Task #1: "The Metric Lax"
Numerical Functional Analysis

Divide et Impera

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Prerequisites

Please form *exactly* 4 groups of *about* 2–3 persons.

Task 1

$\Longrightarrow$ Formulate the Lax (or Lax-Richtmyer) equivalence theorem.

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Something like: *"A* **consistent** *method applied to a* **well-posed** *problem is* **convergent** *if and only if it is* **stable***."*

Tasks

To solve during 15 min of fika

Some notation. We consider *the mathematical problem* (or M-problem for short) *"find $x \in X$ s.t. $Tx = y$"*. Assume metric spaces (X, d) and $(Y, \tilde{d})$. The solution is thus $x = T^{-1}y$.

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Tasks (one per group): in this setting...

1. ...define what is meant by **well-posed**.
2. ...define what is meant by a **consistent** numerical method.
3. ...define what is meant by a **convergent** numerical method.
4. ...define what is meant by a **stable** numerical method.

Task 1/4

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$\Longrightarrow$ Define what is meant by **well-posed** in this setting.

The M-problem is well-posed if T^{-1} is continuous in some neighborhood containing y.

Task 2/4

The M-problem is to be approached numerically by solving (a sequence of) numerical ('N'-) problems *"find $x_n \in X$ s.t. $T_n x_n = y$"*. We can think of n as the resolution of the scheme.

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$\Longrightarrow$ Define what is meant by a **consistent** numerical method in this setting.

A method is consistent if, for any x in the domain of T, $T_n x \to Tx$ as $n \to \infty$.

Task 3/4

$\Longrightarrow$ Define what is meant by a **convergent** numerical method in this setting.

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A method is convergent if $T_n^{-1}y = x_n \to x$ as $n \to \infty$.

Task 4/4

$\Longrightarrow$ Define what is meant by a **stable** numerical method in this setting.

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$\Longrightarrow$ Define what is meant by a **stable** numerical method in this setting.

A method is stable if for all n, T_n^{-1} is continuous in some neighborhood containing y.

“The Metric Lax”

A formulation of the Lax principle in metric spaces

Suppose T^{-1} is continuous and that $T_n x \to Tx$ for any x. Then $T_n^{-1} y \to x$ as $n \to \infty$ if and only if for all n, T_n^{-1} is continuous.

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- Usually T and T_n are not defined on the same space...
- ...so T^{-1} and T_n^{-1} are also defined on different spaces. $y \mapsto y_n$ (some numerical restriction) would be more realistic. *Doable?*
- ...