s) = \begin{bmatrix} 1 & -2 & 0.5 \end{bmatrix} \begin{bmatrix} \frac{1}{s} & 0 & 0 \\ 0 & \frac{1}{s+1} & 0 \\ 0 & 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2} = \frac{2}{s(s+1)(s+2)}.$$

The system has no zeros, and poles in the origin, -1 and -2. No poles or zeros in the right half plane $\Rightarrow$ minimum phase.
(b) The discrete-time model is $x(kh+h) = Fx(kh) + Gu(kh)$, $y(kh) = Cx(kh)$, where $F = e^{Ah}$ and $G = \int_0^h e^{At}Bdt$. Here

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-h} & 0 \\ 0 & 0 & e^{-2h} \end{bmatrix} \quad \text{and} \quad G = \int_0^h \begin{bmatrix} 1 \\ e^{-t} \\ 2e^{-2t} \end{bmatrix} dt = \begin{bmatrix} h \\ 1 - e^{-h} \\ 1 - e^{-2h} \end{bmatrix}.$$

With $h = 0.2 \Rightarrow e^{-h} = 0.819$ the state-space model becomes

$$\begin{aligned} x(kh+h) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.819 & 0 \\ 0 & 0 & 0.670 \end{bmatrix} x(kh) + \begin{bmatrix} 0.2 \\ 0.181 \\ 0.330 \end{bmatrix} u(kh), \\ y(kh) &= \begin{bmatrix} 1 & -2 & 0.5 \end{bmatrix} x(kh). \end{aligned}$$

(c) The transfer function is $G(q) = C(qI - F)^{-1}G$, so

$$\begin{aligned} G(q) &= \begin{bmatrix} 1 & -2 & 0.5 \end{bmatrix} \begin{bmatrix} \frac{1}{q-1} & 0 & 0 \\ 0 & \frac{1}{q-e^{-h}} & 0 \\ 0 & 0 & \frac{1}{q-e^{-2h}} \end{bmatrix} \begin{bmatrix} h \\ 1 - e^{-h} \\ 1 - e^{-2h} \end{bmatrix} \\ &= \frac{h}{q-1} - \frac{2(1-e^{-h})}{q-e^{-h}} + \frac{0.5(1-e^{-2h})}{q-e^{-2h}} = \frac{b_1q^2 + b_2q + b_3}{(q-1)(q-e^{-h})(q-e^{-2h})}, \end{aligned}$$

where
$b_1 = h - 2(1-e^{-h}) + 0.5(1-e^{-2h}) = 2.3015 \cdot 10^{-3}$,
$b_2 = -h(e^{-h}+e^{-2h}) + 2(1-e^{-h})(1+e^{-2h}) - 0.5(1-e^{-2h})(1+e^{-h}) = 7.9456 \cdot 10^{-3}$
and
$b_3 = he^{-3h} - 2(1-e^{-h})e^{-2h} + 0.5(1-e^{-2h})e^{-h} = 1.7051 \cdot 10^{-3}$.
The poles are 1, $e^{-h} = 0.819$ and $e^{-2h} = 0.670$. The zeros are given by $0 = b_1q^2 + b_2q + b_3$, i.e.

$$z = -\frac{b_2}{2b_1} \pm \sqrt{\left[\frac{b_2}{2b_1}\right]^2 - \frac{b_3}{b_1}} = -1.7262 \pm 1.4963.$$

Thus, the zeros are -0.2299 and -3.2225. One zero is outside the unit circle $\Rightarrow$ non-minimum phase.