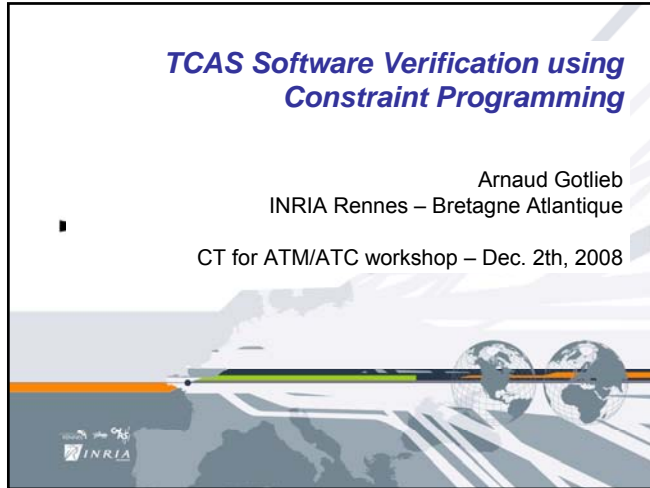


## TCAS Software Verification using Constraint Programming

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CT for ATM/ATC workshop – Dec. 2th, 2008

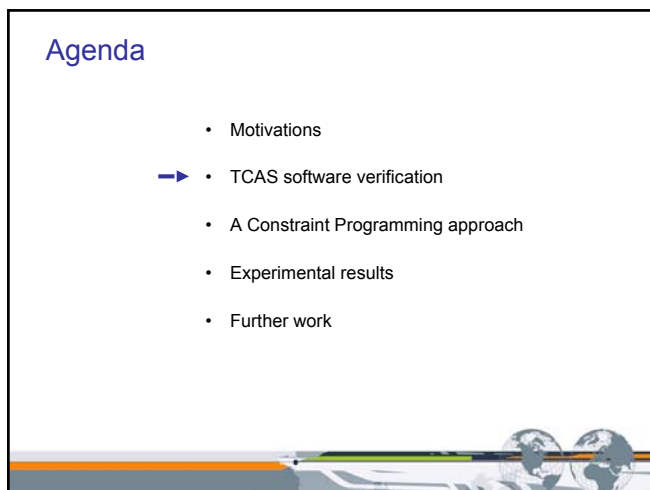


## Motivations

- **The CAVERN project** (ANR, 2008-2010, part: ILOG, INRIA, CEA, U. of Nice):  
To explore the capabilities of Constraint Programming for Automated Program Testing and Verification
- We build a unified framework (called **Euclide**) to perform:
  - test case generation for structural coverage
  - counter-example generation to safety properties
  - (partial) program provingfor safety-critical C programs
- TCAS (Traffic Anti-Collision Avoidance System) software is a real-life example of safety-critical embedded system.  
Strong requirements in terms of verification.

## Agenda

- Motivations
- ➔ • TCAS software verification
- A Constraint Programming approach
- Experimental results
- Further work



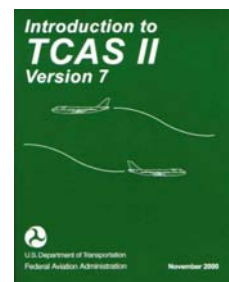
## Traffic alert and Collision Avoidance System

Embedded system on commercial aircrafts

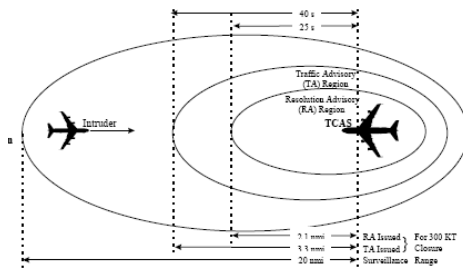
Publicly available implementation for Test and Evaluation (from the *Siemens suite*)

<http://sir.unl.edu/portal/>

DO-178 Level B  
Statement and decision coverage is mandatory



**Tcas.c : component that issues  
Traffic Advisory and Resolution Advisory (170 LOC)**



**The alt\_sep\_test function**

-14 global variables

Own\_Track\_Alt  
Other\_Track\_Alt  
Up\_Separation  
Down\_Separation  
Positive\_RA\_Alt\_Tresh  
...

- Calls 9 distinct functions

- Nested conditionals, logical operators, reifications, ...  
But, no loops, no fp, no pointers

```
int alt_sep_test()
{
    bool enabled = tcas_equipped && intent_not_known;
    bool need_upward_RA, need_downward_RA;
    int alt_sep;

    enabled = High_Confidence && (Own_Tracked_Alt_Rate <= 600) && (Cur_Vertical_Sep > 600);
    tcas_equipped = Other_Capability == 1;
    intent_not_known = Two_of_Three_Reports_Valid && Other_RAC == 0;

    alt_sep = 0;

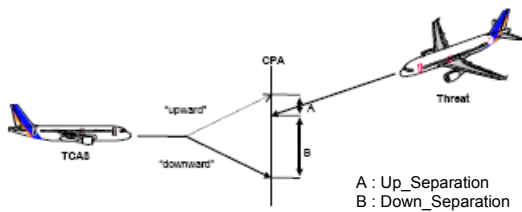
    if (enabled && (tcas_equipped && intent_not_known) || !tcas_equipped)
    {
        need_upward_RA = Non_Crossing_Biased_Climb() && Own_Below_Threat();
        need_downward_RA = Non_Crossing_Biased_Descend() && Own_Above_Threat();
        if (need_upward_RA && need_downward_RA)
            alt_sep = 0;
        else if (need_upward_RA)
            alt_sep = 1;
        else if (need_downward_RA)
            alt_sep = 2;
        else
            alt_sep = 0;
    }

    return alt_sep;
}
```

**A bit of anti-collision Theory**

Safety properties:

- P1: Safe advisory selection
- P2: Best advisory selection
- P3: Avoid unnecessary crossing
- P4: No crossing advisory selection
- P5: Optimal advisory selection (subsumes P1 and P2)



**Safety properties in ACSL**

P1a	/*@ behavior P1a : assumes Up_Separation >= Positive_RA_Alt_Tresh && Down_Separation < Positive_RA_Alt_Tresh; ensures \result != need_Downward_RA;
P1b	/*@ behavior P1b : assumes Up_Separation < Positive_RA_Alt_Tresh && Down_Separation >= Positive_RA_Alt_Tresh; ensures \result != need_Upward_RA;
P2a	/*@ behavior P2a : assumes Up_Separation < Positive_RA_Alt_Tresh && Down_Separation < Positive_RA_Alt_Tresh ; ensures \result != need_Downward_RA;
P2b	/*@ behavior P2b : assumes Up_Separation < Positive_RA_Alt_Tresh && Down_Separation < Positive_RA_Alt_Tresh && Up_Separation < Down_Separation; ensures \result != need_Upward_RA;
P3a	/*@ behavior P3a : assumes Up_Separation >= Positive_RA_Alt_Tresh && Down_Separation >= Positive_RA_Alt_Tresh && Own_Tracked_Alt > Other_Tracked_Alt; ensures \result != need_Downward_RA; @*/
...	

## How prove these properties ?

- In practice, manual code review and testing
- In Theory:
  - Hoare Logic (weakest precondition computations)
  - Model checking
  - Abstract Interpretation-based static analysis
  
  - Constraint-based verification and testing

## Agenda

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## Our CP approach

### Constraint generation

Translate the each C function into a constraint program P  
Translate the property into a constraint S

### Constraint solving

Try to prove that  $\text{sol}(P \wedge \neg S) = \emptyset$

## A constraint model of C functions

Viewing an assignment statement as a relation requires to rename the variables

$i++;$                        $\longrightarrow$                        $i_2 = i_1 + 1$

➔ **Static Single Assignment (SSA)** form (Cytron et al. TOPLAS 1991)  
or **single assignment language**

## SSA form

Each use of a variable refers to a single definition

```
x = x + y;
y = x - y;
x = x - y;
```

```
x1 = x0 + y0;
y1 = x1 - y0;
x2 = x1 - y1;
```

```
if(...)
  i = ... ;
else
  i = ... ;

... = i + ...
```

```
if(...)
  i1 = ... ;
else
  i2 = ... ;

i3 = φ(i1, i2) ;
... = i3 + ...
```

## SSA form



## Constraint Program

Variable declaration

```
unsigned short i;
```

Domain constraint

 $I_0 \in 0..2^{16}-1$ 

Assignment, decision

```
j2 = j1 * i;
i < j;
```

Arithmetical constraints {=, <, ...}

 $J_2 = J_1 * I_0$   
 $I < J$ 

Conditional (SSA)

```
If d then c1, else c2; v3=φ(v1, v2)
```

ITE(D, C<sub>1</sub> ∧ v<sub>3</sub>=v<sub>1</sub>, C<sub>2</sub> ∧ v<sub>3</sub>=v<sub>2</sub>)

Function call (SSA)

```
r = f(a) ;
```

SP\_CALL(f, A, G<sub>1</sub>, ..., G<sub>n</sub>, R)

Implemented as global constraints

## Translating a property into constraints

/\*@ behavior P1b :

```
assumes
Up_Separation < Positive_RA_Alt_Tresh &&
Down_Separation ≥ Positive_RA_Alt_Tresh; ensures \result != need_Upward_RA;
```

S = Preconditions → Post-relations

Then, ¬S = Preconditions ∧ ¬Post-relations

$$\text{Up\_Separation} < \text{Positive\_RA\_Alt\_Tresh} \wedge$$

$$\text{Down\_Separation} \geq \text{Positive\_RA\_Alt\_Tresh} \wedge$$

$$R = \text{need\_Upward\_RA}$$

## Constraint solving

sol( P ∧ ¬S ) ?

- P ∧ ¬S is a non-linear FD constraint problem with global constraints
- We develop our own constraint solver based on:
  - Constraint propagation + bound-consistency filtering
  - Linear Programming techniques over Q
- Why LP ? : capturing linear global behaviour
- Why Q ? : preserving correctness is essential for program verification !
- Property: If the LP relaxed problem does not contain integer points then the original problem is unsatisfiable (but, the converse is false!)
- synchronous cooperation of constraint propagation and simplex over Q through the usage of *Dynamic Linear Relaxations*

## Non-linear expressions in tcas.c

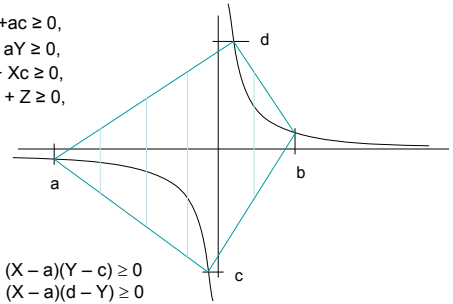
- Multiplication
- Logical operations ( $z > x+y \parallel z < x+y-3$ )
- reification ( $z = x > y$ )
- Conditionals (if then else)

→ Dynamic Linear Relaxations (DLRs)

## DLR of multiplication [McCormick 76]

$Z = X * Y$ ,  $X$  in  $a..b$ ,  $Y$  in  $c..d$ ,  $Z$  in  $e..f$

$$\Leftrightarrow \left\{ \begin{array}{l} Z - Ya - Xc + ac \geq 0, \\ Xd - Z - ad + aY \geq 0, \\ bY - bc - Z + Xc \geq 0, \\ bd - bY - Xd + Z \geq 0, \\ a \leq X \leq b, \\ c \leq Y \leq d, \\ e \leq Z \leq f \end{array} \right\}$$



A consequence of  $(X - a)(Y - c) \geq 0$   
 $(X - a)(d - Y) \geq 0$   
 ...

## DLR of reification

- Reification associates a boolean var. to an expression  
 $Z = (X \leq Y)$  where  $X$  in  $a..b$ ,  $Y$  in  $c..d$  and  $Z$  in  $0..1$

$$\Leftrightarrow \{ 1 - (X - Y) - (1 - a - d) * Z \leq 0, (X - Y) - (b - c) * (1 - Z) \leq 0 \}$$

$$\begin{array}{c} \swarrow \quad \searrow \\ \text{Min}(F(X,Y)) \leq F(X,Y) \leq \text{Max}(F(X,Y)) \\ \uparrow \\ Z = (F(X,Y) \leq 0) \end{array}$$

## DLR of $ITE(Dec, C_1, C_2)$

- Global constraint → is considered iteratively in the constraint store

- Variables of the relation = input-output variables of the conditional
- Awaked when a bound of at least one variable has been pruned
- Filtering algorithm (performed when awaked):

```

if post(Dec ∧ C1) fails
then DLR(¬Dec ∧ C2) and remove ITE
else if post(¬Dec ∧ C2) fails
then DLR(Dec ∧ C1) and remove ITE
else join_dom(Dom1, Dom2) and join_poly(Qpoly1, Qpoly2)
    
```

## How to implement join\_poly(QPoly<sub>1</sub>, QPoly<sub>2</sub>) with a linear solver ?

- ☞ Convex hull computation [Benoy, King, Mesnard TPLP 2004]
- ☞ Big-M relaxation + projection
- ☞ Simplex-based weak\_join operator (from the Abstract Interpretation community) [Sankaranarayanan et al. VMCAI'06]

**NB1:** All these computations are exponential in the number of dimensions in the worst case

**NB2:** switching to the so-called polyhedra « generator representation » is prohibitive in our context

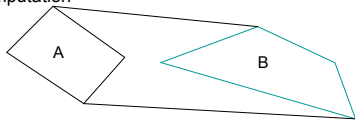
## Weak\_join operator

The disjunction:  $\{g_1^i(x) \geq c_1^i\}_{i \in I} \vee \{g_2^i(x) \geq c_2^i\}_{i \in I}$   
 $x = (x_1, \dots, x_n)$ , where  $x_i \in \mathbb{Z}$

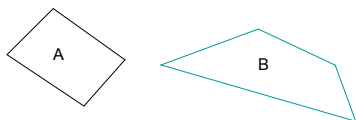
Weak\_join:  $\alpha_1 = \text{Minimize } g_1^1(x) \text{ subject to } \{g_2^i(x)\}_{i \in I}$   
 ...  
 $\alpha_p = \text{Minimize } g_1^{card(I)}(x) \text{ subject to } \{g_2^i(x)\}_{i \in I}$   
 $\alpha_{p+1} = \text{Minimize } g_2^1(x) \text{ subject to } \{g_1^i(x)\}_{i \in I}$   
 ...  
 $\alpha_{2p} = \text{Minimize } g_2^{card(I)}(x) \text{ subject to } \{g_1^i(x)\}_{i \in I}$   
 $g_1^1(x) \geq \text{Min}(\alpha_1, c_1^1)$ ,  
 ...  
 $g_2^{card(I)}(x) \geq \text{Min}(\alpha_{2p}, c_2^{card(I)})$

## Weak\_join operator

convex hull computation

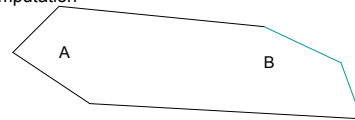


Weak join (Sankaranarayanan et al. VMCAI'06)

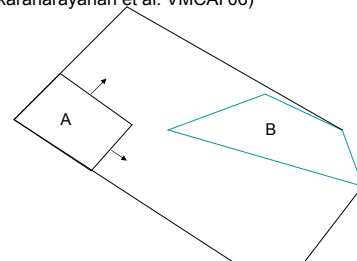


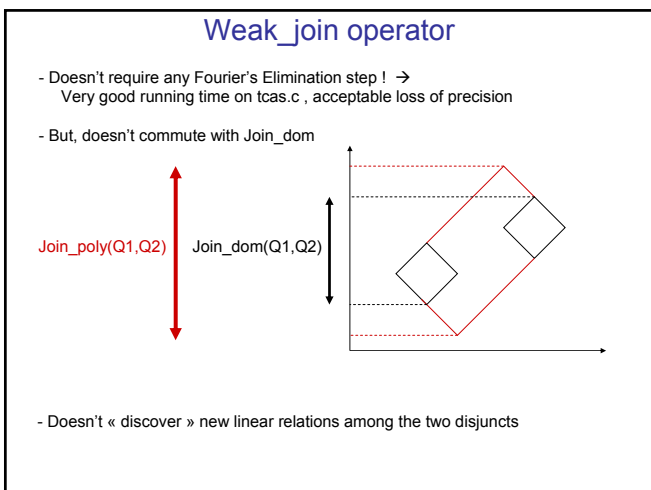
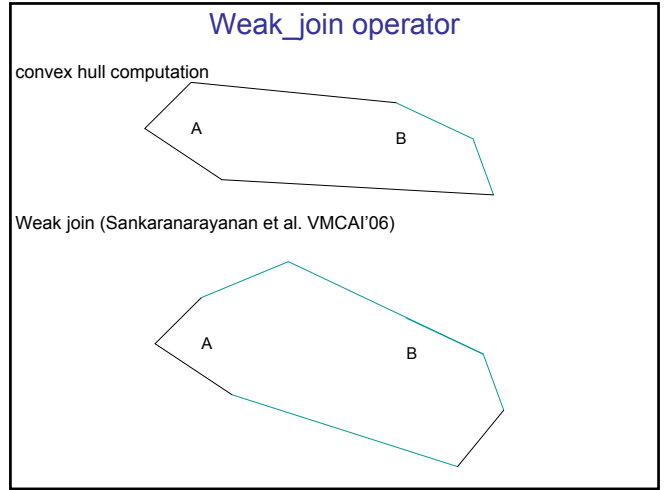
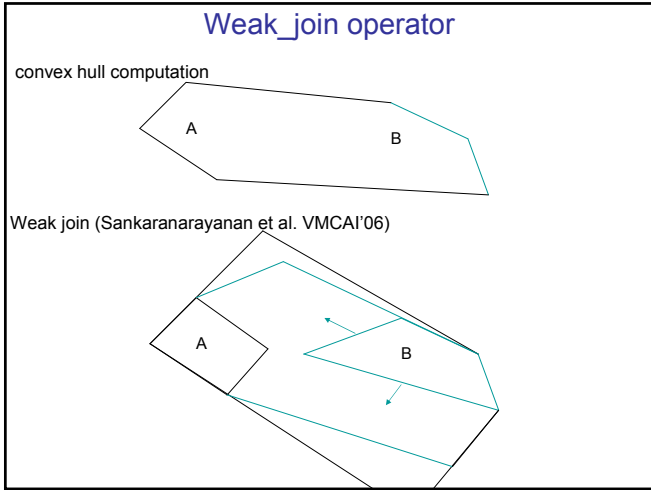
## Weak\_join operator

convex hull computation

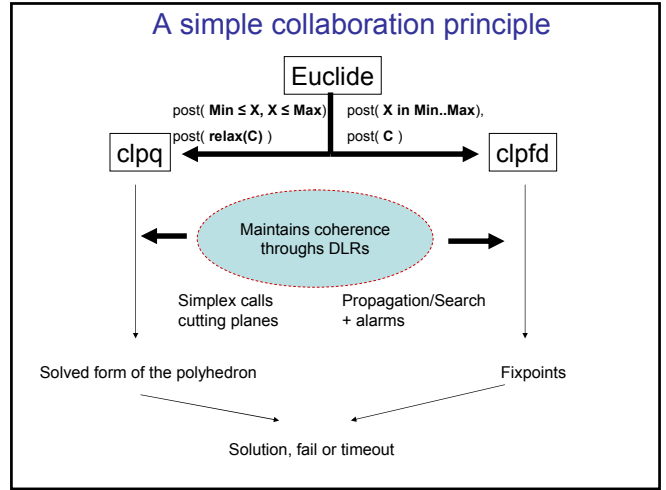
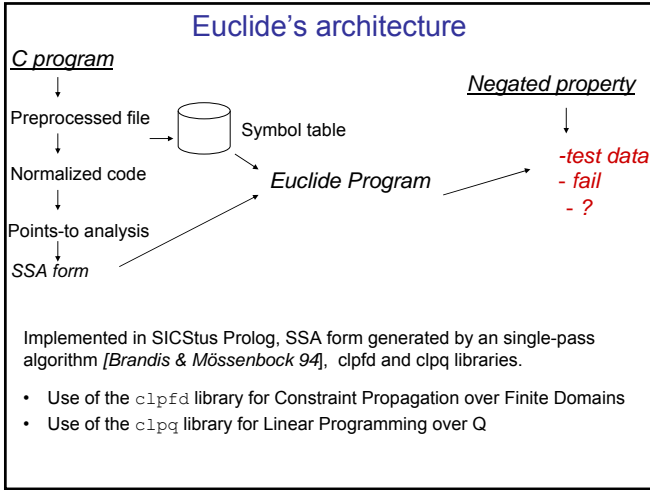


Weak join (Sankaranarayanan et al. VMCAI'06)





- ### Agenda
- Motivations
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-



### First experimental results

Table 2: Results on the verification of safety properties

Num	Results	Time	Mem.
P1a	Property proved	0.7s	4.6Mo
P1b	Property proved	0.7s	4.6Mo
P2a	Property proved	0.6s	4.6Mo
P2b	Counter-example found	0.7s	4.6Mo
P3a	Counter-example found	5.4s	6.3Mo
P3b	Property proved	1.2s	4.6Mo
P4a	Counter-example found	6.8s	6.9Mo
P4b	Counter-example found	2.7s	5.9Mo
P5a	Property proved	0.6s	4.6Mo
P5b	Counter-example found	1.0s	4.6Mo

Intel Core Duo 2.4GHz clocked PC with 2Go of RAM

### And in the Literature !!!

Property	Execution model			Property checking	
	assumptions	Num. of states	Num. of paths	PDL property	Num. of satisfying paths
P1	Up_Separation=Positive_RA_Alt_Thresh and Down_Separation=Positive_FA_Alt_Thresh	4228	58	$\exists p(\text{ASTEa} \rightarrow \text{ASTDownRA})$	0
	Up_Separation=Positive_RA_Alt_Thresh and Down_Separation=Positive_FA_Alt_Thresh	4228	58	$\exists p(\text{ASTEa} \rightarrow \text{ASTUpRA})$	0
P2	Up_Separation=Positive_RA_Alt_Thresh and Down_Separation=Positive_FA_Alt_Thresh and Up_Separation=Down_Separation	4100	62	$\exists p(\text{ASTEa} \rightarrow \text{ASTDownRA})$	0
	Up_Separation=Positive_RA_Alt_Thresh and Down_Separation=Positive_FA_Alt_Thresh and Up_Separation=Down_Separation	4100	62	$\exists p(\text{ASTEa} \rightarrow \text{ASTUpRA})$	0
P3	Up_Separation=Positive_RA_Alt_Thresh and Down_Separation=Positive_FA_Alt_Thresh and Own_Tracked_Alt=Other_Tracked_Alt	2212	38	$\exists p(\text{ASTEa} \rightarrow \text{ASTDownRA})$	0
	Up_Separation=Positive_RA_Alt_Thresh and Down_Separation=Positive_FA_Alt_Thresh and Own_Tracked_Alt=Other_Tracked_Alt	2212	38	$\exists p(\text{ASTEa} \rightarrow \text{ASTUpRA})$	0

Fig. Extracted from « Using Symbolic Execution for Verifying Safety-Critical Systems » ESEC-FSE 2001

Author said :  
 « I think that your analysis of P3a is right. Recently we have redone the TCAS experiment for a workshop paper (attached for your reference) with a different symbolic executor and we found an error in that property too. I did not check your output in detail, but I guess that you bumped in the same error. »



these bugs was previously unreported while the other three had been detected by the developers and fixed.

Finally, we also used MAGIC on the source code of the Air Traffic Collision Avoidance System (TCAS). The source code was obtained from the Siemens suite, a standard set of benchmarks used by the testing and formal verification community. The benchmarks provide both correct and buggy implementations of TCAS. Using assertions, several standard safety properties [56], [57] are hard-coded in the implementation. These safety properties refer to upward/downward resolution advisories, i.e., instructions for an airplane to increase or decrease its altitude. MAGIC successfully detected assertion violations in the buggy implementations and also proved that the correct implementation is safe.

serve as more natural specification mediums for software than logics based purely on either states or events.

There are many interesting research directions for further work. First, we will investigate more powerful abstraction techniques for more precise modeling of the heap and dynamically allocated data structures. Second, we envision an extension of the MAGIC infrastructure to other languages, such as Java and C++. Third, it is important that MAGIC be capable of handling concurrent programs that communicate via shared memory as opposed to message passing. A vast majority of multithreaded C programs fall under this category and we aim at analyzing such programs.

Another important aspect is the analysis of parameterized systems that can consist of an arbitrary number of

Fig. Extracted from « Modular Verification of Software Components in C »

???

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## Further work

- Improving our weak\_join implementation
  - removing spurious equalities
 

```
tmp = ... ,
... = tmp + ...
```

 adds a dimension to the polyhedron !
  - Replacing SICStus clpq library by a verified LP solver (Qsopt\_ex for example [Applegate et al. OR Letters 2007])
- An efficient global constraint for function calls:
  - Abstracting the relations due to function calls (replace the constraints of the callee by a polyhedral abstraction)
- Deal with modular integer computations

Thanks you !