Uppsala University
Department of Information Technology
Division of Systems and Control

Homework Assignment no. 1

Periodogram Methods

The first homework is based on **Exercise C2.20** (old book) / **C2.22** (new book). For your convenience, code for the different spectral estimators can be downloaded at `http://www.prenhall.com/stoica`. Make sure that you use these functions correctly, (use "`help`". If still in doubt how to use the functions, try by inspecting the code directly.) Here follow some explanations and clarifications for the corresponding parts of **Exercise C2.20/C2.22**:

Broadband ARMA process:

(a) Monte-Carlo simulation is a means of gaining information about the statistical properties of a method by applying the algorithm to multiple simulated data sets where the stochastic parts are redrawn from the same probability distribution for each data set. In this problem this is done by using 50 different sequences of the driving noise $e(t)$, yielding 50 data sets $y(t)$. Use "`filter`" to generate the data sequences.

For the different methods and parameters, plot the *sample mean* (experimental average), the sample mean + *sample standard deviation* (experimental standard deviation) and the sample mean − sample standard deviation of the resulting spectral estimates in a superimposed manner (i.e., three curves per plot). Please, use `subplot(2,2,·)` to keep the number of pages down and to make the comparison easier. Also plot the sample variances of the spectral estimates for the different methods and parameters in a superimposed manner to make the comparison easier. Use dB scales in all these plots.

Since **Exercise C2.19/C2.21** is not a part of the homeworks nor a lab you do not need to do the comparison with the result of this exercise. However, try to say something about what you could expect from such a comparison and why (i.e. if you had used the periodogram).

(b) "This agrees with theory"–answers are not sufficient.

(c) Try to describe why you chose this particular window and M as your "best design". Plot the sample mean, the sample mean + sample standard deviation and the sample mean - sample standard deviation of the estimates using your "best design" in the same plot similarly to what you did in (a).

Narrowband ARMA process:

Repeat the experiments with and comparisons of the different methods but now with the narrowband process. Compare the results from the broadband case with the results from the narrowband case. Is there a difference in the conclusions of (b) and if so why?

Exercise C2.22: Refined Methods: Variance–Resolution Tradeoff

In this exercise we apply the Blackman–Tukey and Welch estimators to both a narrowband and broadband random process. We consider the same processes in Chapters 3 and 5 to facilitate comparison with the spectral estimation methods developed in those chapters.

Broadband ARMA Process: Generate realizations of the broadband autoregressive moving–average (ARMA) process

$$y(t) = \frac{B_1(z)}{A_1(z)}\, e(t)$$

with

$$\begin{aligned} A_1(z) &= 1 - 1.3817z^{-1} + 1.5632z^{-2} - 0.8843z^{-3} + 0.4096z^{-4} \\ B_1(z) &= 1 + 0.3544z^{-1} + 0.3508z^{-2} + 0.1736z^{-3} + 0.2401z^{-4} \end{aligned}$$

Choose the number of samples as $N = 256$.

(a) Generate 50 Monte–Carlo data realizations using different noise sequences, and compute the corresponding 50 spectral estimates using the following methods:

- The Blackman–Tukey spectral estimate using the Bartlett window $w_B(t)$. Try both $M = N/4$ and $M = N/16$.
- The Welch spectral estimate using the rectangular window $w_R(t)$, and using both $M = N/4$ and $M = N/16$ and overlap parameter $K = M/2$.

Plot the sample mean, the sample mean plus one sample standard deviation and sample mean minus one sample standard deviation spectral estimate curves. Compare with the periodogram results from Exercise C2.21, and with each other.

(b) Judging from the plots you have obtained, how has the variance decreased in the refined estimates? How does this variance decrease compare to the theoretical expectations?

(c) As discussed in the text, the value of M should be chosen to compromise between low "smearing" and low variance. For the Blackman–Tukey estimate, experiment with different values of M and different window functions to find a "best design" (in your judgment), and plot the corresponding spectral estimates.

Narrowband ARMA Process: Generate realizations of the narrowband ARMA process

$$y(t) = \frac{B_2(z)}{A_2(z)}\, e(t)$$

with

$$\begin{aligned} A_2(z) &= 1 - 1.6408z^{-1} + 2.2044z^{-2} - 1.4808z^{-3} + 0.8145z^{-4} \\ B_2(z) &= 1 + 1.5857z^{-1} + 0.9604z^{-2} \end{aligned}$$

and $N = 256$.

Repeat the experiments and comparisons in the broadband example for the narrowband process.