Nonlinear system identification of the dissolved oxygen to effluent ammonium dynamics in an activated sludge process

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Abstract: Aeration of biological reactors in wastewater treatment plants is important to obtain a high removal of soluble organic matter as well as for nitrification but requires a significant use of energy. It is hence of importance to control the aeration rate, for example, by ammonium feedback control. The goal of this report is to model the dynamics from the set point of an existing dissolved oxygen controller to effluent ammonium using two types of system identification methods for a Hammerstein model, including a newly developed recursive variant. The models are estimated and evaluated using noise corrupted data from a complex mechanistic model (Activated Sludge Model no.1). The performances of the estimated nonlinear models are compared with an estimated linear model and it is shown that the nonlinear models give a significantly better fit to the data. The resulting models may be used for adaptive control (using the recursive Hammerstein variant), gain-scheduling control, $\mathcal{L}_2$ stability analysis, and model based fault detection.

Keywords: System identification, Output error model, Hammerstein model, Wastewater treatment, BSM1.

1. INTRODUCTION

Nonlinear system identification has been studied in many application areas, including neuron models and networks Wigren (2015b), heating systems, Brus (2005), wastewater treatment plants, Ekman (2008), power system control, Åström and Bell (2000), selective catalytic reduction systems, Tayamon and Wigren (2016), and numerous other applications. Along with the state of the art methods implemented in the MATLAB® System Identification Toolbox, a variety of approaches have been developed and implemented as black-box and grey-box identification methods, see e.g. Mattsson and Wigren (2016), Wigren (2006), Pintelon and Schoukens (2012).

Modelling of block oriented systems is an important class within the system identification area. Such systems are used when the input-output relationship can be described by a combination of transfer functions with nonlinear static functions, Ljung (1999) and Giri and Bai (2010). The location of the nonlinear block then defines a model structure: if a linear block follows a static nonlinearity, such models are defined as Hammerstein models, and as Wiener models if vice versa. Also, the combination of both is used, i.e. Hammerstein-Wiener models.

The process of wastewater treatment (WWT) includes a large number of time-varying elements, like different flow rates and flow compositions, time delays, disturbances, and nonlinearities, Olsson and Newell (1999). Such a process complexity may cause problems while controlling and monitoring a plant. One approach is then to create a reduced mathematical model of the process which allows a simplified understanding of nonlinearities and their relation to different control strategies.

The focus of this report is to compare an off-line method and a newly developed recursive method for identification of a Hammerstein model with an affine static input nonlinearity for modelling the nitrification dynamics in an Activated Sludge Process (ASP). To the best of the authors knowledge, Hammerstein modelling has not been applied to this type of process before.

The contribution shows that the Hammerstein model is a suitable model for this task, and that it is likely to be applicable for future controller design and stability analysis of the system. These models can then be used to design a nonlinear controller which can be analysed using the Popov criterion for input-output stability and robustness, Chistiakova et al. (2018). The differences in terms of the practical implementation of the two different Hammerstein based identification methods is pointed out.

The report is organised as follows. Section 2 gives a description of a high fidelity wastewater treatment simulator and presents the motivation for the use of system identification. Methods used for identification are described in Section 3. The identification results are given in Section 4, followed by conclusions in Section 5.

2. AMMONIUM-BASED FEEDBACK CONTROL AND MODELLING OF THE ACTIVATED SLUDGE PROCESS

2.1 Overview

WWT involves a number of complex, nonlinear processes, which control and evaluation is a challenging and demanding task. A typical WWT plant consists of several steps including mechanical, biological and chemical treatment. In this study, the biological treatment is of particular interest.

During the biological treatment, nitrogen and organic materials are removed from the water using microorganisms. A typical process for biological treatment is the ASP, where oxygen supply via aeration is essential for the microorganisms to convert ammonium to nitrate (nitrification), Gerardi (2003). The concentration of Dissolved Oxygen (DO) should be high enough to give a certain removal rate, but a too high DO level requires a high energy consumption and gives little increase in the removal rate due to the inherent nonlinearity of the nitrification process. A classical control strategy is therefore to keep the DO level close to a given and fixed set-point, Olsson et al. (2005). It has also been found that the use of ammonium feedback to control the DO set-point can increase the energy efficiency without producing a higher effluent ammonium load, see Åmand (2014) for some recent results.

2.2 Simulation model

The simulation model used in this study is depicted in Fig. 1. It is a simplified version of the Benchmark Simulation Model no.1 (BSM1), which is a simulation model for evaluating and testing various control strategies, parameter variations and models. BSM1 follows the characteristics of a real plant and is widely used for analysis, Alex et al. (2008).The original BSM1 simulates five biological reactors and a clarifier, for this case study the number of reactors is reduced to one. The volume of the reactor was 3999 m^3, Q_{in} = 18446 m^3/day, $NH_{4,in}$ = 31.5 mg/l, $RAS = Q_{in}$ and WAS was selected so that the sludge retention time was 4 days.

The dynamics of the benchmark model is based on the Activated Sludge Model no.1 (ASM1). ASM1 was created by the International Water Association (IWA) task group in 1983. The idea was to design and promote a simple but still accurate mathematical model for nitrification, carbon oxidation and denitrification in an ASP. The ASM1 contains 13 process variables, 19 model parameters and 8 process equations, Henze (1987). Still today, the ASM1 remains the most commonly used model for dynamical modelling of ASPs.

2.3 Hammerstein model

The dissolved oxygen dynamics in the ASP is nonlinear. To achieve a good controller performance for all working conditions, a simple nonlinear dynamic model is calibrated to data in this report.

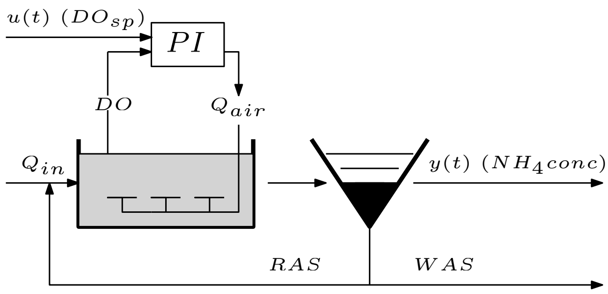

Fig. 1. Simulation model. Q_{in}-inflow, $u(t)$-input signal (DO_{sp}-DO set point), PI-proportional integral DO controller, Q_{air}-airflow rate, $y(t)$-output signal (NH_4conc-ammonium concentration) RAS-return activated sludge, WAS-waste activated sludge.

A nonlinear Hammerstein model is represented by a combination of a nonlinear static and a linear dynamic submodel, see Fig. 2.

The nonlinear block is defined as a static nonlinear function

$$u_n(t, \theta_n) = f(u(t), \theta_n), \tag{1}$$

where θ_n is a vector of unknown parameters and t is discrete time. Then the output $\hat{y}(t, \theta)$ of the model can be described as

$$\hat{y}(t, \theta) = \frac{B(q^{-1})}{A(q^{-1})} u_n(t, \theta_n), \tag{2}$$

where $B(q^{-1})$ and $A(q^{-1})$ are polynomials in the backward shift operator q^{-1} given by

$$B(q^{-1}) = b_0 q^{-n_k} + ... + b_{n_b-1} q^{-n_k-n_b+1}, \tag{3}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_{n_a} q^{-n_a}, \tag{4}$$

and n_k is a time delay specified by the number of samples before the output reacts to the input.

The parameters of the linear block are collected in

$$\theta_l = [a_1 \ \cdots \ a_{n_a} \ b_1 \ \ldots \ b_{n_b}]^\mathsf{T}. \tag{5}$$

The vector θ contains the unknown parameters, i.e.

$$\theta = \begin{bmatrix} \theta_n \\ \theta_l \end{bmatrix}. \tag{6}$$

Note that (2) can be expressed as

$$\hat{y}(t) = \theta_l^\mathsf{T} \varphi(t, \theta), \tag{7}$$

where

$$\varphi(t, \theta) = [-\hat{y}(t-1, \theta) \quad \cdots \quad -\hat{y}(t-n_a, \theta) \\ u_n(t-n_k, \theta_n) \quad \cdots \quad u_n(t-n_k-n_b+1, \theta_n)]^\mathsf{T}. \tag{8}$$

In this contribution, the static function in (1) is modeled as a piecewise affine function. These functions are known for their universal approximation properties, see e.g. Breiman

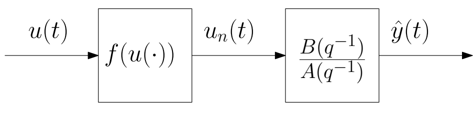

Fig. 2. Hammerstein model.

(1993) and Lin and Unbehauen (1992), and are therefore popular in system identification. Let $\mathcal{G} = [u_1 \cdots u_{n_r}]$ be a set of grid points. Then any function that is piecewise affine between consecutive grid points can be written as

$$f(u, \theta_n) = \sum_{i=1}^{n_r} r_i f_i(u, \mathcal{G}) = \theta_n^{\mathsf{T}} F(u, \mathcal{G}), \tag{9}$$

where $f_i(u, \mathcal{G})$ are piecewise affine basis functions, and

$$\theta_n = [r_1 \ \ldots \ r_{n_r}]^{\mathsf{T}}, \tag{10}$$
$$F(u, \mathcal{G}) = [f_1(u, \mathcal{G}) \ \cdots \ f_{n_r}(u, \mathcal{G})]. \tag{11}$$

Hence, if the grid points $\mathcal{G}$ are fixed, then the nonlinear function is linear in the unknown parameters. For an example of how the basis functions can be constructed, see e.g. Mattsson and Wigren (2016). It is stressed that both evaluated Hammerstein models utilize piecewise affine parametrizations.

2.4 Possible application of the Hammerstein model for ammonium feedback control

Using a Hammerstein model, a main idea of the nonlinear feedback controller design is to introduce global stability by making use of input-output stability theory, Vidyasagar (1978). A lag controller can then be applied so that the Popov criterion is met, cf. Wigren (2015a). Alternatively, a leakage can be added to the integrator of a PI-controller. The controller may therefore be designed as

$$C(s) = K_P + K_I \frac{1}{s + \alpha}, \tag{12}$$

where K_P is a proportional gain, K_I an integral gain and α is a leakage term.

The block diagram of this control system is shown in Fig. 3. The controller $C(s)$ operates on the control error $e(s)$ to give a reference value for the DO level, denoted $u(s)$ in the frequency domain. The saturated output signal $\bar{u}(s)$ corresponds to $u_n(t)$ in discrete time in (1).

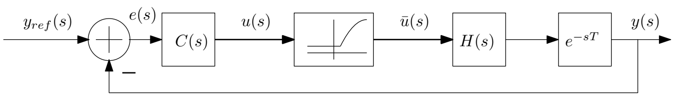

Fig. 3. Block diagram of the control system.

The control signal represented by the DO set point is constrained to be between given minimum and maximum values. The imposed limitations together with the plant model nonlinearity hence give the following saturation type static nonlinearity

$$\bar{u} = \zeta(u) = \begin{cases} f(u_{max}), & u \geq u_{max} \\ f(u), & u_{min} < u < u_{max} \\ f(u_{min}), & u \leq u_{min}. \end{cases} \tag{13}$$

The saturated signal $\bar{u}(s)$ describes the DO-concentration set point. The linear part of the plant dynamics, $H(s)e^{-sT}$, models the dynamic effect of the DO on the ammonium concentration effluent water, denoted $y(s)$. To close the feedback loop, $y(s)$ needs to be measured.

As a time delay in a continuous time system is infinite dimensional, input-output stability theory is suitable to analyse the control system. Since the linear part of the loop gain is given by

$$g(s) = e^{-sT} C(s) H(s), \tag{14}$$

it follows that $g(s)$ has all poles strictly in the interior of the left half of the complex plane provided that $H(s)$ is asymptotically stable. The conclusion is that the Popov criterion is applicable, see Theorem 6.7.63 in Vidyasagar (1978). The details of this controller design and the associated stability results are, however, not within the scope of this report and they are investigated in Chistiakova et al. (2018).

3. IDENTIFICATION METHODS

The scope of the report is to apply and compare one linear and two Hammerstein based identification methods in order to describe the DO to ammonium dynamics in the ASP. These methods are presented next.

3.1 The linear output error method

For identifying the linear output error (OE) model, i.e.(2) with $u_n(t, \theta_n) = u(t)$, the MATLAB® function *oe* was used for the Gauss-Newton optimization. See Ljung (2011) for a detailed explanation of this function.

3.2 The off-line prediction error method for the Hammerstein model

The off-line prediction error method (PEM), see e.g. Ljung (1999) and Söderström and Stoica (1989), can be applied to quite arbitrary model parameterizations, and is one of the most popular methods in system identification. In PEM, with a quadratic criterion, the goal is to minimize the cost function

$$V(\theta) = \frac{1}{N} \sum_{t=1}^{N} \varepsilon^2(t, \theta), \tag{15}$$

where

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t, \theta) \tag{16}$$

is the prediction error.

In the numerical results in Section 4.2 the Gauss-Newton method is applied in order to minimize the non-convex function $V(\theta)$. That is, the algorithm is started with an initial estimate $\hat{\theta}^{(0)}$, and iterated according to

$$\hat{\theta}^{(k+1)} = \hat{\theta}^{(k)} + \alpha_k R^{-1}(\hat{\theta}^{(k)}) \sum_{t=1}^{N} \psi(t, \hat{\theta}^{(k)}) \varepsilon(t, \hat{\theta}^{(k)}), \tag{17}$$

where α_k is the step-length, $\psi(t, \hat{\theta}^{(k)})$ is the negative gradient of $V(\theta)$ and $R(\theta)$ is given by

$$R(\theta) = \frac{1}{N} \sum_{t=1}^{N} \psi(t, \theta) \psi^{\mathsf{T}}(t, \theta). \tag{18}$$

In the numerical examples, the MATLAB® function *nlhw*, see Ljung (2011) for more details, with piecewise affine nonlinearity was used for performing the Gauss-Newton optimization for the off-line PEM. This function also optimizes the location of the grid points which are values of the nonlinear function given the input. The negative gradient is then extended with derivatives with respect to the grid points as in Abd-Elrady (2002). This is a main difference as compared to the method of Section 3.3.

3.3 The recursive prediction error method for the Hammerstein model

In Mattsson and Wigren (2014), a recursive prediction error method (RPEM) for Hammerstein identification was developed by using running estimates of the quantities of (17). A recursive identification method opens up the possibilities for using adaptive control strategies, and also for performing low complexity online fault detection. In Mattsson and Wigren (2014), it was shown that the negative gradient for the Hammerstein model with fixed grid points in the nonlinearity, can be computed as

$$\psi(t,\theta) = \left[\frac{d}{d\theta}\hat{y}(t,\theta)\right]^{\mathsf{T}} = \begin{bmatrix} \dfrac{B(q^{-1})}{A(q^{-1})}F(u(t),\mathcal{G}) \\ \dfrac{1}{A(q^{-1})}\varphi(t,\theta) \end{bmatrix}. \quad (19)$$

The resulting algorithm can be summarized as follows

$$\begin{aligned}
\varepsilon(t) &= y(t) - \hat{y}(t) \\
R(t) &= R(t-1) + \frac{1}{t}\left(\psi(t)\psi^{\mathsf{T}}(t) - R(t-1)\right) \\
\hat{\theta}(t) &= \hat{\theta}(t-1) + \frac{1}{t}R^{-1}(t)\psi(t)\varepsilon(t) \\
\varphi(t+1) &= [-\hat{y}(t) \quad \cdots \quad -\hat{y}(t-n_a+1) \\
&\qquad f(u(t-n_k+1),\hat{\theta}(t)) \quad \cdots \\
&\qquad f(u(t-n_k-n_b+2),\hat{\theta}(t))]^{\mathsf{T}} \qquad (20) \\
\psi(t+1) &= \begin{bmatrix} \dfrac{\hat{B}(q^{-1},t)}{\hat{A}(q^{-1},t)}F(u(t+1),\mathcal{G}) \\ \dfrac{1}{\hat{A}(q^{-1},t)}\varphi(t+1) \end{bmatrix} \\
\hat{y}(t+1) &= \hat{\theta}_l^{\mathsf{T}}(t)\varphi(t+1).
\end{aligned}$$

In order to avoid the computation of $R^{-1}(t)$ in each time-step of the algorithm, it is possible to apply the matrix inversion lemma and compute $P(t) = \frac{1}{t}R^{-1}(t)$ recursively instead, see Mattsson and Wigren (2014) for details about the implementation of the algorithm. The algorithm (20) makes use of fixed preselected grid points, where the first grid value is the function value and the remaining grid points are derivatives of the function. One derivative is locked to fix the overall static gain.

4. NUMERICAL EXAMPLES

4.1 Simulated data

The high fidelity simulator described in Section 2.2 was used for data generation. The DO set point, $u(t)$, was generated as a pseudorandom binary sequence (PRBS) multiplied by a uniformly distributed random process. The so obtained sequence is hence a sequence where each constant interval is multiplied with a uniformly distributed amplitude, see Wigren (2003) for more details.

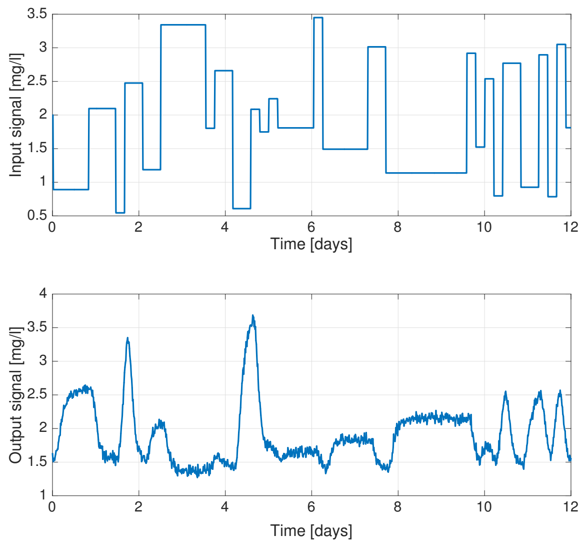

Fig. 4. An example of simulated data.

The generated signal is characterized by a mean, a maximum amplitude and a minimum interval length, which is the duration of a certain input signal level. Due to system characteristics, the mean value is determined to be 2 g/m^3, which is the set point for DO, and since the maximum amplitude can not be higher than the mean value to avoid negative values, it is set to 1.5 g/m^3. The output signal, $y(t)$, is an effluent concentration level of ammonium NH_4.

For the numerical experiment, the plant was simulated using the simplified BSM1 and a constant influent flow. The simulation time was 12 days with 15 minutes sampling time. The noise added to the output signal was a white noise process with standard deviation of 0.1 g/m^3. An example of the generated input and output data sets used for identification is shown in Fig. 4.

Prior to the identification procedure, mean values of the input and output signals were removed in order to obtain the model

$$y(t) - \bar{y} = \frac{B(q^{-1})}{A(q^{-1})}f(u(t)-\bar{u}), \quad (21)$$

where $y(t)$ is the output, $u(t)$ is the input, $\bar{y}$ and $\bar{u}$ are the corresponding means. Therefore, the output signal is reconstructed as

$$y(t) = \bar{y} + \frac{B(q^{-1})}{A(q^{-1})}f(u(t)-\bar{u}). \quad (22)$$

4.2 Identification results

During the identification, several model orders and initial parameters were tested experimentally and the best obtained estimation models were used for a comparison. For the linear OE model and for linear parts of the Hammerstein identification methods, the polynomial orders for (3) and (4) were chosen to be $n_b = 2$ and $n_a = 2$. The time delay was $n_k = 12$.

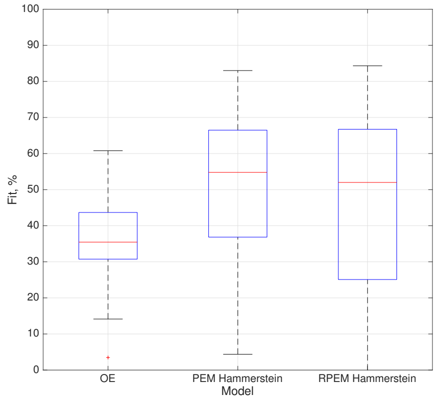

Fig. 5. Fit to the validation data for 100 realizations.

The model parameters were evaluated for 100 different realizations of input-output data. Independently generated validation data sets were used for the performance assessment. The results are given as a fit of the estimated output to the real output, (23),

$$fit = 100 \cdot \left(1 - \frac{||y - \hat{y}||}{||y - \bar{y}||}\right), \tag{23}$$

where y is the output of the validation data, $\hat{y}$ is the output produced by the estimated model and $\bar{y}$ is the mean value of the validation output data.

Fig. 5 shows the distribution of the fit for linear OE model, PEM and RPEM Hammerstein models from 100 signal realizations. The linear OE identification method has a relatively poor performance with a median value of 35.5%, while the identified nonlinear Hammerstein models provide a better fit with median values around 55%.

The validation results for a data set realization where all three models provided a reasonable estimation is presented in Fig. 6. In this case, the fit to the validation data for the linear OE identification model is 41.3%. The modelling error is significant for the linear OE model while nonlinear models show more accurate prediction of output signals and handle the noise and amplitude variations well. The fit is 71.4% for PEM Hammerstein model and 73.1% for RPEM Hammerstein model.

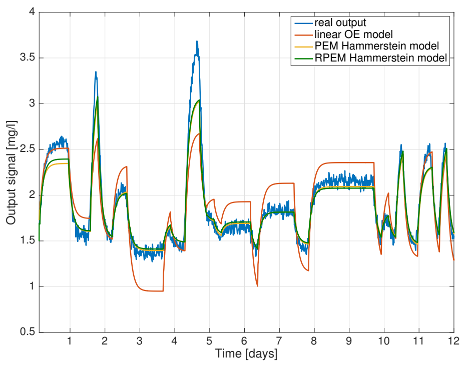

Fig. 6. Example of the validation results.

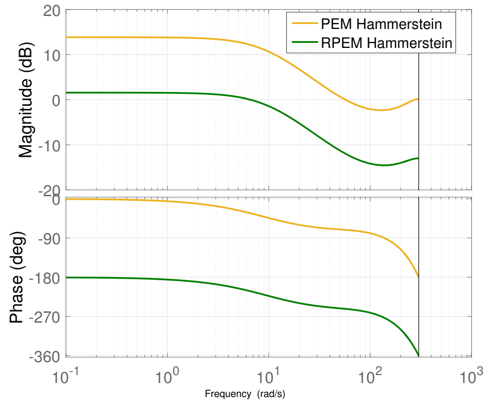

Fig. 7. Bode plot of the linear part obtained with PEM and RPEM Hammerstein models.

An example of the estimated linear blocks obtained by the PEM and RPEM Hammerstein models is given by the transfer function operators

$$H_{PEM}(q^{-1}) = q^{-12}\frac{1 - 0.4q^{-1}}{1 - 0.6q^{-1} - 0.3q^{-2}}, \tag{24}$$

$$H_{RPEM}(q^{-1}) = -q^{-12}\frac{0.2 - 0.08q^{-1}}{1 - 0.7q^{-1} - 0.2q^{-2}}, \tag{25}$$

where q^{-12} corresponds to the output delay q^{-n_k}.

Fig. 7 shows the Bode plot of the linear block of the identified models.

For the PEM Hammerstein model, nonlinear function values were estimated. For the RPEM Hammerstein model, derivatives and one function value were estimated, see Mattsson and Wigren (2014).

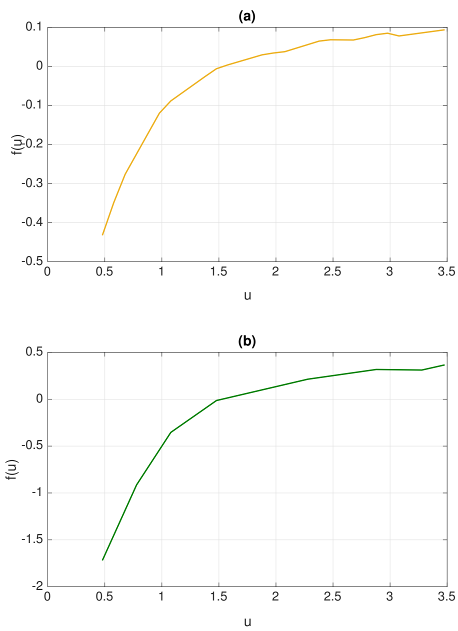

Fig. 8. Static nonlinearity obtained with the PEM (a) and RPEM (b) Hammerstein models.

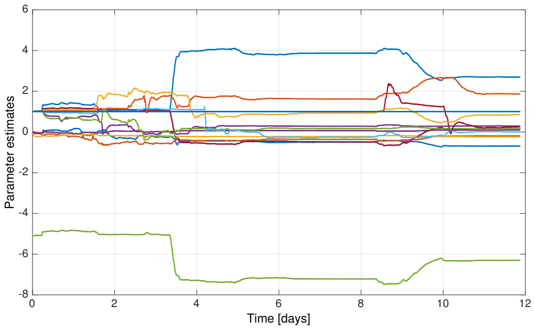

Fig. 9. Parameter estimates obtained with the RPEM Hammerstein model.

The static nonlinearities identified by the algorithms are shown in Fig. 8. The behaviour of the identified nonlinearities are similar. In these plots, the negative gain of the identified plant model has been moved to the linear block for the PEM method so that the shape and differential gain of the PEM and RPEM can be easily compared.

The parameter estimates for the RPEM Hammerstein model as functions of time are shown in Fig. 9.

In summary, the obtained results show that a fairly accurate identification of the nonlinear dynamics of the WWT plant can be obtained by application of Hammerstein identification techniques.

5. CONCLUSIONS

Three system identification methods have been compared: a linear output error method, a nonlinear Hammerstein identification algorithm from the MATLAB® System Identification Toolbox and a recursive prediction error Hammerstein identification algorithm.

The methods were used to identify the dissolved oxygen to effluent ammonium dynamics using noisy data from a simulated nitrifying bioreactor in a WWT plant model. The goal was to find a simplified, but accurate, nonlinear Hammerstein model by application of system identification. This model reduction step, when successful, enables the design of an $\mathcal{L}_2$-stable feedback controller for the plant, this controller being described in a subsequent publication, Chistiakova et al. (2018).

The results of the study indicate that a Hammerstein model can indeed describe the bioreactor dynamics with quite high accuracy. The output error method based on a linear model resulted in a significantly lower accuracy as compared to the nonlinear models. Of the two tested Hammerstein identification methods, the recursive method enjoys advantages for adaptive operation and on-line fault detection purposes. Note, however, that the spread of fit values is larger for the recursive method, Fig. 5.

Topics for further studies include a generalization of the identification to several cascaded reactors. The identified Hammerstein model will then be used as a basis for an $\mathcal{L}_2$-stable controller design.

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