Delay-independent stability criteria for networked control systems

Johannes Nygren, Torbjörn Wigren and Kristiaan Pelckmans [a]

[a] *Uppsala University, SE-75105, Uppsala, SWEDEN*

Abstract

The report analyses a networked control system consisting of a LTI system coupled with a static nonlinearity, subject to large delays in the feedback loop. This model is valid for example in wireless data flow control, where a saturation occurs since the flow is one-directional. The present report extends previous results by proving necessity, in case the loop gain is uniformly less than 1. The results are validated and illustrated in a simulation study.

Key words: Delay; $\mathcal{L}_p$-stability; Networked Control; Nonlinear Control; Saturation;

1 Introduction

Networked control systems (NCS) are becoming increasingly important for optimizing the performance of current and future wireless technologies. As an example, the standardisation of 5G cellular technology is aiming to enable advanced control systems to operate at high bandwidths over new wireless interfaces to the internet [13]. This is manifested e.g. by the focus on delay in the standardization effort as discussed in [4]. The impact of delay on stability of NCSs is the central aspect of the present report.

NCSs are as of now fairly well investigated, at least for the case of linear plants controlled over bandlimited channels. The data rate theorem of NCSs e.g. states the minimum data rate that is needed to stabilize a plant, a result which supports data channel dimensioning [1], [11]. Specific schemes to encode control signals have been developed in [5]. Many applications have been reported over the years. The references [9] and [12] describe power control schemes for wireless systems, while [10] and [18] focus on the effect of delay on data flow control between network nodes. In [17] and [19] a stability analysis inspired by the wireless flow control problem of [18] was performed. A system with general dynamics, subject to long delays and a saturation in the feedback loop was studied. It was proven that unless the static loop gain is bounded and constrained by a relation between the open-loop zeros, the open-loop poles and the slope of the nonlinearity, then $\mathcal{L}_2$ stability *does not follow* from the classical input-output Popov criterion [16]. The very important implication is that stable integrating control may not be feasible for loops dominated by delay in NCS. Rather leaky integration or lead-lag type controllers need to be used, as illustrated e.g. in [18].

The present report focuses on further results on $\mathcal{L}_p$-stability for NCSs subject to long delays and a static nonlinearity in the loop. The main contribution of the report proves equivalence between a number of statements that e.g. imply necessity of the Popov and circle criteria when the loop gain is uniformly less than 1. Although the loop gain is very restrictive performance-wise, the result serves to extend the understanding of e.g. [17]-[19]. Moreover, while earlier work of [17]-[19] concerned saturating nonlinearities, the present report generalises this to nonlinearities satisfying a general sector condition. A further contribution proves that similar necessary and sufficient conditions for $\mathcal{L}_p$-stability hold for the circle criterion, and in the linear case also for the Nyquist criterion. The tools of analysis is

Email address: { `johannes.nygren; torbjorn.wigren; kristiaan.pelckmans` } `@it.uu.se` (Johannes Nygren, Torbjörn Wigren and Kristiaan Pelckmans).

provided by the input-output stability theory as pioneered in [14], [21]. The results are illustrated by a simulation study, using a similar setup as in [18].

The present report is based on the use of linear compensators for control of a nonlinear NCS. This provides controllers with a very low complexity, as required e.g. in [18]. However, this is not an optimal approach. Better performance could therefore be obtained e.g. by employing model predictive control (MPC), accounting for delay [6], [15], [20]. The price would of course be a much higher computational complexity.

There is much prior work on the effect of delay in control systems that is relevant for NCSs: see e.g. [8] for a discussion on robust stabilization, and [2] for a review of delay estimation. To the best of the authors knowledge, prior work does however not arrive at conclusions as those discussed here. The present report is also relevant to other systems than those studied in network control. Other types of flow control constitute one other example as in [7]. Servo control in general is another application since actuators almost always need to be fully utilized to achieve optimal performance [3].

This report is organised as follows. The model that forms the basis for the analysis is introduced in Section 2. The notation and the tools of the stability analysis are defined in Section 3. Section 4 provides the main result and Section 5 provides the proof.

2 System Model

Let $g : [0, \infty) \to \mathbf{R}$ be the impulse response function corresponding to a causal linear time invariant (LTI) system. Consider the corresponding transfer function is

$$G(s) = \int_0^\infty g(\tau) e^{-s\tau} d\tau. \tag{1}$$

This report studies the time delayed system

$$G_T(s) = e^{-Ts} G(s), \tag{2}$$

where $T \geq 0$ is a (time-invariant) delay. The p-operator denotes the derivative of a function, i.e.

$$py(t) = \frac{y(t)}{dt}. \tag{3}$$

Let $y : [0, \infty) \to \mathbf{R}$ and $u : [0, \infty) \to \mathbf{R}$ be continuous-time differentiable signals which are related as

$$\begin{aligned} & y(t) = G_T(p)u(t) \iff \\ & y(t) = \begin{cases} \int_0^{t-\Delta} g(\tau) u\left(t - \tau - T\right) d\tau, & t - \Delta > 0, \\ 0, & t - \Delta \leq 0, \end{cases} \end{aligned} \tag{4}$$

for all $t \geq 0$. Consider a static time-varying nonlineraity $f : \mathbf{R} \times [0, \infty) \to \mathbf{R}$ which satisfies the following conditions:

$$f(0, t) = 0; \quad 0 \leq \frac{f(x, t)}{x} \leq K, \ \forall\, t \in [0, \infty), x \neq 0. \tag{5}$$

The latter condition is referred to a *sector condition* with constant $K \geq 0$. The following closed-loop system is then studied

$$z_{1,T}(t) = G_T(p) \left(u_1(t) + z_{2,T}(t) \right), \tag{6}$$

$$z_{2,T}(t) = f\left(u_2(t) - z_{1,T}(t), t \right), \tag{7}$$

where $z_{1,T}, z_{2,T} : [0, \infty) \to \mathbf{R}$ denote the (delayed) feedback signals before, respectively after applying the static nonlinearity. The signal $u_2 : [0, \infty) \to \mathbf{R}$ denotes often the reference signal, and $u_1 : [0, \infty) \to \mathbf{R}$ can be thought of as the 'disturbance' signal. This system is depicted in Fig. 1.

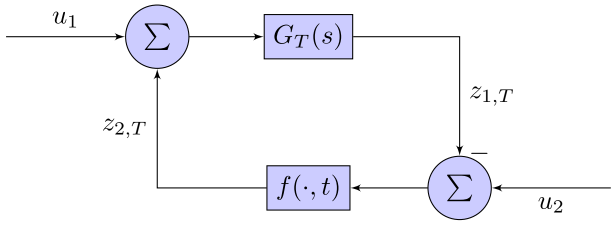

Fig. 1. Block diagram of the closed loop system (6)-(7).

3 Input-output stability

In this section, the input-output stability framework is defined. Then the central assumptions on the linear open loop system is given. The Nyquist criterion for linear closed loop systems, as well as the circle- and Popov criteria for systems with nonlinear feedback, are cited as well.

Definition 1 *For a complex number* $c \in \mathbf{C} \cap \{0\}$, $\arg : \mathbf{C} \cap \{0\} \to \mathbf{R}$ *denotes the multivalued argument* $\arg c = \theta$, *such that* $c = |c|e^{i\theta}$. *The principal value Arg* $: \mathbf{C} \cap \{0\} \to [-\pi, \pi)$ *is the unique* $Arg\ c = \Theta$ *such that* $c = |c|e^{i\Theta}$. *It holds that* $\arg c = Arg\ c + 2\pi n$, *where* $n \in \mathbf{Z}$.

Definition 2 *For a number* $p \in [1, \infty)$, *the set* $\mathcal{L}_p$ *is the set of measurable functions* $y : [0, \infty) \to \mathbf{R}$ *such that*

$$\int_0^\infty |y(t)|^p dt < \infty. \tag{8}$$

The set $\mathcal{L}_\infty$ *is the set of all measurable functions* $y : [0, \infty) \to \mathbf{R}$ *satisfying*

$$\sup_{t\in[0,\infty)} |y(t)| < \infty. \tag{9}$$

Definition 3 *Let* $\mathcal{A}$ *be the set of all functions* $y : [0, \infty) \to \mathbf{R}$ *satisfying*

$$y(t) = \sum_{k=0}^{C} f_k \delta(t - t_k) + y_a(t), \tag{10}$$

where $C \in \{0, 1, 2, \dots\}$ *may be infinite,* δ *is the Dirac delta function,* $0 \le t_0 < t_1 < t_2 < \dots$, $\sum_{k=0}^{\infty} |f_k| < \infty$, *and* $y_a \in \mathcal{L}_1$.

Remark 1 *The set* $\mathcal{A}$ *can be seen as the set of all input-output stable, continuous time impulse responses, since* $g * u \in \mathcal{L}_p$ *for all* $u \in \mathcal{L}_p$ *and all* $p \in [1, \infty]$ *if and only if* $g \in \mathcal{A}$ *([16], Theorem 6.4.30).*

Definition 4 *Let* $\mathbf{\Sigma}_T$ *denote the closed loop mapping from the measurable functions* $u_1 : [0, \infty) \to \mathbf{R}$, $u_2 : [0, \infty) \to \mathbf{R}$ *to the measurable functions* $z_{1,T} : [0, \infty) \to \mathbf{R}$, $z_{2,T} : [0, \infty) \to \mathbf{R}$, *i.e.*

$$(z_{1,T}, z_{2,T}) = \mathbf{\Sigma}_T[u_1, u_2]. \tag{11}$$

The mapping $\mathbf{\Sigma}_T$ *is* $\mathcal{L}_p$-stable *for some* $p \in [1, \infty]$ *if* $u_1, u_2 \in \mathcal{L}_p \implies z_{1,T}, z_{2,T} \in \mathcal{L}_p$.

The following assumptions are made about the transfer function (1):

A1 $g \in \mathcal{A}$,
A2 There exist some w_*, $0 \le \omega_* < \infty$ such that $|G(i\omega_*)| = \sup_{\omega\in\mathbf{R}} |G(i\omega)|$,
A3 $\lim_{\omega\to\infty} G(i\omega) = G_\infty : |G_\infty| < 1/K$, where K defines the sector condition (5).

Remark 2 *Assumption A1 says that* $G(s)$ *is an* $\mathcal{L}_p$*-stable transfer function which is possibly non-rational. Assumption A2 says that the supremum of* $|G(i\omega)|$ *can be found at a point with finite* ω. *Intuitively, this means that the supremum is found after a finite number of rotations around the origin. Note that if* $G(s)$ *is strictly proper and A1 holds, A2 and A3 follows directly since then* $\lim_{\omega\to\infty} |G(i\omega)| = 0$.

Before proceeding to the next section, three stability criteria are cited; the circle criterion for non-linear and possibly time-varying feedback, the less conservative Popov criterion for non-linear and time-invariant feedback, and the necessary and sufficient Nyquist criterion for linear feedback systems. The main result of this report, given in the next section, states that these three criteria are in fact equivalent in the high-delay limit.

Theorem 3 : Nyquist theorem for stable open loop systems. *Consider the system (6)-(7) with linear feedback, i.e.* $f(x,t) = Kx$. *Assume that A1 holds. Then the closed loop system is* $\mathcal{L}_p$*-stable for all* $p \in [1,\infty]$ if and only if *the Nyquist plot* $G_T(iw)$, $\omega \in [0,\infty)$ *does not cross or encircle the point* $-1/K$ *(where clockwise and anti-clockwise encirclements sum up with different signs).*

Theorem 4 : Circle criterion. *Consider the system (6)-(7) with sector condition (5). Assume that A1 holds. Then the closed loop system is* $\mathcal{L}_p$*-stable for all* $p \in [1,\infty]$ *if*

$$1/K + Re\,[G_T(i\omega)] > 0 \tag{12}$$

for all $\omega \in [0,\infty)$.

Theorem 5 : Popov criterion. *Consider the system (6)-(7) with sector condition (5). Assume that both* $g, pg \in \mathcal{A}$. *Also assume that the sector condition is time-invariant, and that the external inputs satisfy* $u_1 \in \mathcal{L}_2$, $u_2 \in \mathcal{L}_2$ *and* $pu_1 \in \mathcal{L}_2$. *If* $\exists\, r \geq 0$ *such that*

$$1/K + Re\,[(1 + i\omega r)G_T(i\omega)] > 0, \tag{13}$$

then $z_{1,T} \in \mathcal{L}_2$ *and* $z_{2,T} \in \mathcal{L}_2$.

4 Main Result

Theorem 6 *Consider a transfer function (1) which satisfies A1-A3. Define the time delayed system (2) where* $T \geq 0$. *Also consider the sector condition (5) with a given* $K \geq 0$. *Then the following statements are equivalent for any* $T_{\min} \geq 0$

(1) $|G(i\omega)| < 1/K$ *for all* $\omega \neq 0$ *and* $-1/K < G(0) \leq 1/K$.
(2) For all $T \geq T_{\min}$, $1/K + Re\,[G_T(i\omega)] > 0\ \forall\, \omega \in \mathbf{R}$ *(circle criterion).*
(3) For all $T \geq T_{\min}$, $\exists r \geq 0$ *such that* $1/K + Re\,[(1 + i\omega r)G_T(i\omega)] > 0$, $\forall\, \omega \in \mathbf{R}$ *(Popov criterion).*
(4) The closed loop system (6)-(7) is $\mathcal{L}_p$*-stable for all* $p \in [1,\infty]$, *all nonlinearities* $f : \mathbf{R} \times [0,\infty) \to \mathbf{R}$ *satisfying the sector condition (5), and all* $T \geq T_{\min}$.
(5) The closed loop system (6)-(7) subject to linear feedback, i.e. $f(x,t) = Kx$ *for all* $t \in [0,\infty)$, *is* $\mathcal{L}_p$*-stable for all* $p \in [1,\infty]$ *and all* $T \geq T_{\min}$.

Remark 7 *In order to understand what purpose* $T_{\min}$ *serves in the theorem, first note that statement 1 does not depend on* $T_{\min}$. *This means that if statements 2-5 hold for* some $T_{\min} \geq 0$, *they hold for* all $T_{\min} \geq 0$. *Hence there cannot be a situation where the closed loop system is stable for all* $T \geq T_{\min}$, *and unstable for some* T *smaller than* $T_{\min}$, *where* $T_{\min}$ *is any finite number. The whole point of having* $T_{\min}$ *in the theorem is to state this fact.*

Remark 8 *Theorem 6 has many peculiarities; e.g equivalence between the Popov and circle criteria, that the Popov criterion imply* $\mathcal{L}_p$*-stability for all* $p \geq 1$ *(and not just* $\mathcal{L}_2$*-stability as in Theorem 5), and furthermore, that the circle- and Popov-criteria are both necessary and sufficient for* $\mathcal{L}_p$*-stability. The key to these peculiar equivalence relations is that the statements hold for all delays* $T \geq T_{\min}$. *The price of delay-independent stability is however quite high, since statement 1 is very conservative. Note that statement 1 is essentially the same as the criterion appearing in the well-known small gain theorem [16].*

5 Proof

The proof is divided into parts as follows. First, it is proven that statement 1 implies statements 2 and 3. Then, it is shown that 2 implies 1, and the same technique is re-used to prove that 3 implies 1. In the last part, it is proven that 2 implies 4 and 5, and that 5 implies 1. The equivalence of statements 1 to 5 then follows. The structure of the proof is illustrated in Figure 2. Note that the implication $2 \implies 1$ is redundant in proving the overall equivalence. However, it is included anyway since the proof of the implication $3 \implies 1$ relies heavily on the proof of $2 \implies 1$.

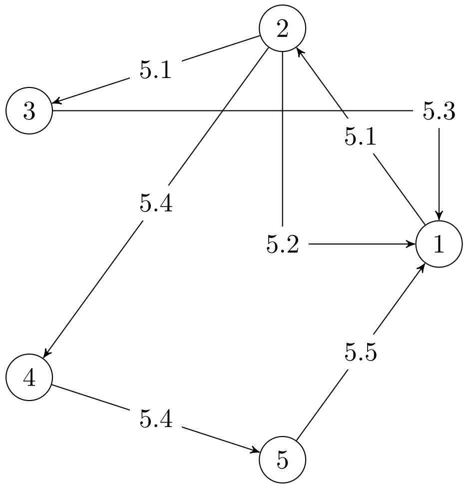

Fig. 2. Structure of the proof of Theorem 6. Balls with numbers indicate statements $1-5$, and arrows represent implication. Arrow labels represent subsections of the proof.

5.1 $(1 \implies 2 \implies 3)$

Statement 2 is implied by statement 1 from the fact that $|G(i\omega)| = |G_T(i\omega)|$ for all $\omega \in \mathbf{R}$ and all $T \geq 0$. Specifically,

$$\begin{aligned} &1/K + \text{Re}\,[G_T(i\omega)] \geq 1/K - |\text{Re}\,[G_T(i\omega)]| \geq \\ &1/K - |G_T(i\omega)| = 1/K - |G(i\omega)| > 1/K - 1/K = 0. \end{aligned} \tag{14}$$

Statement 3 follows directly from statement 2 by choosing $r = 0$.

5.2 $(2 \implies 1)$

Define $\phi_T(\omega) = \text{Arg}\ G_T(i\omega)$. It follows that

$$\arg G_T(i\omega) = \arg G(i\omega) - T\omega = \phi_T(\omega) + 2\pi m, \tag{15}$$
$$\arg G(i\omega) = \text{Arg}\ G(i\omega) + 2\pi k, \tag{16}$$

where $m \in \mathbf{Z}$ and $k \in \mathbf{Z}$. Introduce $n = k - m$. For all $\omega \neq 0$ it holds that

$$T = (\text{Arg}\ G(i\omega) - \phi_T(\omega) + 2\pi n)/\omega. \tag{17}$$

For a given $\phi_T(\omega)$ and $\omega \neq 0$, it is seen that T has an infinite amount of solutions since n can be any integer. It also follows from the definition of the principal value that T is such that $\phi_T(\omega) \in [-\pi, \pi)$. Focusing on the case $\phi_T(\omega) = -\pi$, this above argumentation now proves the following lemma.

Lemma 9 *Consider a transfer function $G(s) = \int_0^\infty p(\tau)e^{-s\tau}d\tau$ which satisfies A1, and let $0 < \omega_* < \infty$. Define the time delayed system $G_T(s) = e^{-Ts}G(s)$ where $T \geq 0$. Then, it holds that $G_T(i\omega_*) = -|G(i\omega_*)|$ for all $T \geq 0$ belonging to the following set $\mathcal{S}_G^{\omega_*}$*

$$\mathcal{S}_G^{\omega_*} = [0, \infty) \cap \{((1 + 2n)\pi + \mathit{Arg}\ G(i\omega_*))/\omega_* : n \in \mathbf{Z}\}. \tag{18}$$

Remark 10 *The statement $G_T(i\omega_*) = -|G(i\omega_*)|$ is exactly the same as saying that $\phi_T(\omega_*) = -\pi$, since $|G(i\omega)| = |G_T(i\omega)|$ for all $\omega \in \mathbf{R}$ and all $T \geq 0$.*

The proof of ($2 \implies 1$) then proceeds. From A2, it follows that there is a ω_*, $0 \le \omega_* < \infty$ such that $|G(i\omega_*)| = \sup_{\omega \ge 0} |G(i\omega)|$. By an argument of contradiction, assume that $\omega_* \ne 0$ and

$$1/K \le |G(i\omega_*)| = |G_T(i\omega_*)|. \tag{19}$$

From Lemma 9, it follows that $G_T(i\omega_*) = -|G(i\omega_*)|$ for all $T \in \mathcal{S}_G^{\omega_*}$. Hence, Re $[G_T(i\omega_*)] = -|G(i\omega_*)| \le -1/K$ which implies that $1/K+$Re $[G_T(i\omega_*)] \le 0$ for each $T \ge T_{\min}$ belonging to the set $\mathcal{S}_G^{\omega_*}$. Note also that the intersection between $\mathcal{S}_G^{\omega_*}$ and $\{T : T \ge T_{\min}\}$ is non-empty for any given $T_{\min}$, since Arg $G(i\omega_*)$ is finite and n is any number in $\mathbf{Z}$.

From the contradiction it is concluded that statement 2 implies that there is no $\omega_* \ne 0$ such that (19) holds. If there is no such ω_*, then $|G(i\omega)| < 1/K$ for all $\omega \ne 0$ and, by continuity in ω, $-1/K \le G(0) \le 1/K$.

It remains to show that statement 2 also implies that $-1/K < G(0)$, and this is done by an argument of contradiction as well. Suppose that $G(0) = -1/K$. Equation (15) then implies that $\arg G_T(0) = \arg G(0)$ and hence, $G_T(0) = -1/K$ for all $T \ge 0$. Therefore, $1/K +$ Re $[G_T(0)] = 0$ for all $T \ge T_{\min}$, implying that statement 2 does not hold. From the contradiction it is concluded that ($2 \implies 1$).

The implication ($2 \implies 1$) is used in the next subsection, and is therefore formulated in the following Lemma for general transfer functions $P(s)$. Note that assumption A3 is not needed here.

Lemma 11 *Consider a transfer function $P(s) = \int_0^\infty p(\tau)e^{-s\tau}d\tau$ which satisfies A1 and A2. Define the time delayed system $P_T(s) = e^{-Ts}P(s)$ where $T \ge 0$. Let $K \ge 0$ be given. If there is a $T_{\min} > 0$ such that $1/K +$ Re $[P_T(i\omega)] > 0$ for all $T \ge T_{\min}$, $\omega \in \mathbf{R}$ (statement 2), then $|P(i\omega)| < 1/K$ for all $\omega \ne 0$ and $-1/K < P(0) \le 1/K$ (statement 1).*

5.3 ($3 \implies 1$)

Statement 3 for the special case $r = 0$ is exactly the same as statement 2. Therefore, assume that statement 3 holds for some $r > 0$ and define $G_{r,T}(s) = (1 + sr)G_T(s)$. This is exactly the same as assuming that statement 2 holds for $G_T(i\omega)$ replaced by $G_{r,T}(i\omega)$. Using Lemma 11 with $P(s) = G_{r,0}(s)$, it follows that $|G_{r,0}(i\omega)| < 1/K$ for all $\omega \ne 0$ and $-1/K < G_{r,0}(0) \le 1/K$.

Furthermore, it holds that $|G_{r,T}(i\omega)| = |1+i\omega r||G_T(i\omega)|$ for all $\omega \in \mathbf{R}$ and all $T \ge 0$. Therefore, $|G_{r,T}(i\omega)| \ge |G_T(i\omega)|$ for all $r \ge 0$, $\omega \in \mathbf{R}$ and all $T \ge 0$. Specifically, this implies that $|G(i\omega)| \le |G_{r,0}(i\omega)| < 1/K$ for all $\omega \ne 0$. From the definition it directly follows that $G_{r,0}(0) = G(0)$, implying that $-1/K < G(0) \le 1/K$. Statement 1 then follows.

5.4 ($2 \implies 4 \implies 5$)

If 2 holds, statement 4 is implied by the circle stability criterion, see Theorem 4. If 4 holds, 5 follows directly since the linear feedback $f(x, t) = Kx$ is a special case of the sector condition (5).

5.5 ($5 \implies 1$)

This is shown by an argument of contradiction. Suppose that statement 1 does not hold and that f is a linear gain, i.e. $f(x, t) = Kx$. The closed loop system (6)-(7) can then be rewritten in the following way

$$z_{1,T}(t) = S_T(p)\{G_T(p)u_{1,T}(t) + KG_T(p)u_{2,T}(t)\}, \tag{20}$$
$$z_{2,T}(t) = S_T(p)\{Ku_{2,T}(t) - KG_T(p)u_{1,T}(t)\}, \tag{21}$$
$$S_T(s) = 1/(1 + KG_T(s)). \tag{22}$$

Since statement 1 does not hold, it follows by continuity and assumption A3 that there is either a $\omega_0 \ne 0$ such that $|G(i\omega_0)| = 1/K$ or a $\omega_0 = 0$ such that $G(i\omega_0) = G(0) = -1/K$.

If $\omega_0 = 0$, then $G_T(i\omega_0) = G_T(0) = -1/K$ for all $T \ge 0$. Equation (22) implies that the transfer function $S_T(s)$ has a singularity at $s = 0$. Therefore, $S_T(s)$ is not L^2-stable, meaning that statement 5 does not hold.

If $\omega_0 \neq 0$, then it can be concluded (using Lemma 9 with $\omega_* = \omega_0$) that $G_T(i\omega_0) = -|G(i\omega_0)| = -1/K$ for all $T \in \mathcal{S}_G^{\omega_0}$. For these T, it follows from (22) that $S_T(s)$ has singularities in $s = \pm i\omega_0$, which implies that $S_T(s)$ is not L^2-stable. Therefore, statement 5 does not hold. By contradiction, it is concluded that statement 5 implies statement 1.

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