Exam in Automatic Control II Reglerteknik II 5hp

Date: August 20, 2016

Venue: Polacksbacken

Responsible teacher: Hans Rosth.

Aiding material: Calculator, mathematical handbooks, textbooks by Glad & Ljung (Reglerteori/Control theory & Reglerteknik). Additional notes in the textbooks are allowed.

Preliminary grades: 23p for grade 3, 33p for grade 4, 43p for grade 5.

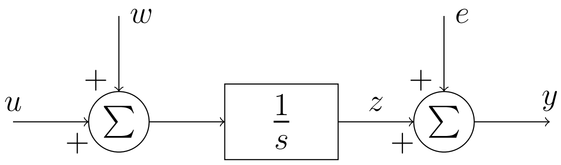
Use separate sheets for each problem, i.e. no more than one problem per sheet. Write your exam code on every sheet.

Important: Your solutions should be well motivated unless else is stated in the problem formulation! Vague or lacking motivations may lead to a reduced number of points.

Problem 6 is an alternative to the homework assignments. (In case you choose to hand in a solution to Problem 6 you will be accounted for the best performance of the homework assignments and Problem 6.)

Good luck!

Problem 1 A simple model describing variations of the level in a water reservoir is given in the block diagram below.



Here the input u is the controlled net flow into the reservoir, z is the level and y is the measured level. There are two independent, zero mean noise sources: the measurement noise, e, and uncontrolled variations in the net flow, w.

It is desirable to keep the variations of the level as small as possible, at a minimal cost in terms of control power. Therefore the LQG control law $u = -L\hat{x}$, which minimizes the cost function

$$V = E\left[z^2 + \rho^2 u^2\right], \qquad \rho > 0,$$

is used. (The estimate $\hat{x}$ is obtained from a Kalman filter, which is not considered in this problem.)

(a) In a first approximation it is assumed that both w and e are white noise. Find the optimal feedback gain L, expressed in ρ. (Use $x = z$ as state variable.) **(3p)**

(b) It turns out that

$$w(t) = \frac{1}{p+1} v(t),$$

where v is white noise, is a much better description of w. The measurement noise, e, is still assumed to be white. Give a state space representation of the total model, where the dynamics of w are incorporated. Use the "standard" form, i.e. determine the matrices and vectors A, B, N, M and C in

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Nv_1(t), \\ z(t) = Mx(t), \\ y(t) = Cx(t) + v_2(t). \end{cases}$$

Also, relate v_1 and v_2 to v and e. **(4p)**

(c) Find the optimal state feedback gain L for the total model in (b). **(4p)**

Problem 2 The continuous-time system

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t), \\ y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(t), \end{cases} \tag{1}$$

is controlled by the state feedback $u(t) = -L_c x(t)$, where $L_c = \begin{bmatrix} 2 & 0 \end{bmatrix}$.

(a) Determine the poles of the closed loop system. **(1p)**

(b) The control system is modernized, and the continuous-time controller is replaced by a sampling controller, implemented in a PC. Zero-order hold (ZOH) sampling is used. In order to analyze the controlled system a discrete-time (ZOH sampled) model of the system (1) is needed. Determine the matrices and vectors F, G and H in the discrete-time state space model

$$\begin{cases} x(kh+h) = Fx(kh) + Gu(kh), \\ y(kh) = Hx(kh), \end{cases} \tag{2}$$

where $k = 0, 1, \ldots$ and $h > 0$ is the sampling interval. **(2p)**

(c) Initially the same state feedback is used as for the continuous-time case, i.e. the control law $u(kh) = -L_c x(kh)$, with the same L_c as above, is implemented. Use the discrete-time model (2) and determine the closed loop poles for this case. Also determine for which $h > 0$ the closed loop system is stable. **(3p)**

(d) It turned out that the performance of the sampling controller in (c) was not satisfactory (due to a rather long sampling interval h). Redesign the controller by use of the sampled model (2). That is, find a vector L_d so that the state feedback $u(kh) = -L_d x(kh)$ gives a closed loop sampled system which behaves in the same (similar) way as the original continuous-time closed loop system in (a). (L_d will depend on h.)
Hint: Find out what the corresponding poles of the closed loop sampled system should be. **(4p)**

Problem 3 A certain industrial process is described by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v(k),$$

where $v(k)$ is zero mean white noise, and $Ev(k)^2 = 1$. Initially both state variables were measured, but due to a malfunctioning sensor the only available measurement now is

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + e(k),$$

where $e(k)$ is zero mean white noise, uncorrelated to $v(k)$ and with intensity $Ee(k)^2 = r$.

(a) Since the value of $x_1(k)$ is required for subsequent calculations needed in the production line, the missing measurements are replaced by the estimate $\hat{x}_1(k) = Ex_1(k) = 0$ (for all k). What is the variance of the estimation error $\tilde{x}_1(k) = x_1(k) - \hat{x}_1(k)$ for this estimate? **(3p)**

(b) Is $x_1(k)$ observable from $y(k)$? **(1p)**

(c) A clever student from Uppsala suggests that a Kalman filter should be used instead for estimation of $x_1(k)$[1]. Show that

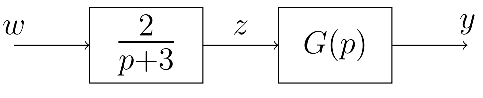
$$P = \begin{bmatrix} p_1 & 1 \\ 1 & 1 \end{bmatrix}$$

is the solution of the associated algebraic Riccati equation, and determine p_1. **(4p)**

(d) Determine the variance of the estimation error $\tilde{x}_1(k)$ when the Kalman filter in (c) is used. Is the estimate improved? **(2p)**

Problem 4 The block diagram below represents a stationary continuous-time stochastic process. The transfer operator $G(p)$ is minimum phase (with $G(0) \geq 0$), and w is zero mean white noise.

$\Phi_w(\omega) = 4$



$\Phi_y(\omega) = \frac{4\omega^2+8}{\omega^4+25\omega^2+144}$

(a) Determine the spectrum for z. **(2p)**

(b) Determine the transfer operator $G(p)$. **(4p)**

[1]The Kalman filter will of course provide an estimate of the full state vector, but then x_1 is part of that.

Problem 5 Specify for each of the following statements whether it is true or false. No motivations required — only answers "true"/"false" are considered!

(a) The discrete-time system $y(k) = \frac{0.2}{q-0.8}u(k)$ has unit static gain.

(b) A Kalman filter is always stable.

(c) For a Kalman filter, based on a correct model and full knowledge of the noise, the *output innovations* are white noise.

(d) For a linear time-invariant (LTI) system LQG design yields an LTI controller.

(e) For an LTI system MPC yields an LTI controller.

(f) With MPC it is not possible to handle bounds on the control input.

Each correct answer scores +1, each incorrect answer scores −1, and omitted answers score 0 points. (Minimal total score is 0 points.) **(6p)**

Problem 6 *The HW bonus points are exchangeable for this problem.*
A system with one input and two outputs is described by

$$\begin{cases} \dot{y}_1(t) + 2y_2(t) = 2\dot{u}(t) + u(t), \\ \dot{y}_2(t) + y_2(t) + 2y_1(t) = u(t). \end{cases}$$

(a) Give a state space representation for the system. **(4p)**

(b) Is your state space representation in (a) a minimal realization? **(2p)**

(c) Is the system stable? **(1p)**

Solutions to the exam in Automatic Control II, 2016-08-20:

1. **(a)** A state space representation is

$$\dot{x} = u + w, \quad z = x, \quad y = x + e.$$

Theorem 9.1 $\Rightarrow L = Q_2^{-1}B^T S$, where $S = S^T > 0$ solves the CARE $0 = A^T S + SA + M^T Q_1 M - SBQ_2^{-1}B^T S$. Here $Q_1 = 1$, $Q_2 = \rho^2$, $A = 0$, $B = 1$ and $M = 1$, leading to

$$0 = 1 - S^2/\rho^2 \qquad \Rightarrow \qquad S = \rho,$$

and then $L = \rho^{-2}\rho = 1/\rho$.
(b) We have that $pw = -w + v$, and therefore

$$\begin{cases} \dot{z} = w + u, \\ \dot{w} = -w + v, \\ z = z, \\ y = z + e \end{cases} \qquad \Leftrightarrow \qquad \begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v, \\ z = \begin{bmatrix} 1 & 0 \end{bmatrix} x, \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + e, \end{cases}$$

with $x = \begin{bmatrix} z & w \end{bmatrix}^T$.
(c) The CARE becomes

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} s_1 & s_{12} \\ s_{12} & s_2 \end{bmatrix} + \begin{bmatrix} s_1 & s_{12} \\ s_{12} & s_2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$
$$- \begin{bmatrix} s_1 & s_{12} \\ s_{12} & s_2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rho^{-2} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s_1 & s_{12} \\ s_{12} & s_2 \end{bmatrix},$$

which, taken element by element, leads to the equation system

$$\begin{cases} 0 = 1 - s_1^2/\rho^2, \\ 0 = s_1 - s_{12} - s_1 s_{12}/\rho^2, \\ 0 = 2s_{12} - 2s_2 - s_{12}^2/\rho^2, \end{cases} \qquad \Rightarrow \qquad \begin{cases} s_1 = \rho, \\ s_{12} = \dfrac{s_1}{1 + s_1/\rho^2} = \dfrac{\rho^2}{\rho + 1}, \\ s_2 = \left(2 - s_{12}/\rho^2\right) s_{12}/2 = \dfrac{\rho^2(2\rho + 1)}{2(\rho + 1)^2}. \end{cases}$$

The state feedback gain is then $L = \rho^{-2} \begin{bmatrix} s_1 & s_{12} \end{bmatrix} = \begin{bmatrix} 1/\rho & 1/(\rho + 1) \end{bmatrix}$.

2. **(a)** The closed loop poles are given by the characteristic equation $0 = \det(sI - A + BL_c)$, which here is

$$0 = \det \begin{bmatrix} s + 1 + l_1 & l_2 \\ l_1 & s + 2 + l_2 \end{bmatrix} = s^2 + (3 + l_1 + l_2)s + 2 + 2l_1 + l_2 = s^2 + 5s + 6.$$

The poles are -2 and -3.
(b) For a ZOH sampled system we have $F = e^{Ah}$, $G = \int_0^h e^{At} B dt$ and $H = C$. Thus

$$F = \begin{bmatrix} e^{-h} & 0 \\ 0 & e^{-2h} \end{bmatrix}, \qquad G = \int_0^h \begin{bmatrix} e^{-t} \\ e^{-2t} \end{bmatrix} dt = \begin{bmatrix} 1 - e^{-h} \\ 0.5(1 - e^{-2h}) \end{bmatrix}, \qquad H = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

(c) Set $\alpha = e^{-h}$ and $L = \begin{bmatrix} l_1 & 0 \end{bmatrix}$. The poles of the closed loop sampled system are given by

$$0 = \det(qI - F + GL) = \det\left\{\begin{bmatrix} q & 0 \\ 0 & q \end{bmatrix} - \begin{bmatrix} \alpha & 0 \\ 0 & \alpha^2 \end{bmatrix} + \begin{bmatrix} 1-\alpha \\ 0.5(1-\alpha^2) \end{bmatrix}\begin{bmatrix} l_1 & 0 \end{bmatrix}\right\}$$

$$= \det\begin{bmatrix} q - \alpha + (1-\alpha)l_1 & 0 \\ 0.5(1-\alpha^2)l_1 & q-\alpha^2 \end{bmatrix} = (q - \alpha + (1-\alpha)l_1)(q-\alpha^2).$$

Thus, the poles are $\alpha^2 = e^{-2h}$ and $(l_1+1)\alpha - l_1 = 3e^{-h} - 2$ (since $l_1 = 2$). For stability both poles must lie inside the unit circle. Since $0 < e^{-2h} < 1$ for all $h > 0$, it is only the other pole that can cause an unstable closed loop system. The condition for stability is

$$-1 < 3e^{-h} - 2 < 1 \qquad \Leftrightarrow \qquad 1 < 3e^{-h} < 3.$$

Again, since $e^{-h} < 1$ for all $h > 0$, it is only the lower bound that can be violated. Hence, the condition for stability is

$$1 < 3e^{-h} \qquad \Leftrightarrow \qquad e^h < 3 \qquad \Leftrightarrow \qquad h < \log 3 \approx 1.0986.$$

(d) The continuous-time poles are -2 and -3, and the corresponding sampled poles are $e^{-2h} = \alpha^2$ and $e^{-3h} = \alpha^3$ (see Sec 4.3 in Glad/Ljung). The desired pole polynomial for the sampled system is then $(q-\alpha^2)(q-\alpha^3)$. Compare with the obtained pole polynomial obtained in (c), $(q-\alpha+(1-\alpha)l_1)(q-\alpha^2)$. Here we see that the desired poles can be obtained with $l_2 = 0$ (as in (c)) and $l_1 = \frac{\alpha-\alpha^3}{1-\alpha} = \frac{\alpha(1-\alpha^2)}{1-\alpha} = \alpha(1+\alpha) = e^{-h} + e^{-2h}$. Then $L_d = \begin{bmatrix} l_1 & 0 \end{bmatrix}$.

3. (a) Since $\hat{x}_1(k) = 0$, we have $\tilde{x}_1(k) = x_1(k)$, and thus

$$\tilde{x}_1(k+1) = 0.5\tilde{x}_1(k) + v(k).$$

The variance $E\tilde{x}_1(k)^2 = \Pi_{\tilde{x}}$ is then obtained from the discrete-time Lyapunov equation $\Pi_{\tilde{x}} = F\Pi_{\tilde{x}}F^T + GR_vG^T$. Here $F = 0.5$, $G = 1$ and $R_v = 1$, so

$$\Pi_{\tilde{x}} = 0.5^2\Pi_{\tilde{x}} + 1 \quad \Leftrightarrow \quad \Pi_{\tilde{x}} = \frac{1}{0.75} = \frac{4}{3}.$$

(b) The observability matrix is

$$\mathcal{O} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \text{and} \quad \mathcal{O}\begin{bmatrix} x_1 \\ 0 \end{bmatrix} = 0$$

regardless of x_1. Therefore it is *not* observable.

(c) The associated DARE is $P = FPF^T + NR_1N^T - FPH^T(HPH^T + R_2)^{-1}HPF^T$ (since $R_{12} = 0$), and here $R_1 = 1$ and $R_2 = r$, so written out the DARE becomes

$$\begin{bmatrix} p_1 & p_{12} \\ p_{12} & p_2 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} p_1 & p_{12} \\ p_{12} & p_2 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$-\begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} p_1 & p_{12} \\ p_{12} & p_2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} p_1 & p_{12} \\ p_{12} & p_2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + r\right)^{-1}\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} p_1 & p_{12} \\ p_{12} & p_2 \end{bmatrix}\begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.25p_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \frac{0.25p_{12}^2}{p_2+r} & 0 \\ 0 & 0 \end{bmatrix}.$$

This gives the equation system

$$\begin{cases} p_1 = 0.25p_1 + 1 - \dfrac{0.25p_{12}^2}{p_2 + r}, \\ p_{12} = 1, \\ p_2 = 1, \end{cases} \quad \Rightarrow \quad P = \begin{bmatrix} p_1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Then

$$p_1 = 0.25p_1 + 1 - \frac{0.25}{1+r} \quad \Leftrightarrow \quad p_1 = \frac{4}{3} - \frac{1}{3(1+r)}.$$

(d) Since $P = E\tilde{x}\tilde{x}^T$, and $\tilde{x}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \tilde{x}$, we get

$$E\tilde{x}_1^2 = \begin{bmatrix} 1 & 0 \end{bmatrix} P \begin{bmatrix} 1 \\ 0 \end{bmatrix} = p_1 = \frac{4}{3} - \frac{1}{3(1+r)}.$$

Furthermore, $1 < p_1 < \frac{4}{3}$ for every $r > 0$, so the estimate from the Kalman filter *is* better than the estimate $\hat{x}_1 = 0$!

4. (a) We have

$$\Phi_z(\omega) = \left| \frac{2}{i\omega + 3} \right|^2 \Phi_w(\omega) = \frac{2}{i\omega + 3} \cdot \frac{2}{-i\omega + 3} \cdot 4 = \frac{16}{\omega^2 + 9}.$$

(b) Again we can use that $\Phi_y(\omega) = |G(i\omega)|^2 \Phi_z(\omega)$, and from (a) we have $\Phi_z(\omega)$. Thus,

$$\begin{aligned} |G(i\omega)|^2 = \frac{\Phi_y(\omega)}{\Phi_z(\omega)} = \frac{\frac{4\omega^2+8}{\omega^4+25\omega^2+144}}{\frac{16}{\omega^2+9}} &= \frac{(4\omega^2+8)(\omega^2+9)}{16(\omega^4+25\omega^2+144)} \\ &= \frac{(4\omega^2+8)(\omega^2+9)}{16(\omega^2+9)(\omega^2+16)} = \frac{0.25\omega^2+0.5}{\omega^2+16}. \end{aligned}$$

Based on the degrees of ω^2 in numerator and denominator we try with

$$G(p) = \frac{b_1 p + b_2}{p + a} \quad \Rightarrow \quad |G(i\omega)|^2 = \frac{(ib_1\omega + b_2)(-ib_1\omega + b_2)}{(i\omega + a)(-i\omega + a)} = \frac{b_1^2\omega^2 + b_2^2}{\omega^2 + a^2}.$$

Comparison with the expression above gives

$$\begin{cases} b_1^2 = 0.25, \\ b_2^2 = 0.5, \\ a^2 = 16, \end{cases} \quad \Rightarrow \quad \begin{cases} b_1 = 0.5, \\ b_2 = \sqrt{0.5}, \\ a = 4, \end{cases} \quad \Rightarrow \quad G(p) = \frac{0.5p + \sqrt{0.5}}{p + 4}.$$

The negative roots are omitted since stationarity $\Leftrightarrow$ stability $\Leftrightarrow a > 0$, minimum phase and $G(0) \geq 0 \Leftrightarrow b_1, b_2 \geq 0$.

5. (a) True (set $q = 1$); **(b)** True (see Lemma 5.1); **(c)** True (see Theorem 5.5); **(d)** True (see Theorem 9.1); **(e)** False (MPC is time-varying and in general non-linear); **(f)** False (MPC handles bounds and constraints);

6. (a) There are several (infinitely many) possible state space representations. One possibility is the following choice of state variables: $x_1 = y_1 - 2u$ and $x_2 = y_2$. Then we can rewrite the differential equations as

$$\begin{cases} \dot{x}_1 + 2\dot{u} + 2x_2 = 2\dot{u} + u, \\ \dot{x}_2 + x_2 + 2x_1 + 4u = u \end{cases} \Rightarrow \begin{cases} \dot{x}_1 = -2x_2 + u, \\ \dot{x}_2 = -2x_1 - x_2 - 3u, \\ y_1 = x_1 + 2u, \\ y_2 = x_2, \end{cases}$$

which in vector form, with $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and $y = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T$, is

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & -2 \\ -2 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -3 \end{bmatrix} u, \\ y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u. \end{cases}$$

Another possibility is to use the controller canonical form (which always works when there is only one input). Then we need to start by finding the transfer operator/function:

$$\begin{cases} py_1 + 2y_2 = (2p+1)u, \\ 2y_1 + (p+1)y_2 = u \end{cases} \Leftrightarrow \begin{bmatrix} p & 2 \\ 2 & p+1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2p+1 \\ 1 \end{bmatrix} u$$

$$\Leftrightarrow \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} p & 2 \\ 2 & p+1 \end{bmatrix}^{-1} \begin{bmatrix} 2p+1 \\ 1 \end{bmatrix} u = \frac{1}{p(p+1)-4} \begin{bmatrix} p+1 & -2 \\ -2 & p \end{bmatrix} \begin{bmatrix} 2p+1 \\ 1 \end{bmatrix} u$$

$$= \begin{bmatrix} \frac{2p^2+3p-1}{p^2+p-4} \\ \frac{-3p-2}{p^2+p-4} \end{bmatrix} u = \begin{bmatrix} 2 + \frac{p+7}{p^2+p-4} \\ \frac{-3p-2}{p^2+p-4} \end{bmatrix} u$$

The controller canonical form then gives us the state space representation

$$\begin{cases} \dot{x} = \begin{bmatrix} -1 & 4 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \\ y = \begin{bmatrix} 1 & 7 \\ -3 & -2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u. \end{cases}$$

(b) A minimal realization is both controllable and observable. For the first state space representation in (a) we have

$$\mathcal{S} = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -3 & 1 \end{bmatrix}, \quad \mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -2 \\ -2 & -1 \end{bmatrix}.$$

Obviously both $\mathcal{S}$ and $\mathcal{O}$ have full rank $\Leftrightarrow$ the state space representation is both controllable and observable, and hence a minimal realization.

The controller canonical form is obviously controllable. Thus we only need to check observability for that:

$$\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ -3 & -2 \\ 6 & 4 \\ 1 & -12 \end{bmatrix}.$$

Again $\mathcal{O}$ has full rank (eg. the first two rows are linearly independent), and we have observability, and thereby a minimal realization.
(c) The pole polynomial is $s^2 + s - 4$, and one pole is in the right half plane $\Rightarrow$ unstable.