

Task #1: “The Metric Lax”

Numerical Functional Analysis

Divide et Impera

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Prerequisites

Please form *exactly* 4 groups of *about* 3–4 persons.

Task 1

\implies Formulate the Lax (or Lax-Richtmyer) equivalence theorem.

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⇒ Formulate the Lax (or Lax-Richtmyer) equivalence theorem.

Something like: “A **consistent** method applied to a **well-posed** problem is **convergent** if and only if it is **stable**.”

Tasks

To solve during 15 min of fika

Some notation. We consider *the mathematical problem* (or M-problem for short) “find $x \in X$ s.t. $Tx = y$ ”. Assume metric spaces (X, d) and (Y, \tilde{d}) . The solution is thus $x = T^{-1}y$.

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The M-problem is to be approached numerically by solving (a sequence of) numerical ('N'-) problems “find $x_n \in X$ s.t. $T_n x_n = y$ ”. We can think of n as the resolution of the scheme.

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Tasks (one per group): in this setting...

1. ...define what is meant by **well-posed**.
2. ...define what is meant by a **consistent** numerical method.
3. ...define what is meant by a **convergent** numerical method.
4. ...define what is meant by a **stable** numerical method.

Task 1/4

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\implies Define what is meant by **well-posed** in this setting.

The M-problem is well-posed if T^{-1} is continuous in some neighborhood containing y .

Task 2/4

The M-problem is to be approached numerically by solving (a sequence of) numerical ('N'-) problems “*find* $x_n \in X$ s.t. $T_n x_n = y$ ”. We can think of n as the resolution of the scheme.

⇒ Define what is meant by a **consistent** numerical method in this setting.

Task 2/4

The M-problem is to be approached numerically by solving (a sequence of) numerical ('N'-) problems “find $x_n \in X$ s.t. $T_n x_n = y$ ”. We can think of n as the resolution of the scheme.

⇒ Define what is meant by a **consistent** numerical method in this setting.

A method is consistent if, for any x in the domain of T , $T_n x \rightarrow Tx$ as $n \rightarrow \infty$.

Task 3/4

⇒ Define what is meant by a **convergent** numerical method in this setting.

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A method is convergent if $T_n^{-1}y = x_n \rightarrow x$ as $n \rightarrow \infty$.

Task 4/4

⇒ Define what is meant by a **stable** numerical method in this setting.

Task 4/4

⇒ Define what is meant by a **stable** numerical method in this setting.

A method is stable if for all n , T_n^{-1} is continuous in some neighborhood containing y .

“The Metric Lax”

A formulation of the Lax principle in metric spaces

Suppose T^{-1} is continuous and that $T_n x \rightarrow T x$ for any x . Then $T_n^{-1} y \rightarrow x$ as $n \rightarrow \infty$ if and only if for all n , T_n^{-1} is continuous.

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⇒ Try to find weaknesses with this formulation. -Can you straightforwardly apply it in a setting with which you are familiar?

- ▶ $y \mapsto y_n$ would be more realistic.
- ▶ Usually T and T_n are not defined on the same space.
- ▶ ...