Final Exam in Algorithms and Data Structures 1 (1DL210)

Department of Information Technology
Uppsala University
2013–08–20
Lecturers: Parosh Aziz Abdulla, Jonathan Cederberg and Jari Stenman
Location: Polacksbacken
Time: 08:00 - 13:00
No books or calculator allowed.

Directions:

1. Do not write on the back of the paper
2. Do not use red ink
3. Write your **anonymous exam code** on each sheet of paper
4. **Important:** Unless explicitly stated otherwise, justify you answer carefully! Answers without justification do not give any credits.

Good Luck!

Problem 1 (10p)

Order these functions in order of increasing asymptotic growth rate[1]. If two of them have the same asymptotic growth rate, state that fact. No justification is needed.

$$\left(\frac{1000}{999}\right)^n \qquad 2^n \qquad \left(\frac{999}{1000}\right)^n \qquad 2^{2\cdot n} \qquad n^2 \qquad \log n \qquad n^2 + n\log n \qquad n$$

Problem 2 (10p)

For each of the propositions below, state whether that proposition is true or false. No justification is needed.

a) If $f(n) = \mathcal{O}(g(n))$ and $f(n) = \Omega(g(n))$, then we have $(f(n))^2 = \Theta((g(n))^2)$

b) If $f(n) = \mathcal{O}(g(n))$ and $f(n) = \Omega(g(n))$, then we have $f(n) = g(n)$

[1]Here, log denotes the binary logarithm.

Problem 3 (15p)

The performance of QUICKSORT depends heavily on the choice of the so called *pivot* element. Assume that you have an implementation of QUICKSORT that simply chooses the leftmost element as pivot. Also assume that QUICKSORT returns the sorted array when finished.

a) What is the asymptotic running time of QUICKSORT(A) in the *average case*?

b) What is the asymptotic running time of QUICKSORT(QUICKSORT(A)) in the *average case*?

Problem 4 (10p)

a) Draw the two input arrays for the *final* call to MERGE in MERGESORT($[2, 4, 8, 3, 5, 1, 7, 6]$).

b) What is the worst case complexity of MERGE in MERGESORT? Note that the function takes as arguments two arrays A_1 and A_2.

c) What is the worst case complexity of MERGESORT?

Problem 5 (10p)

Assume that we have the set $S = \{14, 23, 32, 41, 50, 59, 68\}$ and we want to insert them into a hash table T of size at most 10, using chaining to resolve collisions.

a) Provide a size of T and a suitable hash function h, such that

   - the distribution of elements in T by using h would be good for random input
   - h performs badly for the elements in S

b) Provide a size of T and a suitable hash function h, such that

- the distribution of elements in T by using h would be good for random input
- h performs well for the elements in S

Justify each answer in at most 3 lines.

Problem 6 (15p)

a) What is the largest possible height (i.e. number of levels) of a Binary Search Tree with n elements? Why?

b) What is the worst case complexity of printing the elements of a Binary Search Tree in sorted order? Why?

c) What is the *minimal* size of the rightmost subtree in a Max-Heap containing 10 elements?

d) What is the *maximal* size of the rightmost subtree in a Max-Heap containing 10 elements?

Problem 7 (15p)

Give one possible BFS traversal (i.e. a sequence of nodes) starting from node A of the graph shown in Figure 1. Print the nodes *only* when they are finished. No justification needed.

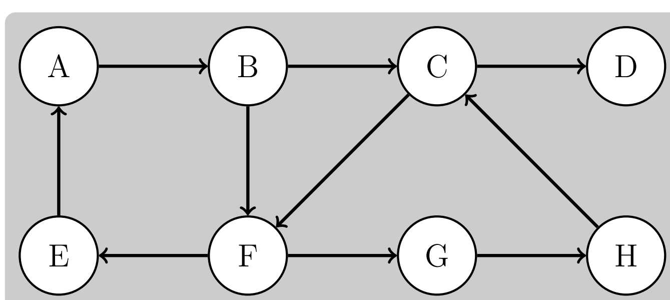

Figure 1: Graph for Problem 7

Problem 8 (15p)

Consider the algorithm VÄINÄMÖINEN, which operates on an array A containing the elements $A[0]$ to $A[n-1]$:

```
VÄINÄMÖINEN(A)
n = LENGTH(A)
for j = 0 to n - 1
    target = j
    for i = j + 1 to n - 1
        if A[i] = A[target]
            return TRUE
return FALSE
```

a) Describe, in 1 line, what VÄINÄMÖINEN does.

b) Give a (tight) asymptotic upper bound on the average case running time of VÄINÄMÖINEN. (Is it $\mathcal{O}(1)$, $\mathcal{O}(\log n)$, $\mathcal{O}(n)$, etc?) Justify your answer in at most 5 lines.

c) Propose a different way of doing the same thing that is asymptotically faster in the average case. Justify your answer in at most 5 lines.