**Uppsala University**
**Department of Information Technology**
**Division of Systems and Control**



# Homework Assignment no. 4

## Parametric Methods for Line Spectra

This fourth homework is based on **Exercise C4.10** (old book) / **C4.14** (new book) (and **Exercise C2.21/C2.23** and **Exercise C3.18/C3.20** in parts).
For your convenience, code for the different spectral estimators can be downloaded at `http://www.prenhall.com/stoica`. Make sure that you use these functions correctly (use "`help`". If still in doubt how to use the functions, try by inspecting the code directly). Here follow some explanations and clarifications for the corresponding parts of **Exercise C4.10/C4.14**:

We recommend the following approach to simplify the interpretation of the results:

1. Compute frequency estimates for even orders between 4 and 18, with HOYW for $L = M = N/6$ and $L = M = N/4$, MUSIC, Min-Norm, and ESPRIT each for $m = n + 1$ and $m = 2n$.
2. Compute the least-squares (LS) amplitude and phase estimates $\hat{\beta}$, by using Eq. (4.3.8) in the book, where $\{\hat{\omega}_j\}_{j=1}^{n}$ are given by the frequency estimation methods above.
3. Reconstruct the signal using the estimated frequencies, amplitudes and phases.
4. Compute and plot the mean-squared errors between the data and reconstructed signals, versus model order.
5. Choose the model order such that no significant improvement can be achieved by further increasing the model order.
6. Present the relevant plots (**not** for all tested orders) using `subplot(2,2,·)` to keep the number of pages down and to make the comparison easier.

Discuss the questions and remarks raised in the book. Single sentence answers are not sufficient. For the discussion on line spectral methods vs. nonparametric and ARMA methods, you need to solve **Exercise C2.21/C2.23** and **Exercise C3.18/C3.20** partly as well (see homework no. 2). If you have done homework no. 2, only little extra work should be required for this part.

**Exercise C4.14: Line Spectral Methods applied to Measured Data**

Apply the Min–Norm, MUSIC, ESPRIT, and HOYW frequency estimators to the data in the files `sunspotdata.mat` and `lynxdata.mat` (use both the original `lynx` data and the logarithmically transformed data as in Exercise C2.23). These files can be obtained from the text web site `www.prenhall.com/stoica`. Try to answer the following questions:

- **(a)** Is the sinusoidal model appropriate for the data sets under study?
- **(b)** Suggest how to choose the number of sinusoids in the model (see Exercise C4.13).
- **(c)** What periodicities can you find in the two data sets?

Compare the results you obtain here to the AR(MA) and nonparametric spectral estimation results you obtained in Exercises C2.23 and C3.20.

**Exercise C3.20: AR and ARMA Estimators applied to Measured Data**

Consider the data sets in the files `sunspotdata.mat` and `lynxdata.mat`. These files can be obtained from the text web site `www.prenhall.com/stoica`.

Apply your favorite AR and ARMA estimator(s) (for the `lynx` data, use both the original data and the logarithmically transformed data as in Exercise C2.23) to estimate the spectral content of these data. You will also need to determine appropriate model orders $m$ and $n$ (see, *e.g.*, Exercise C3.19). As in Exercise C2.23, try to answer the following questions: Are there sinusoidal components (or periodic structure) in the data? If so, how many components and at what frequencies? Discuss the relative strengths and weaknesses of parametric and nonparametric estimators for understanding the spectral content of these data. In particular, discuss how a combination of the two techniques can be used to estimate the spectral and periodic structure of the data.

**Exercise C2.23: Periodogram–Based Estimators applied to Measured Data**

Consider the data sets in the files `sunspotdata.mat` and `lynxdata.mat`. These files can be obtained from the text web site `www.prenhall.com/stoica`. Apply periodogram–based estimation techniques (possibly after some preprocessing; see the following) to estimate the spectral content of these data. Try to answer the following questions:

- **(a)** Are there sinusoidal components (or periodic structure) in the data? If so, how many components and at what frequencies?
- **(b)** Nonlinear transformations and linear or polynomial trend removal are often applied before spectral analysis of a time series. For the `lynx` data, compare your spectral analysis results from the original data, and the data transformed first by taking the logarithm of each sample and then by subtracting the sample mean of this logarithmic data. Does the logarithmic transformation make the data more sinusoidal in nature?