Fast Quiz #3
Numerical Functional Analysis

Præparatus supervivet

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Question 1

True/False: $\|x + y\|^2 \leq \|x\|^2 + \|y\|^2$.

Question 2

True/False: One way of defining the l^p inner product is

$$(x, y) = \sum_{j \geq 1} \xi_j^{p/2} \eta_j^{p/2},$$

where $x = (\xi_j)$ and $y = (\eta_j)$.

Question 3

True/False: *Unfortunately*, the norm $\|x\| = \max_{t\in[a,b]} |x(t)|$ for $x \in C[a,b]$ can not be obtained from an inner product.

Question 4

True/False: Let Y be an open subspace of a Hilbert space H. Then Y is complete.

Question 5

True/False: Suppose $(Sv, v) = 0$ for all v in a complex Hilbert space H, where S is a bounded linear operator. Then $S = 0$.

Question 6

True/False: A bounded bilinear form $a(\cdot, \cdot) \geq 0$ defines a norm by $\|v\|^2 := a(v, v)$.

Question 7

True/False: If a Hilbert space H contains a total orthogonal sequence, then H is separable.

Question 8

True/False: Let $f \in L^2[0,1]$. Assume that (e_k) is an orthonormal sequence in $L^2[0,1]$. Put

$$\tilde{f} = \sum_{k \geq 1} (f, e_k) e_k.$$

Then $f = \tilde{f}$.

Question 9

True/False: If $x \perp y$, then $\|x + y\|^2 = \|x\|^2 + \|y\|^2$.

Question 10

True/False: Suppose $(Sv, v) = 0$ for some v in a complex Hilbert space H, where S is a bounded linear operator. Then $Sv = 0$.

Question 11

True/False: A bounded coercive bilinear form defines an inner product by $(u, v) := a(u, v)$.