

# Fast Quiz #2

## Numerical Functional Analysis

*Præparatus supervivet*

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## Question 1

**True/False:** In a normed vector space  $(X, \|\cdot\|)$ , the map  $x : X \rightarrow \mathbf{R}$  defined by  $x \mapsto \sqrt{1 + \|x\|^2}$  is continuous.

## Question 2

**True/False:** When considered on  $\mathbf{R}^n$ , the norms  $\|\cdot\|_\infty$  and  $\|\cdot\|_1$  imply the same topology, but they are not equivalent.

## Question 3

**True/False:** All subspaces of the (sequence-)  $l^2$ -space are complete.

## Question 4

**True/False:** Define  $f_z(x) = \|x - z\|$  for arguments  $x$  in some normed vector space  $X$ . Then there is a solution to  $x = \arg \max_{x \in M} f_z(x)$  for any compact subset  $M \subseteq X$ .

## Question 5

**True/False:** All linear operators on  $C[0, 1]$  are continuous.

## Question 6

**True/False:** If there is a Schauder basis, then the space is separable.

## Question 7

**True/False:**  $L^2[0, 1]$  is separable.



## Question 8

**True/False:** The convergence of a Schauder expansion is always in the absolute sense.

## Question 9

**True/False:** If the space is separable, then there is a Schauder basis.

## Question 10

**True/False:** A closed and bounded subset of a metric space is compact.