

Exam in Automatic Control II

Reglerteknik II 5hp (1RT495)

Date: October 18, 2016

Venue: Bergsbrunnagatan 15, room 2

Responsible teacher: Hans Rosth.

Aiding material: Calculator, mathematical handbooks, textbooks by Glad & Ljung (Reglerteori/Control theory & Reglerteknik). Additional notes in the textbooks are allowed.

Preliminary grades: 23p for grade 3, 33p for grade 4, 43p for grade 5.

Use separate sheets for each problem, i.e. no more than one problem per sheet. Write your exam code on every sheet.

Important: Your solutions should be well motivated unless else is stated in the problem formulation! Vague or lacking motivations may lead to a reduced number of points.

Problem 6 is an alternative to the homework assignments. (In case you choose to hand in a solution to Problem 6 you will be accounted for the best performance of the homework assignments and Problem 6.)

Please use English in your solutions when possible, that would be appreciated!

Good luck!

Problem 1 A continuous-time harmonic oscillator can be modeled as

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & \pi \\ -\pi & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \\ y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t), \end{cases} \quad \Leftrightarrow \quad y(t) = \frac{s}{s^2 + \pi^2} u(t). \quad (1)$$

(a) Show that it is possible to stabilize this harmonic oscillator by use of proportional feedback, $u(t) = K(r(t) - y(t))$. **(1p)**

Now assume that a sampling controller is used, operating with zero-order hold (ZOH)¹.

(b) Compute the transfer function $G_{ZOH}(z)$ for the ZOH sampled version of (1) when the sampling interval is $h = 1$ second. **(2p)**

(c) Next, compute the transfer function $G_{ZOH}(z)$ for the ZOH sampled version of (1) when the sampling interval is $h = 0.5$ seconds. **(2p)**

(d) Explain why the results in (b) and (c) differ. **(1p)**

(e) Assume the sampling period is chosen to $h = 0.5$ seconds. Is it possible to stabilize (1) by use of a ZOH sampling proportional controller, $u(kh) = K(r(kh) - y(kh))$, with this h ? If you answer yes, suggest a stabilizing value for K , if you answer no, show that it is not possible. **(3p)**

Problem 2 Specify for each of the following statements whether it is true or false. No motivations required — only answers “true”/“false” are considered!

(a) When sampling a signal the Nyquist frequency is always proportional to the sampling frequency.

(b) White noise is characterized by that its covariance function $r(\tau)$ is constant and non-zero for all time lags τ .

(c) White noise processes always have a constant spectrum.

(d) An advantage with MPC is that it is an open loop control algorithm.

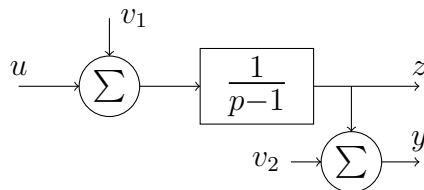
(e) An advantage with MPC is that it can account for bounds and constraints, for example of the type $|u| \leq U_{max}$.

(f) A disadvantage with LQG-design is that it never results in an LTI controller.

Each correct answer scores +1, each incorrect answer scores -1, and omitted answers score 0 points. (Minimal total score is 0 points.) **(6p)**

¹The control input, $u(t)$, is held constant between the sampling instants.

Problem 3 The block diagram below shows an unstable system.



The system is affected by the process noise v_1 and the measurement noise v_2 . The noise processes, v_1 and v_2 , are zero mean, uncorrelated white noise processes with intensities $R_1 = 8$ and $R_2 = 1$.

(a) Give a state space representation, in the “standard form”, for the system. Use $x_1 = z$ as state variable. **(1p)**

(b) The system is to be stabilized by output feedback,

$$u(t) = -F_y(p)y(t), \quad F_y(p) = \frac{b}{p+a}. \quad (2)$$

Give a state space representation for the controller (2), with y as input and u as output. Use $x_2 = u$ as state variable. **(1p)**

(c) Determine the controller that minimizes the cost function

$$V = 3Ez^2 + Eu^2. \quad (3)$$

(4p)

(d) Show that the controller in (c) can be represented as in (2). Give the corresponding values of a and b . **(3p)**

(e) Determine the value of the cost function (3) with the controller in (c).

Hint: Set up a state space representation for the closed loop system based on your results in (a) and (b) (and (d)). Then compute the variances Ez^2 and Eu^2 individually. **(2p)**

Problem 4 Consider the discrete-time stochastic process

$$\begin{cases} x(k+1) = \begin{bmatrix} 0.4 & 1 \\ -0.6 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 0.6 \\ 0 \end{bmatrix} v(k), \\ y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) + v(k), \end{cases} \quad Ev(k) = 0, \Phi_v(\omega) = 1. \quad (4)$$

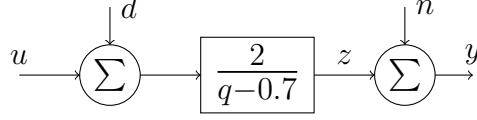
(a) Determine the covariance matrix of the state vector, $\Pi_x = Ex(k)x(k)^T$, and the variance of the output, $Ey(k)^2$. **(3p)**

(b) What is the spectrum, $\Phi_y(\omega)$, of the output? **(3p)**

(c) Give the Kalman filter gain, K , for the $\hat{x}(k|k-1)$ estimator for (4).

Hint: If you find it necessary you may use that $P = E\tilde{x}\tilde{x}^T$ is diagonal in this case. **(5p)**

Problem 5 The block diagram below shows a discrete-time system which is affected by a process noise d and a measurement noise n , which are mutually uncorrelated.



The process noise is a stochastic process modeled as

$$d(k) = \frac{4}{q+0.6}w(k), \quad Ew(k) = 0, \quad \Phi_w(\omega) = 1.$$

The measurement noise is a stochastic process described by

$$\begin{cases} \chi(k+1) = 0.4\chi(k) + 3e(k), \\ n(k) = \chi(k) + e(k), \end{cases}$$

where $e(k)$ is zero mean white noise with intensity $R_e = 2$.

(a) Set up a state space model for the system in the “standard form”,

$$\begin{cases} x(k+1) = Fx(k) + Gu(k) + Nv_1(k), \\ z(k) = Mx(k), \\ y(k) = Hx(k) + v_2(k), \end{cases} \quad (5)$$

where $v_1(k)$ and $v_2(k)$ are white noise processes. That is, give the numerical values for the matrices and vectors F , G , N , M and H . Use $x(k) = [x_1(k) \ x_2(k) \ x_3(k)]^T = [z(k) \ d(k) \ \chi(k)]^T$ as state vector. **(4p)**

(b) Specify what $v_1(k)$ and $v_2(k)$ in (5) are in terms of $w(k)$ and $e(k)$, and give the covariance matrices $R_1 = Ev_1(k)v_1(k)^T$, $R_2 = Ev_2(k)v_2(k)^T$ and $R_{12} = Ev_1(k)v_2(k)^T$. **(2p)**

Problem 6 The HW bonus points are exchangeable for this problem.

(a) The stationary continuous-time stochastic process $y(t)$ is modeled as $y(t) = G(p)u(t)$, where $G(p)$ is minimum phase and $G(0) > 0$. The input $u(t)$ is zero mean white noise with intensity $\Phi_u(\omega) = 1$. The spectrum of $y(t)$ is

$$\Phi_y(\omega) = \frac{16\omega^2 + 4}{\omega^4 - 15\omega^2 + 64}.$$

Determine the transfer operator $G(p)$. **(4p)**

(b) Compute the Kalman filter estimate $\hat{x}(k|k)$ for the system

$$\begin{cases} x(k+1) = -x(k) + v_1(k), \\ y(k) = x(k) + v_2(k), \end{cases} \quad R_1 = 1, \ R_2 = 6, \ R_{12} = 0,$$

when $\hat{x}(k|k-1) = -0.7$ and $y(k) = 0.2$. **(3p)**

Solutions to the exam in Automatic Control II, 2016-10-18:

1. (a) The closed loop pole polynomial, which can be obtained eg. by $0 = 1 + KG(s)$ or by $\det(sI - A + CK)$, becomes $s^2 + Ks + \pi^2$. The closed loop system is stable (all poles in LHP) for $K > 0$.

(b) The ZOH sampled system is (by Theorem 4.1)

$$\begin{cases} qx = Fx + Gu, \\ y = Cx, \end{cases} \quad \text{where } F = e^{Ah}, \quad G = \int_0^h e^{At} B dt.$$

Use that $e^{At} = \mathcal{L}^{-1}[(sI - A)^{-1}]$:

$$\begin{aligned} e^{At} &= \mathcal{L}^{-1} \left[\begin{bmatrix} s & -\pi \\ \pi & s \end{bmatrix}^{-1} \right] = \mathcal{L}^{-1} \left[\begin{bmatrix} \frac{s}{s^2 + \pi^2} & \frac{\pi}{s^2 + \pi^2} \\ \frac{-\pi}{s^2 + \pi^2} & \frac{s}{s^2 + \pi^2} \end{bmatrix} \right] = \begin{bmatrix} \cos \pi t & \sin \pi t \\ -\sin \pi t & \cos \pi t \end{bmatrix} \\ \Rightarrow F &= \begin{bmatrix} \cos \pi h & \sin \pi h \\ -\sin \pi h & \cos \pi h \end{bmatrix}, \quad G = \int_0^h \begin{bmatrix} \sin \pi t \\ \cos \pi t \end{bmatrix} dt = \frac{1}{\pi} \begin{bmatrix} 1 - \cos \pi h \\ \sin \pi h \end{bmatrix}. \end{aligned}$$

The transfer function is $G_{ZOH}(z) = C(zI - F)^{-1}G$, and with $h = 1$ we get

$$G_{ZOH}(z) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z + 1 & 0 \\ 0 & z + 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{2}{\pi} \\ 0 \end{bmatrix} = 0.$$

(c) With $h = 0.5$ we instead get

$$G_{ZOH}(z) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z & -1 \\ -1 & z \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\pi} \\ \frac{1}{\pi} \end{bmatrix} = \frac{1}{\pi} \cdot \frac{z - 1}{z^2 + 1}.$$

(d) Checking for controllability and observability for the case $h = 1$:

$$\mathcal{S} = [G \quad FG] = \begin{bmatrix} \frac{2}{\pi} & -\frac{2}{\pi} \\ 0 & 0 \end{bmatrix}, \quad \mathcal{O} = \begin{bmatrix} C \\ CF \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}.$$

Both \mathcal{S} and \mathcal{O} are rank-deficient, meaning that the system is neither controllable, nor observable. This is due to the particular choice of sampling period h . Because of this the system order decrease (here to order zero). (What happens here is that the sampling period coincides exactly with the systems natural frequency, so that the sampling instants occurs exactly in the zero crossings of the system's impulse response.)

(e) The closed loop pole polynomial becomes $z^2 + 1 + \frac{K}{\pi}(z - 1) = z^2 + \frac{K}{\pi}z + 1 - \frac{K}{\pi}$. A polynomial $z^2 + az + b$ has both its zeroes inside the unit circle if and only if $|a| - 1 < b < 1$. Here:

$$\begin{aligned} b < 1 &: \quad 1 - \frac{K}{\pi} < 1 & \Leftrightarrow & \quad K > 0, \\ a - 1 < b &: \quad \frac{K}{\pi} - 1 < 1 - \frac{K}{\pi} & \Leftrightarrow & \quad K < \pi, \\ -a - 1 < b &: \quad -\frac{K}{\pi} - 1 < 1 - \frac{K}{\pi} & \Leftrightarrow & \quad -1 < 1. \end{aligned}$$

Thus, the closed loop system is stable for $0 < K < \pi$.

2. (a) True ($\omega_n = 0.5\omega_s$ by definition); **(b)** False (For white noise $r(\tau) = 0$ for $\tau \neq 0$); **(c)** True (Definition 5.2); **(d)** False (MPC is a feedback control algorithm); **(e)** True; **(f)** False (The stationary LQG controller is LTI.)

3. (a) From the block diagram we get (with $x_1 = z$)

$$x_1 = \frac{1}{p-1}(u + v_1) \Leftrightarrow px_1 = x_1 + u + v_1.$$

The state space representation becomes

$$\begin{cases} \dot{x}_1 = x_1 + u + v_1, \\ x = x_1, \\ y = x_1 + v_2. \end{cases}$$

(b) With $x_2 = u$ we get

$$(p+a)x_2 = -by \Rightarrow \begin{cases} \dot{x}_2 = -ax_2 - by, \\ u = x_2. \end{cases}$$

(c) This is an LQG problem, so the control law is (Theorem 9.1) $u = -L\hat{x}$, where $L = Q_2^{-1}B^T S$, and $S = S^T \geq 0$ solves the CARE $0 = A^T S + SA + M^T Q_1 M - SBQ_2^{-1}B^T S$, and \hat{x} comes from the Kalman filter

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x}). \quad (6)$$

Here $A = B = M = 1$, $Q_1 = 3$ and $Q_2 = 1 \Rightarrow$ the CARE becomes

$$0 = 2S + 3 - S^2 \Leftrightarrow S = 1 + \sqrt{1+3} = 3 \Rightarrow L = 3.$$

The Kalman gain is $K = PC^T R_2^{-1}$, where $P = P^T \geq 0$ solves the CARE $0 = AP + PA^T + NR_1 N^T - PC^T R_2^{-1} CP$ (when $R_{12} = 0$, like here). Here $N = C = 1$, $R_1 = 8$ and $R_2 = 1$, so the CARE becomes

$$0 = 2P + 8 - P^2 \Leftrightarrow P = 1 + \sqrt{1+8} = 4 \Rightarrow K = 4.$$

(d) Putting the control law $u = -L\hat{x} = -3\hat{x}$ into the Kalman filter (6) gives

$$\dot{\hat{x}} = \hat{x} + (-3\hat{x}) + 4(y - \hat{x}) = -6\hat{x} + 4y \Rightarrow \begin{cases} \dot{\hat{x}} = -6\hat{x} + 4y, \\ u = -3\hat{x}. \end{cases}$$

Thus,

$$u = -\frac{4 \cdot 3}{p+6}y = -\frac{12}{p+6}y \Rightarrow \begin{cases} a = 6, \\ b = 12, \end{cases}$$

by comparison with (2).

(e) Combining the state space representations in (a) and (b) gives

$$\left\{ \begin{array}{l} \dot{x}_1 = x_1 + u + v_1 \\ \quad = x_1 + x_2 + v_2, \\ \dot{x}_2 = -ax_2 - by \\ \quad = -12x_1 - 6x_2 - 12v_2, \\ z = x_1, \\ u = x_2, \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \dot{x} = \begin{bmatrix} 1 & 1 \\ -12 & -6 \end{bmatrix}^{\bar{A}} x + \begin{bmatrix} 1 & 0 \\ 0 & -12 \end{bmatrix}^{\bar{N}} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \\ z = \begin{bmatrix} 1 & 0 \end{bmatrix}^{\bar{M}} x, \\ u = \begin{bmatrix} 0 & 1 \end{bmatrix}^{\bar{C}} x. \end{array} \right.$$

The covariance matrix of the state vector, $\Pi_x = Exx^T$, solves the continuous-time Lyapunov equation $0 = \bar{A}\Pi_x + \Pi_x\bar{A}^T + \bar{N}R\bar{N}^T$, where

$$R = E \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix}. \quad \text{Setting } \Pi_x = \begin{bmatrix} \pi_1 & \pi_{12} \\ \pi_{21} & \pi_2 \end{bmatrix}$$

and spelling out the Lyapunov equation yields the equation system

$$\left\{ \begin{array}{l} 0 = 2(\pi_1 + \pi_{12}) + 8, \\ 0 = -12\pi_1 - 5\pi_{12} + \pi_2, \\ 0 = 2(-12\pi_{12} - 6\pi_2) + 144, \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \pi_1 = 8, \\ \pi_{12} = -12, \\ \pi_2 = 36. \end{array} \right.$$

Furthermore, $Ez^2 = Ex_1^2 = \pi_1 = 8$ and $Eu^2 = Ex_2^2 = \pi_2 = 36$, and hence

$$V = 3Ez^2 + Eu^2 = 3\pi_1 + \pi_2 = 3 \cdot 8 + 36 = 60.$$

4. (a) The covariance matrix solves the discrete-time Lyapunov equation, $\Pi_x = F\Pi_x F^T + NRN^T$ (can be used directly since v is white noise), where $R = 1$ in this case. Here we have

$$F = \begin{bmatrix} 0.4 & 1 \\ -0.6 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0.6 \\ 0 \end{bmatrix}. \quad \text{Setting } \Pi_x = \begin{bmatrix} \pi_1 & \pi_{12} \\ \pi_{12} & \pi_2 \end{bmatrix}$$

and spelling out the Lyapunov equation, the following equation system is obtained:

$$\left\{ \begin{array}{l} \pi_1 = 0.16\pi_1 + 0.8\pi_{12} + \pi_2 + 0.36, \\ \pi_{12} = -0.24\pi_1 - 0.6\pi_{12}, \\ \pi_2 = 0.36\pi_1, \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \pi_1 = \frac{0.36}{0.6} = 0.6, \\ \pi_{12} = -\frac{0.24}{1.6} \cdot 0.6 = -0.09, \\ \pi_2 = 0.36 \cdot 0.6 = 0.216. \end{array} \right.$$

Since $y = Hx + v$ we get $Ey^2 = E[(Hx + v)(Hx + v)^T] = H\Pi_x H^T + R_v = \pi_1 + R_v$ (since $x(k)$ and $v(k)$ are uncorrelated). Hence

$$\Pi_x = \begin{bmatrix} 0.6 & -0.09 \\ -0.09 & 0.216 \end{bmatrix}, \quad Ey^2 = \pi_1 + R_v = 0.6 + 1 = 1.6.$$

(b) The output spectrum is $\Phi_y(\omega) = |G(e^{i\omega})|^2 \Phi_v(\omega)$. The system (4) is on observer canonical form so we get $G(q) = \frac{0.6q}{q^2 - 0.4q + 0.6} + 1 = \frac{q^2 + 0.2q + 0.6}{q^2 - 0.4q + 0.6}$ (can also use $G(q) = H(qI - F)^{-1}N + 1$), and since $\Phi_v(\omega) = 1$ we have

$$\begin{aligned} \Phi_y(\omega) &= \frac{e^{-i2\omega} + 0.2e^{-i\omega} + 0.6}{e^{-i2\omega} - 0.4e^{-i\omega} + 0.6} \cdot \frac{e^{i2\omega} + 0.2e^{i\omega} + 0.6}{e^{i2\omega} - 0.4e^{i\omega} + 0.6} \\ &= \frac{1 + 0.2^2 + 0.6^2 + (0.2 + 0.2 \cdot 0.6)(e^{i\omega} + e^{-i\omega}) + 0.6(e^{i2\omega} + e^{-i2\omega})}{1 + (-0.4)^2 + 0.6^2 + (-0.4 - 0.4 \cdot 0.6)(e^{i\omega} + e^{-i\omega}) + 0.6(e^{i2\omega} + e^{-i2\omega})} \\ &= \frac{1.40 + 0.32(e^{i\omega} + e^{-i\omega}) + 0.6(e^{i2\omega} + e^{-i2\omega})}{1.52 - 0.64(e^{i\omega} + e^{-i\omega}) + 0.6(e^{i2\omega} + e^{-i2\omega})} = \frac{1.40 + 0.64 \cos \omega + 1.2 \cos 2\omega}{1.52 - 1.28 \cos \omega + 1.2 \cos 2\omega}. \end{aligned}$$

(c) Notice that (4) is on *innovations form* if $F - NH$ is stable (has all eigenvalues inside the unit circle). Here $\det(qI - F + NH) = q^2 + 0.2q + 0.6 \Rightarrow$ eigenvalues in $-0.1 \pm i\sqrt{0.59}$, which is inside the unit circle. Hence the system is on innovations form, and then the Kalman gain is $K = N$ — see Eq. (5.83) in Glad/Ljung (this holds for discrete-time systems as well). Thus, without solving any DARE we can directly state that

$$K = N = \begin{bmatrix} 0.6 \\ 0 \end{bmatrix}.$$

(Of course it is possible and correct to obtain K by solving the DARE. The solution then is $P = 0$! Note though that $R_1 = R_2 = R_{12} = 1$!)

5. (a) From the block diagram we have, with $x_1 = z$ and $x_2 = d$, that

$$x_1 = \frac{2}{q - 0.7}(u + x_2) \quad \Leftrightarrow \quad qx_1 = 0.7x_1 + 2x_2 + 2u.$$

Then, again with $x_2 = d$,

$$x_2 = \frac{4}{q + 0.6}w \quad \Leftrightarrow \quad qx_2 = -0.6x_2 + 4w.$$

Combining this together with the state space model for $\chi = x_3$, and that $y = z + n$, we get

$$\begin{cases} qx_1 = 0.7x_1 + 2x_2 + 2u, \\ qx_2 = -0.6x_2 + 4w, \\ qx_3 = 0.4x_3 + 3e, \\ z = x_1, \\ y = x_1 + x_3 + e, \end{cases} \quad \Leftrightarrow \quad \begin{cases} qx = \begin{bmatrix} 0.7 & 2 & 0 \\ 0 & -0.6 & 0 \\ 0 & 0 & 0.4 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 4 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} w \\ e \end{bmatrix}, \\ z = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x, \\ y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x + e. \end{cases}$$

(b) From the state space model we see that $v_1 = [w \ e]^T$ and $v_2 = e$. Hence,

$$R_1 = E \begin{bmatrix} w \\ e \end{bmatrix} [w \ e] = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad R_2 = Ee^2 = 2, \quad R_{12} = E \begin{bmatrix} w \\ e \end{bmatrix} e = \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$$

6. (a) $G(p)$ stationary and minimum phase \Leftrightarrow all poles in the left half plane and no zeros in the right half plane. The numerator is a first order polynomial, and the denominator is a second order polynomial in $\omega^2 \Rightarrow$ try with $G(p) = \frac{b_1 p + b_2}{p^2 + a_1 p + a_2}$, with $b_1, b_2, a_1, a_2 > 0$ (to also have $G(0) > 0$). The spectrum then is

$$\begin{aligned}\Phi_y(\omega) = G(i\omega)G(-i\omega) &= \frac{(b_2 + ib_1\omega)(b_2 - ib_1\omega)}{(-\omega^2 + a_2 + ia_1\omega)(-\omega^2 + a_2 - ia_1\omega)} \\ &= \frac{b_2^2 + b_1^2\omega^2}{(-\omega^2 + a_2)^2 + a_1^2\omega^2} = \frac{b_2^2 + b_1^2\omega^2}{\omega^4 + (a_1^2 - 2a_2)\omega^2 + a_2^2}\end{aligned}$$

Comparing with the given spectrum, and equating equal powers of ω^2 in both numerator and denominator, gives

$$\begin{cases} b_1^2 = 16, \\ b_2^2 = 4, \\ a_1^2 - 2a_2 = -15, \\ a_2^2 = 64, \end{cases} \Rightarrow \begin{cases} b_1 = 4, \\ b_2 = 2, \\ a_1 = 1, \\ a_2 = 8. \end{cases}$$

Thus $G(p) = \frac{4p+2}{p^2+p+8}$.

(b) Theorem 5.6, Eqs. (5.100)–(5.101) \Rightarrow

$$\hat{x}(k|k) = \hat{x}(k|k-1) + \tilde{K}(y(k) - H\hat{x}(k|k-1)), \quad \tilde{K} = PH^T(HPH^T + R_2)^{-1}.$$

Here $F = -1$, $N = H = 1$, $R_1 = 1$, $R_2 = 6$ and $R_{12} = 0$ so the associated DARE is

$$P = P + 1 - \frac{P^2}{P+6} \quad \Leftrightarrow \quad P^2 - P - 6 = 0,$$

with solution $P = 0.5 + \sqrt{0.5^2 + 6} = 3 \Rightarrow \tilde{K} = \frac{P}{P+6} = \frac{1}{3}$. Hence

$$\hat{x}(k|k) = -0.7 + \frac{1}{3}(0.2 - (-0.7)) = -0.4.$$