Fast Quiz #3
Numerical Functional Analysis, 5.0 hp

Præparatus supervivet

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1. **True/False:** $\|x+y\|^2 \leq \|x\|^2 + \|y\|^2$.

2. **True/False:** One way of defining the l^p inner product is

$$(x,y) = \sum_{j\geq 1} \xi_j^{p/2} \eta_j^{p/2},$$

   where $x = (\xi_j)$ and $y = (\eta_j)$.

3. **True/False:** *Unfortunately*, the norm $\|x\| = \max_{t\in[a,b]} |x(t)|$ for $x \in C[a,b]$ can not be obtained from an inner product.

4. **True/False:** Let Y be an open subspace of a Hilbert space H. Then Y is complete.

5. **True/False:** Suppose $(Sv, v) = 0$ for all v in a complex Hilbert space H, where S is a bounded linear operator. Then $S = 0$.

6. **True/False:** A bounded bilinear form $a(\cdot,\cdot) \geq 0$ defines a norm by $\|v\|^2 := a(v,v)$.

7. **True/False:** If a Hilbert space H contains a total orthogonal sequence, then H is separable.

8. **True/False:** Let $f \in L^2[0,1]$. Assume that (e_k) is an orthonormal sequence in $L^2[0,1]$. Put

$$\tilde{f} = \sum_{k \geq 1} (f, e_k) e_k.$$

Then $f = \tilde{f}$.

9. **True/False:** If $x \perp y$, then $\|x + y\|^2 = \|x\|^2 + \|y\|^2$.

10. **True/False:** Suppose $(Sv, v) = 0$ for some v in a complex Hilbert space H, where S is a bounded linear operator. Then $Sv = 0$.

11. **True/False:** A bounded coercive bilinear form defines an inner product by $(u, v) := a(u, v)$.